



महाराष्ट्र शासन

शालेय शिक्षण व क्रीडा विभाग

राज्य शैक्षणिक संशोधन व प्रशिक्षण परिषद, महाराष्ट्र पुणे

७०८ सदाशिव पेठ, कुमठेकर मार्ग, पुणे ४११०३०

संपर्क क्रमांक (०२०) २४४७ ६९३८

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दि.

इयत्ता:- बारावी

विषय:- गणित (Arts and Science Stream)

सूचना-

१. सदर प्रश्नपेढी ही १००% अभ्यासक्रमावर तयार करण्यात आली आहे.
२. सदर प्रश्नपेढीतील प्रश्न हे अधिकच्या सरावासाठी असून प्रश्नसंचातील प्रश्न बोर्डाच्या प्रश्नपत्रिकेत येतीलच असे नाही, याची नोंद घ्यावी.

**Maharashtra State Council of Educational research and Training
Pune**

QUESTION BANK

STD XII Arts and Science Stream

MATHEMATICS AND STATISTICS (40)

Part-I

1. MATHEMATICAL LOGIC

Q1) Select and write the most appropriate answer from the given alternatives:

- i) Which of the following statement is true?
 - a) $3 + 7 = 4$ or $3 - 7 = 4$
 - b) If Pune is in Maharashtra, then Hyderabad is in Kerala
 - c) It is false that 12 is not divisible by 3
 - d) The square of any odd integer is even.
- ii) Which of the following is not a statement?
 - a) $2+2 =4$
 - b) 2 is the only even prime number
 - c) Come here
 - d) Mumbai is not in Maharashtra
- iii) If p is any statement then $(p \vee \sim p)$ is a
 - a) Contingency
 - b) Contradiction
 - c) Tautology
 - d) None of these
- iv) If p and q are two statements , then $(p \rightarrow q) \leftrightarrow (\sim q \rightarrow \sim p)$ is
 - a) Contradiction
 - b) Tautology
 - c) Neither (i) nor (ii)
 - d) None of these
- v) Negation of $p \rightarrow (p \vee \sim q)$ is
 - a) $\sim p \rightarrow (\sim p \vee q)$
 - b) $p \wedge (\sim p \wedge q)$
 - c) $\sim p \vee (\sim p \vee \sim q)$

- d) $\sim p \rightarrow (\sim p \rightarrow q)$
- vi) If p : He is intelligent
 q : He is strong
 Then, symbolic form of statement “It is wrong that, he is intelligent or strong “ is
- a) $\sim p \vee \sim q$
 b) $\sim(p \wedge q)$
 c) $\sim(p \vee q)$
 d) $p \vee \sim q$
- vii) A biconditional statement is the conjunction of two ----- statements
- a) Negative
 b) Compound
 c) Connective
 d) Conditional
- viii) If $p \rightarrow q$ is an implication , then the implication $\sim q \rightarrow \sim p$ is called its
- a) Converse
 b) Contrapositive
 c) Inverse
 d) Alternative
- ix) The negation of the statement $(p \wedge q) \rightarrow (r \vee \sim p)$
- a) $p \wedge q \wedge \sim r$
 b) $(p \wedge q) \vee r$
 c) $p \vee q \vee \sim r$
 d) $(p \vee q) \wedge (r \vee s)$
- x) The false statement in the following is
- a) $p \wedge (\sim p)$ is contradiction
 b) $(p \rightarrow q) \leftrightarrow (\sim q \rightarrow \sim p)$ is a contradiction
 c) $\sim(\sim p) \leftrightarrow p$ is a tautology
 d) $p \vee (\sim p) \leftrightarrow p$ is a tautology
- xi) The dual of statement $p \wedge [\sim q \vee (p \wedge q) \vee \sim r]$ is
- A) $p \wedge [\sim q \wedge (p \wedge q) \wedge \sim r]$ B) $p \vee [\sim q \wedge (p \vee q) \wedge \sim r]$
 C) $p \wedge [\sim q \wedge (p \vee q) \wedge \sim r]$ D) $p \vee [\sim q \wedge (p \wedge q) \wedge \sim r]$
- xii) The dual of statement $p \rightarrow q$ is

A) $p \wedge \sim q$

B) $q \rightarrow p$

C) $p \vee \sim q$

D) $\sim p \wedge q$

xiii) The dual of a statement $p \wedge \sim p$ is

A) a tautology

B) a contradiction

C) Neither tautology nor contradiction

D) tautology as well as

contradiction

Q 2) Attempt the following 1 marks

i) Find the negation of $10 + 20 = 30$

ii) State the truth Value of $x^2 = 25$

iii) Write the negation of $p \rightarrow q$

iv) State the truth value of $\sqrt{3}$ is not an irrational number

v) State the truth value of $(p \vee \sim p)$

vi) State the truth value of $(p \wedge \sim p)$

Q3) Attempt the following 2 marks

i) : If statements p, q are true and r, s are false, determine the truth values of the following.

a) $\sim p \wedge (q \vee \sim r)$

b) $(p \wedge \sim r) \wedge (\sim q \vee s)$

ii) Write the following compound statements symbolically.

a) Nagpur is in Maharashtra and Chennai is in Tamilnadu.

b) Triangle is equilateral or isosceles.

iii) . Write the converse and contrapositive of the following statements.

“If a function is differentiable then it is continuous”.

iv) Without using truth table prove that :

$$\sim (p \vee q) \vee (\sim p \wedge q) \equiv \sim p$$

Answers

- i) a) F b) F ii) a) $p \wedge q$ b) $p \vee q$
ii) converse: If function is continuous then it is differentiable.
Contrapositive: If function is not continuous then it is not differentiable.

Q4) Answer the following questions

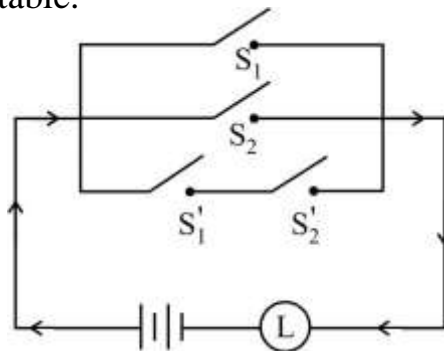
- i) Write the negation of the statement “ An angle is a right angle if and only if it is of measure 90^0 ”
- ii) Write the following statements in symbolic form
a) Milk is white if and only if the sky is not blue
b) If Kutab – Minar is in Delhi then Taj- Mahal is in Agra
c) Even though it is not cloudy , it is still raining
- iii) Use quantifiers to convert the given open sentence defined on N into a true statement
a) $n^2 \geq 1$
b) $3x - 4 < 9$
c) $Y + 4 > 6$
- iv) Examine whether the statement pattern is a tautology, contradiction or contingency
 $(p \wedge \sim q) \rightarrow (\sim p \wedge \sim q)$
- v) Using truth table prove that $\sim p \wedge q \equiv (p \vee q) \wedge \sim p$
- vi) Write the dual of the following
a) 13 is prime number and India is a democratic country
b) $(p \wedge \sim q) \vee (\sim p \wedge q) \equiv (p \vee q) \wedge \sim (p \wedge q)$
- vii) Write the converse, inverse and contrapositive of the statement “If it snows, then they do not drive the car”
- viii) Write the dual of the following statement
7 is a prime number but 9 is not divisible by 2.

Q5) Answer the following questions

- i) Examine whether the statement pattern

$[p \rightarrow (\sim q \vee r)] \leftrightarrow \sim[p \rightarrow (q \rightarrow r)]$ is a tautology, contradiction or contingency.

- ii) Using truth table prove that $p \vee (q \wedge r) \equiv (p \vee q) \wedge (p \vee r)$
- iii) Without using truth table show that $(p \vee q) \wedge (\sim p \vee \sim q) \equiv (p \vee \sim q) \wedge (\sim p \vee q)$
- iv) With proper justification state the negation of $(p \leftrightarrow q) \vee (\sim q \rightarrow \sim r)$
- v) Prepare truth table for $(p \wedge q) \vee \sim r$
- vi) Express the following circuits in the symbolic form of logic and write the input-output table.



2. MATRICES

I. MCQ (2 marks each)

Q. 1. The adjoint matrix of $\begin{bmatrix} 3 & -3 & 4 \\ 2 & -3 & 4 \\ 0 & -1 & 1 \end{bmatrix}$ is

a) $\begin{bmatrix} 4 & 8 & 3 \\ 2 & 1 & 6 \\ 0 & 2 & 1 \end{bmatrix}$ b) $\begin{bmatrix} 1 & -1 & 0 \\ -2 & 3 & -4 \\ -2 & 3 & -3 \end{bmatrix}$ c) $\begin{bmatrix} 11 & 9 & 3 \\ 1 & 2 & 8 \\ 6 & 9 & 1 \end{bmatrix}$ d) $\begin{bmatrix} 1 & -2 & 1 \\ -1 & 3 & 3 \\ -2 & 3 & -3 \end{bmatrix}$

Q. 2. $A = \begin{bmatrix} \cos \alpha & -\sin \alpha & 0 \\ \sin \alpha & \cos \alpha & 0 \\ 0 & 0 & 1 \end{bmatrix}$, then A^{-1} is

a) A b) $-A$ c) $\text{adj}(A)$ d) $-\text{adj}(A)$

Q. 3. The solution (x, y, z) of the equation

$$\begin{bmatrix} 1 & 0 & 1 \\ -1 & 1 & 0 \\ 0 & -1 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \\ 2 \end{bmatrix} \text{ is } (x, y, z) =$$

a) $(1, 1, 1)$ b) $(0, -1, 2)$ c) $(-1, 2, 2)$ d) $(-1, 0, 2)$

Q. 4. If ω is a complex cube root of unity, then the matrix $A = \begin{bmatrix} 1 & \omega^2 & \omega \\ \omega^2 & \omega & 1 \\ \omega & 1 & \omega^2 \end{bmatrix}$ is

a) Singular matrix b) Non-symmetric matrix
c) Skew-symmetric matrix d) Non- Singular matrix

Q. 5. If $A = \begin{bmatrix} 4 & -1 \\ -1 & k \end{bmatrix}$ such that $A^2 - 6A + 7I = 0$, then $k = \dots$

a) 1 b) 3 c) 2 d) 4

Q. 6. $\cos \theta \begin{bmatrix} \cos \theta & \sin \theta \\ -\sin \theta & \cos \theta \end{bmatrix} + \sin \theta \begin{bmatrix} \sin \theta & -\cos \theta \\ \cos \theta & \sin \theta \end{bmatrix} = \dots$

a) $\begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$ b) $\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$ c) $\begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}$ d) $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

Q. 7. If $A = \begin{bmatrix} 0 & 0 & -1 \\ 0 & -1 & 0 \\ -1 & 0 & 0 \end{bmatrix}$, then the only correct statement about the matrix A is....

a) $A^2 = I$ b) A is a zero matrix c) A^{-1} does not exist

d) $A = (-1)I$, where I is a unit matrix.

Q. 8. If $A = \begin{bmatrix} \cos \alpha & \sin \alpha \\ -\sin \alpha & \cos \alpha \end{bmatrix}$, then $A^{10} = \dots$

a) $\begin{bmatrix} \cos 10\alpha & -\sin 10\alpha \\ \sin 10\alpha & \cos 10\alpha \end{bmatrix}$

b) $\begin{bmatrix} \cos 10\alpha & \sin 10\alpha \\ -\sin 10\alpha & \cos 10\alpha \end{bmatrix}$

c) $\begin{bmatrix} \cos 10\alpha & \sin 10\alpha \\ -\sin 10\alpha & -\cos 10\alpha \end{bmatrix}$

d) $\begin{bmatrix} \cos 10\alpha & -\sin 10\alpha \\ -\sin 10\alpha & -\cos 10\alpha \end{bmatrix}$

Q. 9. The element of second row and third column in the inverse of $\begin{bmatrix} 1 & 2 & 1 \\ 2 & 1 & 0 \\ -1 & 0 & 1 \end{bmatrix}$ is...

a) -2

b) -1

c) 1

d) 2

Q. 10. If $A = \begin{bmatrix} 4 & 5 \\ 2 & 5 \end{bmatrix}$, then $|(2A)^{-1}| = \dots$

a) $\frac{1}{30}$

b) $\frac{1}{20}$

c) $\frac{1}{60}$

d) $\frac{1}{40}$

Q. 11. If $\begin{bmatrix} x - y - z \\ -y + z \\ z \end{bmatrix} = \begin{bmatrix} 0 \\ 5 \\ 3 \end{bmatrix}$, then the value of x, y and z are respectively...

a) 0, -3, 3

b) 1, -2, 3

c) 5, 2, 2

d) 11, 8, 3

Q. 12 The value of x, y, z for the following system of equations

$$x + y + z = 6, \quad x - y + 2z = 5, \quad 2x + y - z = 1 \text{ are...}$$

a) $x = 1, y = 2, z = 3$

b) $x = 2, y = 1, z = 3$

c) $x = -1, y = 2, z = 3$

d) $x = y = z = 3$

Q. 13. If $A = \begin{bmatrix} 3 & 0 & 0 \\ 0 & 3 & 0 \\ 0 & 0 & 3 \end{bmatrix}$, then $|A||adjA| = \dots$

a) 3^3

b) 3^9

c) 3^6

a) 3^{27}

Q. 14. System of equations $x + y = 2, 2x + 2y = 3$ has....

a) no solution

b) only one solution

c) many finite solutions.

d) infinite solutions.

Q. 15. If $A = \begin{bmatrix} 1 & -1 & 1 \\ 2 & 1 & -3 \\ 1 & 1 & 1 \end{bmatrix}$, $10B = \begin{bmatrix} 4 & 2 & 2 \\ -5 & 0 & \alpha \\ 1 & -2 & 3 \end{bmatrix}$ and B is the inverse of matrix A, then $\alpha = \dots$

- a) -2 b) -1 c) 2 d) 5

II. Very Short Answers (1 mark)

Q. 1. If $A = \begin{bmatrix} 1 & 2 & 3 \\ 1 & 1 & 5 \\ 2 & 4 & 7 \end{bmatrix}$, then find the value of $a_{31}A_{31} + a_{32}A_{32} + a_{33}A_{33}$

Q. 2. For an invertible matrix A, if $A \cdot (\text{adj}A) = \begin{bmatrix} 10 & 0 \\ 0 & 10 \end{bmatrix}$, then find the value of $|A|$.

Q. 3. If the inverse of the matrix $\begin{bmatrix} \alpha & 14 & -1 \\ 2 & 3 & 1 \\ 6 & 2 & 3 \end{bmatrix}$ does not exist then find the

value of α .

Q. 4. If $A = \begin{bmatrix} 2 & 2 \\ -3 & 2 \end{bmatrix}$ and $B = \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}$, then find the matrix $(B^{-1}A^{-1})^{-1}$

Q. 5. $A = \begin{bmatrix} \cos\theta & -\sin\theta \\ -\sin\theta & -\cos\theta \end{bmatrix}$ then find A^{-1} .

Q. 6. If $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$ then find the value of $|A|^{-1}$

Q. 7. If $A = \begin{bmatrix} 3 & 1 \\ 5 & 2 \end{bmatrix}$, and $AB = BA = I$, then find the matrix B.

Q. 8. If $A(\alpha) = \begin{bmatrix} \cos\alpha & \sin\alpha \\ -\sin\alpha & \cos\alpha \end{bmatrix}$ then prove that $A^2(\alpha) = A(2\alpha)$

Q. 9. If $A = \begin{bmatrix} 1 & 2 \\ 3 & -2 \\ -1 & 0 \end{bmatrix}$ and $B = \begin{bmatrix} 1 & 3 & 2 \\ 4 & -1 & 3 \end{bmatrix}$ then find the order of AB.

Q. 10. $A+I = \begin{bmatrix} 3 & -2 \\ 4 & 1 \end{bmatrix}$ then find the value of $(A+I)(A-I)$

Q. 11. If $A = \begin{bmatrix} 2 & -1 & 1 \\ -2 & 3 & -2 \\ -4 & 4 & -3 \end{bmatrix}$ then find A^2

Q. 12. If $A = \begin{bmatrix} -2 & 4 \\ -1 & 2 \end{bmatrix}$ then find A^2

Q. 13. If $A = \begin{bmatrix} 0 & 3 & 3 \\ -3 & 0 & -4 \\ -3 & 4 & 0 \end{bmatrix}$ and $B = \begin{bmatrix} x \\ y \\ z \end{bmatrix}$, find the matrix $B'(AB)$

III. Short Answers (2 marks)

Q. 1. If $f(x) = x^2 - 2x - 3$ then find $f(A)$ when $A = \begin{bmatrix} 1 & 2 \\ 2 & 1 \end{bmatrix}$

Q. 2. If $A = \begin{bmatrix} -1 \\ 2 \\ 3 \end{bmatrix}$, $B = [3 \quad 1 \quad -2]$, find $B'A'$

Q. 3 If A is invertible matrix of order 3 and $|A| = 5$, then find $|adjA|$

Q. 4. If $A = \begin{bmatrix} 6 & 5 \\ 5 & 6 \end{bmatrix}$ and $B = \begin{bmatrix} 11 & 0 \\ 0 & 11 \end{bmatrix}$ then find $A'B'$

Q. 5. If $A = \begin{bmatrix} 2 & 4 \\ 1 & 3 \end{bmatrix}$ and $B = \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}$ then find $(A^{-1}B^{-1})$

Q. 6. If $A = \begin{bmatrix} 2 & 0 \\ 0 & 1 \end{bmatrix}$ and $B = \begin{bmatrix} 1 \\ 2 \end{bmatrix}$ then find the matrix X such that $A^{-1}X = B$

Q. 7. Find the matrix X such that $AX = I$ where $A = \begin{bmatrix} 6 & 17 \\ 1 & 3 \end{bmatrix}$

Q. 8. Find A^{-1} using adjoint method, where $A = \begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix}$

Q. 9. Find A^{-1} using column transformations :

i) $A = \begin{bmatrix} 5 & 3 \\ 3 & -2 \end{bmatrix}$ ii) $A = \begin{bmatrix} 2 & -3 \\ -1 & 2 \end{bmatrix}$

Q. 10. Find the adjoint of matrix $A = \begin{bmatrix} 6 & 5 \\ 3 & 4 \end{bmatrix}$

Q. 11. Transform $\begin{bmatrix} 1 & 2 & 4 \\ 3 & -1 & 5 \\ 2 & 4 & 6 \end{bmatrix}$ into an upper triangular matrix by using suitable row transformations.

IV. Short answers (3 Marks)

Q. 1. If $A = \begin{bmatrix} 0 & 4 & 3 \\ 1 & -3 & -3 \\ -1 & 4 & 4 \end{bmatrix}$, then find A^2 and hence find A^{-1}

Q. 2 If $A = \begin{bmatrix} 0 & 1 \\ 2 & 3 \\ 1 & -1 \end{bmatrix}$ and $B = \begin{bmatrix} 1 & 2 & 1 \\ 2 & 1 & 0 \end{bmatrix}$, then find $(AB)^{-1}$

Q. 3. If $A = \begin{bmatrix} -4 & -3 & -3 \\ 1 & 0 & 1 \\ 4 & 4 & 3 \end{bmatrix}$, find $\text{adj}(A)$

Q. 4. Solve the following by inversion method $2x + y = 5$, $3x + 5y = -3$

Q. 5. If $A = \begin{bmatrix} 1 & 2 & -1 \\ 3 & -2 & 5 \end{bmatrix}$, apply $R_1 \leftrightarrow R_2$ and then $C_1 \rightarrow C_1 + 2C_3$ on A.

Q. 6. Three chairs and two tables costs ₹ 1850. Five chairs and three tables costs ₹2850. Find the cost of four chairs and one table by using matrices.

Q. 7. If $A = \begin{bmatrix} 4 & 5 \\ 2 & 1 \end{bmatrix}$, show that $A^{-1} = \frac{1}{6} (A - 5I)$

Q. 8. Find the adjoint of matrix $A = \begin{bmatrix} 2 & 0 & -1 \\ 3 & 1 & 2 \\ -1 & 1 & 2 \end{bmatrix}$

Q. 9. Find the matrix X such that $\begin{bmatrix} 1 & 2 & 3 \\ 2 & 3 & 2 \\ 1 & 2 & 2 \end{bmatrix} X = \begin{bmatrix} 2 & 2 & -5 \\ -2 & -1 & 4 \\ 1 & 0 & -1 \end{bmatrix}$,

Q. 10. Find the inverse of $A = \begin{bmatrix} \sec\theta & \tan\theta & 0 \\ \tan\theta & \sec\theta & 0 \\ 0 & 0 & 1 \end{bmatrix}$

Q. 11. Transform $\begin{bmatrix} 1 & 2 & 4 \\ 3 & -1 & 5 \\ 2 & 4 & 6 \end{bmatrix}$ into an upper triangular matrix by using suitable row transformations.

Q. 12. If $A = \begin{bmatrix} 1 & 0 & 1 \\ 0 & 2 & 3 \\ 1 & 2 & 1 \end{bmatrix}$ and $B = \begin{bmatrix} 1 & 2 & 3 \\ 1 & 1 & 5 \\ 2 & 4 & 7 \end{bmatrix}$, then find the matrix X such that $XA = B$

V. Long answers (4 Marks)

Q. 1. Find the inverse of $A = \begin{bmatrix} 2 & -3 & 3 \\ 2 & 2 & 3 \\ 3 & -2 & 2 \end{bmatrix}$ by using elementary row transformations.

Q. 2. If $A = \begin{bmatrix} 1 & 0 & 0 \\ 3 & 3 & 0 \\ 5 & 2 & -1 \end{bmatrix}$, find A^{-1} by the adjoint method.

Q. 3. Solve the following equations by using inversion method.

$$x + y + z = -1, \quad x - y + z = 2 \quad \text{and} \quad x + y - z = 3$$

Q. 4. If three numbers are added, their sum is 2. If 2 times the second number is subtracted from the sum of first and third numbers, we get 8. If three times the first number is added to the sum of second and third numbers, we get 4. Find the numbers using matrices.

Q. 5. Find the inverse of $A = \begin{bmatrix} 1 & 0 & 1 \\ 0 & 2 & 3 \\ 1 & 2 & 1 \end{bmatrix}$ by elementary column transformations.

Q. 6. If $A = \begin{bmatrix} x & 0 & 0 \\ 0 & y & 0 \\ 0 & 0 & z \end{bmatrix}$ is a non-singular matrix, then find A^{-1} by using elementary row transformations. Hence, write the inverse of $\begin{bmatrix} 2 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -1 \end{bmatrix}$.

Q. 7. Find the inverse of $A = \begin{bmatrix} \cos \theta & -\sin \theta & 0 \\ \sin \theta & \cos \theta & 0 \\ 0 & 0 & 1 \end{bmatrix}$ by using elementary row transformations.

Q. 8. If $A = \begin{bmatrix} 1 & 1 \\ 1 & 2 \end{bmatrix}$, $B = \begin{bmatrix} 4 & 1 \\ 3 & 1 \end{bmatrix}$, and $C = \begin{bmatrix} 24 & 7 \\ 31 & 9 \end{bmatrix}$, then find the matrix X such that $AXB = C$.

Q. 9. If $A = \begin{bmatrix} 1 & -1 & 2 \\ 3 & 0 & -2 \\ 1 & 0 & 3 \end{bmatrix}$, verify that $A(\text{adj } A) = (\text{adj } A)A$.

Q. 10. If $A = \begin{bmatrix} 2 & 3 \\ 1 & 2 \end{bmatrix}$, $B = \begin{bmatrix} 1 & 0 \\ 3 & 1 \end{bmatrix}$, find AB and $(AB)^{-1}$.

Q. 11. Solve the following system of equations by using inversion method.

$$x + y = 1, \quad y + z = \frac{5}{3} \quad \text{and} \quad z + x = \frac{4}{3}$$

Q12. Express the following equations in the matrix form and solve them by the method of reduction.

$$x + y + z = 6, \quad 3x - y + 3z = 6, \quad 5x + 5y - 4z = 3.$$

Q. 13. The cost of 4 dozen pencils, 3 dozen pens and 2 dozen erasers is ₹ 60. The cost of 2 dozen pencils, 4 dozen pens and 6 dozen erasers is ₹ 90. Whereas the cost of 6 dozen pencils, 2 dozen pens and 3 dozen erasers is ₹ 70. Find the cost of each item per dozen by using matrices.

Q14. The total cost of 3 televisions and 2 VCR's is Rs.35,000. The shopkeeper wants profit of Rs.1000 per television and Rs.500 per VCR. He sells 2 televisions and 1 VCR and gets the total revenue as Rs.21,500. Find the cost of a television and a VCR by using reduction method.

3. TRIGONOMETRIC FUNCTIONS

I. MCQ (2 marks each)

- 1) The principal solutions of $\sqrt{3} \sec x - 2 = 0$ are _____.
- a) $\frac{\pi}{3}, \frac{11\pi}{6}$ b) $\frac{\pi}{6}, \frac{11\pi}{6}$ c) $\frac{\pi}{4}, \frac{11\pi}{4}$ d) $\frac{\pi}{6}, \frac{11\pi}{3}$
- 2) In ΔABC , if $\cos A = \frac{\sin B}{2 \sin C}$, then ΔABC is _____.
- a) an equilateral triangle. b) a right angled triangle.
c) an isosceles triangle. d) an isosceles right angled triangle.
- 3) $\sin^{-1} x - \cos^{-1} x = \frac{\pi}{6}$, then $x =$ _____.
- a) $\frac{1}{2}$ b) $\frac{\sqrt{3}}{2}$ c) $-\frac{1}{2}$ d) $-\frac{\sqrt{3}}{2}$
- 4) The principal value of $\sin^{-1}\left(\frac{1}{2}\right)$ is _____.
- a) $\frac{\pi}{3}$ b) $\frac{\pi}{6}$ c) $\frac{2\pi}{3}$ d) $\frac{3\pi}{2}$
- 5) The principal value of $\cos^{-1}\left(-\frac{1}{2}\right)$ is _____.
- a) $\frac{\pi}{3}$ b) $\frac{\pi}{6}$ c) $\frac{2\pi}{3}$ d) $\frac{3\pi}{2}$
- 6) In ΔABC , if $\angle A = 30^\circ$, $\angle B = 60^\circ$, then the ratio of sides is _____.
- a) $1:\sqrt{3}:2$ b) $2:\sqrt{3}:1$ c) $\sqrt{3}:1:2$ d) $\sqrt{3}:2:1$
- 7) In ΔABC , if $b^2 + c^2 - a^2 = bc$, then $\angle A =$ _____.
- a) $\frac{\pi}{4}$ b) $\frac{\pi}{3}$ c) $\frac{\pi}{2}$ d) $\frac{\pi}{6}$
- 8) If polar co-ordinates of a point are $\left(\frac{3}{4}, \frac{3\pi}{4}\right)$, then its Cartesian co-ordinate are _____.
- a) $\left(\frac{3}{4\sqrt{2}}, -\frac{3}{4\sqrt{2}}\right)$ b) $\left(\frac{3}{4\sqrt{2}}, \frac{3}{4\sqrt{2}}\right)$ c) $\left(-\frac{3}{4\sqrt{2}}, \frac{3}{4\sqrt{2}}\right)$ d) $\left(-\frac{3}{4\sqrt{2}}, -\frac{3}{4\sqrt{2}}\right)$

9) $\tan^{-1}\left(\tan\frac{7\pi}{6}\right) = \underline{\hspace{2cm}}$.

a) $-\frac{\pi}{6}$

b) $\frac{\pi}{6}$

c) $\frac{13\pi}{6}$

d) $\frac{5\pi}{6}$

10) If $\sin(\sin^{-1}\left(\frac{1}{5}\right) + \cos^{-1}(x)) = 1$, then $x = \underline{\hspace{2cm}}$.

a) $\frac{1}{5}$

b) $-\frac{1}{5}$

c) 5

d) -5

11) If in ΔABC , $\sin\frac{A}{2} \cdot \sin\frac{C}{2} = \sin\frac{B}{2}$ and $2s$ is the perimeter of the Δ , then $s =$

A) $2b$

B) b

C) $3b$

D) $4b$

II. Very Short Answers (1 mark)

1) Evaluate $\cot(\tan^{-1}(2x) + \cot^{-1}(2x))$.

2) In ΔABC , prove that $ac \cos B - bc \cos A = a^2 - b^2$.

3) In ΔABC , if $\sin^2 A + \sin^2 B = \sin^2 C$, then show that $a^2 + b^2 = c^2$.

4) Find the polar co-ordinates of point whose Cartesian co-ordinates are $(1, \sqrt{3})$.

5) Prove that $2 \tan^{-1}\left(\frac{3}{4}\right) = \tan^{-1}\left(\frac{24}{7}\right)$.

6) Evaluate $\sin[\cos^{-1}\left(\frac{3}{5}\right)]$.

7) In ΔABC , $a = 3, b = 4$ and $\sin A = \frac{3}{4}$, find $\angle B$.

8) Find the principal solutions of $\operatorname{cosec} x = 2$.

9) Find the principal solutions of $\sin x - 1 = 0$.

10) Find the Cartesian co-ordinates of point whose polar co-ordinates are $\left(4, \frac{\pi}{3}\right)$.

III. Short Answer Questions (2 marks each):

- 1) With usual notations, prove that $\frac{\cos A}{a} + \frac{\cos B}{b} + \frac{\cos C}{c} = \frac{a^2+b^2+c^2}{2abc}$.
- 2) Find the principal solutions of $\cos 2x = 1$.
- 3) In ΔABC , prove that $(b - c)^2 \cos^2 \left(\frac{A}{2}\right) + (b + c)^2 \sin^2 \left(\frac{A}{2}\right) = a^2$.
- 4) Find the principal solutions of $\sin x = -\frac{1}{2}$.
- 5) Find the value of $\cos^{-1} \left(\frac{1}{2}\right) + \tan^{-1} \left(\frac{1}{\sqrt{3}}\right)$.
- 6) In ΔABC , if $a = 13, b = 14, c = 15$, then find the value of $\cos B$.
- 7) In ΔABC , if $\frac{\cos A}{a} = \frac{\cos B}{b}$, then show that it is an isosceles triangle.
- 8) Find the principal solution of $\tan x = -\sqrt{3}$.
- 9) Evaluate $\cos\left[\frac{\pi}{6} + \cos^{-1}\left(-\frac{\sqrt{3}}{2}\right)\right]$.
- 10) Find the general solution of the equation $\sin 2x = \frac{\sqrt{3}}{2}$
- 11) Find the general solution of the equation $\cos 2\theta = \cos \theta$
- 12) Find the general solution of the equation $4 \tan^2 x = 3$

IV. Short Answer Questions (3 marks each):

- 1) In ΔABC , if $a \cos A = b \cos B$, then prove that ΔABC is either a right angled or an isosceles triangle.
- 2) In ΔABC , prove that $\frac{\cos 2A}{a^2} - \frac{\cos 2C}{c^2} = \frac{1}{a^2} - \frac{1}{c^2}$.
- 3) If $\tan^{-1} x + \tan^{-1} y + \tan^{-1} z = \pi$, then show that $\frac{1}{xy} + \frac{1}{yz} + \frac{1}{zx} = 1$.

- 4) Prove that $\sin \left[\tan^{-1} \left(\frac{1-x^2}{2x} \right) + \cos^{-1} \left(\frac{1-x^2}{1+x^2} \right) \right] = 1$.
- 5) In ΔABC , if $\frac{2\cos A}{a} + \frac{\cos B}{b} + \frac{2\cos C}{c} = \frac{a}{bc} + \frac{b}{ca}$, then show that the triangle is a right angled.
- 6) In ΔABC , prove that $\sin \left(\frac{A-B}{2} \right) = \left(\frac{a-b}{c} \right) \cos \left(\frac{C}{2} \right)$.
- 7) If the angles A, B, C of ΔABC are in A.P. and its sides a, b, c are in G.P., then show that a^2, b^2, c^2 are in A.P.
- 8) Prove that $\cot^{-1}(7) + 2 \cot^{-1}(3) = \frac{\pi}{4}$.
- 9) Find the general solution of the equation $\tan \theta \tan 2\theta = 1$
- 10) Find the general solution of the equation $\frac{\sin 3x}{2 \cos 2x+1} = \frac{1}{2}$
- 11) Find the general solution of the equation $2 \cot \frac{\theta}{2} = (1 + \cot \theta)^2$
- 12) Find the general solution of the equation $\cos 2\theta = (\sqrt{2} + 1) \left(\cos \theta - \frac{1}{\sqrt{2}} \right)$
- 13) Find the general solution of the equation $4\cos^2 x \sin x - 2\sin^2 x = 3 \sin x$

V. Long Answer Questions (4 marks each):

- 1) In ΔABC , prove that $\frac{\cos^2 A - \cos^2 B}{a+b} + \frac{\cos^2 B - \cos^2 C}{b+c} + \frac{\cos^2 C - \cos^2 A}{c+a} = 0$.
- 2) Show that $\sin^{-1} \left(\frac{3}{5} \right) + \sin^{-1} \left(\frac{8}{17} \right) = \cos^{-1} \left(\frac{36}{85} \right)$.
- 3) In ΔABC , prove that $\frac{a^2 \sin(B-C)}{\sin A} + \frac{b^2 \sin(C-A)}{\sin B} + \frac{c^2 \sin(A-B)}{\sin C} = 0$.
- 4) In ΔABC , prove that $\frac{b^2 - c^2}{a} \cos A + \frac{c^2 - a^2}{b} \cos B + \frac{a^2 - b^2}{c} \cos C = 0$.
- 5) Prove that $2 \tan^{-1} \left(\frac{1}{8} \right) + \tan^{-1} \left(\frac{1}{7} \right) + 2 \tan^{-1} \left(\frac{1}{5} \right) = \frac{\pi}{4}$.
- 6) In ΔABC , if $\angle A = \frac{\pi}{2}$, then prove that $\sin(B - C) = \frac{b^2 - c^2}{b^2 + c^2}$.
- 7) If $\cos^{-1} x + \cos^{-1} y - \cos^{-1} z = 0$, then show that

$$x^2 + y^2 + z^2 - 2xyz = 1.$$

8) Find the general solution of the equation $\cos \theta + \sin \theta = \frac{1}{\sqrt{2}}$

9) Find the general solution of $\tan x + \tan 2x + \sqrt{3} \tan x \tan 2x = \sqrt{3}$

10) In a ΔABC , if $a = 25$, $b = 52$, $c = 63$ then

Find the values of i) $\cos C$ ii) $\sin \frac{C}{2}$ iii) $\tan \frac{C}{2}$ iv) $A(\Delta ABC)$

11) In ΔABC , prove that $\left(\cot \frac{A}{2} + \cot \frac{B}{2}\right) \left(a \sin^2 \frac{B}{2} + b \sin^2 \frac{A}{2}\right) = c \cot \frac{C}{2}$

12) In ΔABC , prove that $\frac{1}{a} \cos^2 \frac{A}{2} + \frac{1}{b} \cos^2 \frac{B}{2} + \frac{1}{c} \cos^2 \frac{C}{2} = \frac{1}{abc}$

4. PAIR OF LINES

I. MCQ (2 marks each)

1. The combined equation of the two lines passing through the origin, each making angle 45° and 135° with the positive X axis is ...

- A. $x^2 + y^2 = 0$ B. $xy = 1$
C. $x^2 - y^2 = 0$ D. $x^2 + xy = 0$

2. The separate equations of the lines represented by $3x^2 - 2\sqrt{3}xy - 3y^2 = 0$ are...

- A. $x + \sqrt{3}y = 0$ and $\sqrt{3}x + y = 0$ B. $x - \sqrt{3}y = 0$ and $\sqrt{3}x - y = 0$
C. $x - \sqrt{3}y = 0$ and $\sqrt{3}x + y = 0$ D. $x + \sqrt{3}y = 0$ and $\sqrt{3}x - y = 0$

3. The equation $4x^2 + 4xy + y^2 = 0$ represents two....

- A. real and distinct lines B. real and coincident lines
C. imaginary lines D. perpendicular lines

4. If the lines represented by $kx^2 - 3xy + 6y^2 = 0$ are perpendicular to each other then.....

- A. $k = 6$ B. $k = -6$ C. $k = 3$ D. $k = -3$

5. Auxillary equation of $2x^2 + 3xy - 9y^2 = 0$ is.....

- A. $2m^2 + 3m - 9 = 0$ B. $9m^2 - 3m - 2 = 0$
C. $2m^2 - 3m + 9 = 0$ D. $-9m^2 - 3m + 2 = 0$

6. The combined equation of the lines through origin and perpendicular to the pair of lines $3x^2 + 4xy - 5y^2 = 0$ is.....

- A. $5x^2 + 4xy - 3y^2 = 0$ B. $3x^2 + 4xy - 5y^2 = 0$
C. $3x^2 - 4xy + 5y^2 = 0$ D. $5x^2 + 4xy + 3y^2 = 0$

7. The acute angle between the lines represented by $x^2 + xy = 0$ is.....

- A. $\frac{\pi}{2}$ B. $\frac{\pi}{4}$ C. $\frac{\pi}{6}$ D. $\frac{\pi}{3}$

8. If $2x + y = 0$ is one of the line represented by $3x^2 + kxy + 2y^2 = 0$

then $k = \dots$

- A. $\frac{1}{2}$ B. $\frac{11}{2}$ C. $\frac{2}{3}$ D. $\frac{3}{2}$

9. If the equation $3x^2 + 10xy + 3y^2 + 16y + k = 0$ represents a pair of lines , then $k = \dots$
- A) 21 B) -12 C) 12 D) -21
10. The line $5x + y - 1 = 0$ coincides with one of the lines given by $5x^2 + xy - kx - 2y + 2 = 0$, then $k = \dots$
- A) -11 B) 31 C) 11 D) -31

II. Very Short Answers (2 mark)

- 1) Find the combine equation of the pair of lines passing through the point (2,3) and parallel to the coordinate axes.
- 2) Find the separate equations of the lines given by $x^2 + 2xy + y^2 - 4x - 6y + 5 = 0$
- 3) Find k, if the sum of the slopes of the lines represented by $x^2 + kxy - 3y^2 = 0$ is twice their products.
- 4) Find the measure of acute angle between the lines given by $x^2 - 4xy + y^2 = 0$
- 5) Find the value of h , if the measure of the angle between the lines $3x^2 + 2hxy + 2y^2 = 0$ is 45° .

III. Short Answers (3 marks)

- 1) Find the combine equation of pair of lines passing through (-1,2), one is parallel to $x+3y-1=0$ and other is perpendicular to $2x-3y-1=0$.
- 2) Find the joint equation of pair of lines through the origin which are perpendicular to the lines represented by $5x^2 + 2xy - 3y^2 = 0$
- 3) Find the condition that the line $4x+5y=0$ coincides with one of the lines given by $ax^2 + 2hxy + by^2 = 0$
- 4) Find the measure of acute angle between the lines represented by $3x^2 - 4\sqrt{3}xy + 3y^2 = 0$

IV. Short answers (4 Marks)

- 1) Show that the combine equation of pair of lines passing through the origin is a homogeneous equation of degree 2 in x and y. Hence find the combined equation of the lines $2x+3y=0$ and $x-2y=0$
- 2) Show that the homogeneous equation of degree 2 in x and y represents a pair of lines passing through the origin if $h^2 - ab \geq 0$
- 3) If θ is the acute angle between the lines given by $ax^2 + 2hxy + by^2 = 0$

then prove that $\tan \theta = \left| \frac{2\sqrt{h^2 - ab}}{a+b} \right|$. Hence find acute angle between the lines $2x^2 + 7xy + 3y^2 = 0$

4) If the angle between the lines represented by $ax^2 + 2hxy + by^2 = 0$ is equal to the angle between the lines $2x^2 - 5xy + 3y^2 = 0$ then show that $100(h^2 - ab) = (a + b)^2$

5) The equation $4x^2 + 2pxy + 25y^2 + 2x + 5y - 1 = 0$ represents a pair of parallel lines, then find value of p .

6. Find the acute angle between the pair of straight lines $x^2 - 6xy + 5y^2 + 10x - 4y + 9 = 0$.

5. VECTOR AND THREE DIMENSIONAL GEOMETRY

I. MCQ (2 marks each)

1) If $|\vec{a}| = 3$, $|\vec{b}| = 4$, then the value of λ for which $\vec{a} + \lambda \vec{b}$ is perpendicular to

$\vec{a} - \lambda \vec{b}$ is

- A) $\frac{9}{16}$ B) $\frac{3}{4}$ C) $\frac{3}{2}$ D) $\frac{4}{3}$

2) $(\hat{i} + \hat{j} - \hat{k}) \cdot (\hat{i} - \hat{j} + \hat{k}) =$ _____

- A) $\hat{i} - \hat{j} - \hat{k}$ B) 1 C) -1 D) $-\hat{j} + \hat{k}$

3) The angle θ between two non-zero vectors \vec{a} & \vec{b} is given by $\cos \theta = \dots$

- A) $\frac{\vec{a} \cdot \vec{b}}{|\vec{a}| |\vec{b}|}$ B) $\vec{a} \cdot \vec{b}$ C) $|\vec{a}| |\vec{b}|$ D) $\frac{|\vec{a}| |\vec{b}|}{\vec{a} \cdot \vec{b}}$

4) If sum of two unit vectors is itself a unit vector, then the magnitude of their difference is...

- A) $\sqrt{2}$ B) $\sqrt{3}$ C) 1 D) 2

5) If α, β, γ are direction angles of a line and $\alpha = 60^\circ$, $\beta = 45^\circ$, then $\gamma =$ _____

- A) 30° or 90° B) 45° or 60° C) 90° or 30° D)

60° or 120°

6) The distance of the point (3, 4, 5) from Y- axis is _____

- A) 3 B) 5 C) $\sqrt{34}$ D) $\sqrt{41}$

7) If $\cos \alpha$, $\cos \beta$, $\cos \gamma$ are the direction cosines of a line then the value of

$\sin^2 \alpha + \sin^2 \beta + \sin^2 \gamma$ is _____

- A) 1 B) 2 C) 3 D) 4

8) If $|\vec{a}| = 2$, $|\vec{b}| = 5$, and $\vec{a} \cdot \vec{b} = 8$ then $|\vec{a} - \vec{b}| =$ _____

- A) 13 B) 12 C) $\sqrt{13}$ D) $\sqrt{21}$

9) If $\vec{AB} = 2\hat{i} + \hat{j} - 3\hat{k}$, and A(1, 2, -1) is given point then coordinates of B are _____

- A) (3, 3, -4) B) (-3, 3, -2) C) (3, 3, 2) D) (-3, 3, 4)

10) If l, m, n are direction cosines of a line then $l\hat{i} + m\hat{j} + n\hat{k}$ is ____

- A) Null vector B) the unit vector along the line.
C) Any vector along the line D) a vector perpendicular to the line.

11) The values of c that satisfy $|c\bar{u}| = 3$, $\bar{u} = \hat{i} + 2\hat{j} + 3\hat{k}$ is ____

- A) $\sqrt{14}$ B) $3\sqrt{14}$ C) $\frac{3}{\sqrt{14}}$ D) 3

12. The value of $\hat{i} \cdot (\hat{j} \times \hat{k}) + \hat{j} \cdot (\hat{i} \times \hat{k}) + \hat{k} \cdot (\hat{i} \times \hat{j})$

- A) 0 B) -1 C) 1 D) 3

13. The two vectors $\hat{j} + \hat{k}$ & $3\hat{i} - \hat{j} + 4\hat{k}$ represents the two sides AB and AC, respectively of a ΔABC . The length of the median through A is

- A) $\frac{\sqrt{34}}{2}$ B) $\frac{\sqrt{48}}{2}$ C) $\sqrt{18}$ D) $\sqrt{34}$

14. If \bar{a}, \bar{b} and \bar{c} are non coplanar unit vectors, and $\bar{a} \times (\bar{b} \times \bar{c}) = \frac{\bar{b} + \sqrt{3}\bar{c}}{2}$, then the angle between \bar{a} and \bar{c} is ____

- a) $\frac{5\pi}{3}$ b) $\frac{5\pi}{6}$ c) $\frac{\pi}{3}$ d) π

II. Very Short Answers (1 mark)

- Find the magnitude of a vector with initial point : $(1, -3, 4)$; terminal point : $(1, 0, -1)$.
- Find the coordinates of the point which is located Three units behind the YZ-plane, four units to the right of the XZ-plane and five units above the XY-Plane.
- $A(2,3), B(-1,5), C(-1,1)$ and $D(-7,5)$ are four points in the Cartesian plane, Check if, \overline{CD} is parallel to \overline{AB} .
- Find a unit vector in the opposite direction of \bar{u} . Where $\bar{u} = 8\hat{i} + 3\hat{j} - \hat{k}$.
- The non zero vectors \bar{a} and \bar{b} are not collinear find the value of λ and μ :
if $\bar{a} + 3\bar{b} = 2\lambda\bar{a} - \mu\bar{b}$

6. If $\bar{a} = 4\hat{i} + 3\hat{k}$ and $\bar{b} = -2\hat{i} + \hat{j} + 5\hat{k}$ then find $2\bar{a} + 5\bar{b}$
7. Find the distance from $(4, -2, 6)$ to the XZ- Plane.
8. If the vectors $2\hat{i} - q\hat{j} + 3\hat{k}$ and $4\hat{i} - 5\hat{j} + 6\hat{k}$ are collinear then find the value of q .
9. Find $\bar{a} \cdot \bar{b} \times \bar{c}$, if $\bar{a} = 3\hat{i} - \hat{j} + 4\hat{k}$, $\bar{b} = 2\hat{i} + 3\hat{j} - \hat{k}$, $\bar{c} = -5\hat{i} + 2\hat{j} + 3\hat{k}$
10. If a line makes angle 90° , 60° and 30° with the positive direction of X, Y and Z axes respectively, find its direction cosines.
11. Find $\hat{k} \times (\hat{j} \times \hat{i})$

III. Short Answers (2 mark)

1. The vector \bar{a} is directed due north and $|\bar{a}| = 24$. The vector \bar{b} is directed due west and $|\bar{b}| = 7$. find $|\bar{a} + \bar{b}|$.
2. Show that following points are collinear $P(4,5,2), Q(3,2,4), R(5,8,0)$
3. If a vector has direction angles 45° and 60° find the third direction angle.
4. If $\bar{c} = 3\bar{a} - 2\bar{b}$ then prove that $[\bar{a} \ \bar{b} \ \bar{c}] = 0$
5. If $|\bar{a} \cdot \bar{b}| = |\bar{a} \times \bar{b}|$ & $\bar{a} \cdot \bar{b} < 0$, then find the angle between \bar{a} & \bar{b} .
6. Find the direction ratios of a vector perpendicular to the two lines whose direction ratios are 1, 3, 2 and $-1, 1, 2$
7. If \bar{a} , \bar{b} and \bar{c} are position vectors of the points A, B, C respectively and $5\bar{a} - 3\bar{b} - 2\bar{c} = \bar{0}$, then find the ratio in which the point C divides the line segment BA.
8. If \bar{a} and \bar{b} are two vectors perpendicular each other, prove that

$$(\bar{a} + \bar{b})^2 = (\bar{a} - \bar{b})^2$$
9. Find the position vector of point R which divides the line joining the points P and Q whose position vectors are $2\hat{i} - \hat{j} + 3\hat{k}$ and $-5\hat{i} + 2\hat{j} - 5\hat{k}$ in the ratio 3 : 2
 - (i) internally (ii) externally.
10. Find a unit vector perpendicular to the vectors $\hat{j} + 2\hat{k}$ & $\hat{i} + \hat{j}$

IV. Short Answers (3 mark)

1. If two of the vertices of the triangle are $A(3,1,4)$ and $B(-4,5,-3)$ and the centroid of a triangle is $G(-1,2,1)$, then find the co-ordinates of the third vertex C of the triangle.
2. Find the centroid of tetrahedron with vertices $K(5,-7,0)$, $L(1,5,3)$, $M(4,-6,3)$, $N(6,-4,2)$?
3. If a line has the direction ratios , $4, -12, 18$ then find its direction cosines.
4. Show that the points $A(2,-1,0)$, $B(-3,0,4)$, $C(-1,-1,4)$ and $D(0,-5,2)$ are non coplanar.
5. Using properties of scalar triple product, prove that

$$[\bar{a} + \bar{b} \quad \bar{b} + \bar{c} \quad \bar{c} + \bar{a}] = 2 [\bar{a} \quad \bar{b} \quad \bar{c}]$$

6. The direction ratios of \overline{AB} are $-2, 2, 1$. If $A = (4,1,5)$ and $l(AB) = 6$ units, Then find B .
7. If $G(a, 2, -1)$ is the centroid of the triangle with vertices $P(1,2,3)$, $Q(3, b, -4)$ and $R(5,1, c)$ then find the values of a, b and c .
8. If $A(5,1, p)$, $B(1, q, p)$ and $C(1, -2, 3)$ are vertices of triangle and $G\left(r, -\frac{4}{3}, \frac{1}{3}\right)$ is its centroid then find the values of p, q & r .
9. Prove by vector method that the angle subtended on semicircle is a right angle.
10. Prove that medians of a triangle are concurrent.
11. Prove that altitudes of a triangle are concurrent.

V. Long Answers (4 mark)

1. Express $-\hat{i} - 3\hat{j} + 4\hat{k}$ as linear combination of the vectors $2\hat{i} + \hat{j} - 4\hat{k}$, $2\hat{i} - \hat{j} + 3\hat{k}$ and $3\hat{i} + \hat{j} - 2\hat{k}$.
2. If Q is the foot of the perpendicular from $P(2,4,3)$ on the line joining the points $A(1,2,4)$ and $B(3,4,5)$, find coordinates of Q .
3. Prove that the angle bisectors of a triangle are concurrent.
4. Using vector method, find the incenter of the triangle whose vertices are $A(0,3,0)$, $B(0,0,4)$ and $C(0,3,4)$.

5. Find the angles between the lines whose direction cosines l, m, n satisfy the equations $5l + m + 3n = 0$ and $5mn - 2nl + 6lm = 0$
6. Let $A(\bar{a})$ and $B(\bar{b})$ be any two points in the space and $R(\bar{r})$ be a point on the line segment AB dividing it internally in the ratio $m : n$ then prove that $\bar{r} = \frac{m\bar{b} + n\bar{a}}{m+n}$.
7. D and E divides sides BC and CA of a triangle ABC in the ratio 2 : 3 respectively. Find the position vector of the point of intersection of AD and BE and the ratio in which this point divides AD and BE.
8. If $\bar{u} = \hat{i} - 2\hat{j} + \hat{k}$, $\bar{r} = 3\hat{i} + \hat{k}$ & $\bar{w} = \hat{j}, \hat{k}$ are given vectors, then find $[\bar{u} + \bar{w}] \cdot [(\bar{w} \times \bar{r}) \times (\bar{r} \times \bar{w})]$
9. Find the volume of a tetrahedron whose vertices are $A(-1, 2, 3)$, $B(3, -2, 1)$, $C(2, 1, 3)$ and $D(-1, -2, 4)$
10. If four points $A(\bar{a})$, $B(\bar{b})$, $C(\bar{c})$ & $D(\bar{d})$ are coplanar then show that $[\bar{a} \ \bar{b} \ \bar{d}] + [\bar{b} \ \bar{c} \ \bar{d}] + [\bar{c} \ \bar{a} \ \bar{d}] = [\bar{a} \ \bar{b} \ \bar{c}]$
11. Prove that the volume of parallelepiped with coterminous edges as \bar{a}, \bar{b} and \bar{c} is $[\bar{a} \ \bar{b} \ \bar{c}]$.
12. Prove that the volume of tetrahedron with coterminous edges as \bar{a}, \bar{b} and \bar{c} is $\frac{1}{6}[\bar{a} \ \bar{b} \ \bar{c}]$

6. LINE AND PLANE

I. Multiple choice questions.....(2 marks)

1)The equation of X axis is...

A) $x = y = z$ (B) $y = z$ (C) $y = 0, z = 0$ (D) $x = 0, y = 0$

2)If the perpendicular distance of the plane $2x+3y-z = k$ from the origin is $\sqrt{14}$ units, then $k = \dots$

A) 14 (B) 196 (C) $2\sqrt{14}$ (D) $\frac{\sqrt{14}}{2}$

3) The equation of the plane passing through the points $(1, -1, 1)$, $(3, 2, 4)$ and parallel to Y axis is...

A) $3x + 2z - 1 = 0$ (B) $3x - 2z = 1$ (C) $3x+2z+1 = 0$ (D) $3x+2z = 2$

4)The direction ratios of the line $3x+1 = 6y -2 = 1 - z$ are

A) 2, 1, 6 (B) 2, 1, -6 (C) 2, -1, 6 (D) -2, 1, 6

5)If the planes $2x - my + z = 3$ and $4x - y + 2z = 5$ are parallel then $m = \dots$

A) -2 (B) 2 (C) $-\frac{1}{2}$ (D) $\frac{1}{2}$

6) The direction cosines of the normal to the plane $2x - y + 2z = 3$ are

A) $\frac{2}{3}, \frac{-1}{3}, \frac{2}{3}$ (B) $\frac{-2}{3}, \frac{1}{3}, \frac{-2}{3}$ (C) 2, -1, 2 (D) -2, 1, -2

7)If the foot of the perpendicular drawn from the origin to the plane is $(4, -2, 5)$, then the equation of plane is...

A) $4x+y+5z = 14$ (B) $4x-2y-5z = 45$ (C) $x-2y-5z = 10$ (D) $4x+y+6z = 11$

8)The perpendicular distance of the origin from the plane $x-3y+4z=6$ is....

A) 6 (B) $\frac{6}{\sqrt{26}}$ (C) 36 (D) $\frac{1}{\sqrt{26}}$

9)The coordinates of the foot of perpendicular drawn from the origin to the plane $2x+y-2z = 18$ are...

A) $(4, 2, 4)$ (B) $(-4, 2, 4)$ (C) $(-4, -2, 4)$ (D) $(4, 2, -4)$

II. Very Short answers (1 marks)

1) Find the Cartesian equation of a plane passing through A(1, 2, 3) and direction ratios of its normal are 3, 2, 5.

2) Find the direction ratios of the normal to the plane $2x+3y+z=7$.

3) Find the vector equation of the line

$$\frac{x}{1} = \frac{y-1}{2} = \frac{z-2}{3}$$

4) Verify if the point having position vector $4\hat{i}-11\hat{j}+2\hat{k}$ lies on the line

$$\vec{r} = (6\hat{i}-4\hat{j}+5\hat{k}) + \mu (2\hat{i} + 7\hat{j}+3\hat{k}),$$

5) Find the Cartesian equation of the line passing through

A (1, 2, 3) and having direction ratios 2, 3, 7.

6) Find the vector equation of the line passing through the point having position vector $4\hat{i} - \hat{j}+2\hat{k}$ and parallel to the vector $-2\hat{i} - \hat{j}+\hat{k}$.

7) Find the Cartesian equation of the plane passing through the points (3, 2, 1) and (1, 3, 1)

8) What is the condition that the line $\vec{r} = \vec{a} + \lambda\vec{b}$ parallel to the plane $\vec{r} \cdot \vec{n} = \vec{a} \cdot \vec{n}$?

III Short answer questions (2 marks)

1) Find the direction ratios of the line perpendicular to the lines

$$\frac{x-7}{2} = \frac{y+7}{-3} = \frac{z-6}{1} \quad \text{and} \quad \frac{x+5}{1} = \frac{y+3}{2} = \frac{z-6}{-2}$$

2) Find direction cosines of the normal to the plane $\vec{r} \cdot (3\hat{i}+4\hat{k}) = 5$

3) If the normal to the plane has direction ratios 2, -1, 2 and its perpendicular distance from origin is 6, find its equation.

4) Reduce the equation $\vec{r} \cdot (3\hat{i} + 4\hat{j} + 12\hat{k}) = 8$ to normal form.

5) Find the Cartesian equation of the line passing through A(1, 2, 3) and

B (2, 3, 4)

6) Find the perpendicular distance of origin from the plane $6x-2y+3z-7=0$

7) Find the acute angle between the lines $x=y$; $z=0$ and $x=0$ $z=0$.

IV. Short answer questions (3 marks)

1) Find Cartesian equation of the line passing through the point $A(2, 1, -3)$ and perpendicular to vectors $\hat{i} + \hat{j} + \hat{k}$ and $\hat{i} + 2\hat{j} - \hat{k}$

2) Find the vector equation of the line passing through the point having position vector $-\hat{i} - \hat{j} + 2\hat{k}$ and parallel to the line $\vec{r} = (\hat{i} + 2\hat{j} + 3\hat{k}) + \mu (3\hat{i} + 2\hat{j} + \hat{k})$; μ is a parameter.

3) Find the Cartesian equation of the line passing through $(-1, -1, 2)$ and parallel to the line $2x - 2 = 3y + 1 = 6z - 2$.

4) Find the Cartesian equation of the plane passing through $A(7, 8, 6)$ and parallel to XY plane.

5) Find the coordinates of the foot of perpendicular from the origin to the plane $2x + 6y - 3z = 63$.

6) Find the vector equation of a plane at a distance 6 units from the origin and to which vector $2\hat{i} - \hat{j} + 2\hat{k}$ is normal.

7) Find the Cartesian equation of the plane passing through the points $A(1, 1, 2)$, $B(0, 2, 3)$ $C(4, 5, 6)$.

8) Find acute angle between the lines $\frac{x-1}{1} = \frac{y-2}{-1} = \frac{z-3}{2}$ and $\frac{x-1}{2} = \frac{y-1}{1} = \frac{z-3}{1}$

9) Find the distance between the parallel lines $\frac{x}{2} = \frac{y}{-1} = \frac{z}{2}$ and $\frac{x-1}{2} = \frac{y-1}{-1} = \frac{z-3}{2}$.

10) Find the equation of the plane passing through the point $(7, 8, 6)$ and parallel to the plane $\vec{r} \cdot (6\hat{i} + 8\hat{j} + 7\hat{k}) = 0$

11) Find m , if the lines $\frac{1-x}{3} = \frac{7y-14}{2m} = \frac{z-3}{2}$ and $\frac{7-7x}{3m} = \frac{y-5}{1} = \frac{6-z}{5}$ are at right angles.

13) Find the equation of the plane containing lines $\frac{x-5}{4} = \frac{y-7}{4} = \frac{z+3}{-5}$ and $\frac{x-8}{7} = \frac{y-4}{1} = \frac{z-5}{3}$

14) Find the equation of the plane which is parallel to the plane $x - 2y + 2z = 5$ and whose distance from the point $(1, 2, 3)$ is 1

V. Long answer questions (4 marks)

1) show that the lines $\frac{x+1}{-10} = \frac{y+3}{-1} = \frac{z-4}{1}$ and $\frac{x+10}{-1} = \frac{y+1}{-3} = \frac{z-1}{4}$ intersect each other. also find the coordinates of the point of intersection.

2) A(-2, 3, 4) B(1, 1, 2) C(4, -1, 0) are three points. Find the Cartesian equation of line AB and show that points A, B, C are collinear.

3) find the Cartesian and vector equation of the line passing through the point having position vector $\hat{i} + 2\hat{j} + 3\hat{k}$ and perpendicular to vectors $\hat{i} + \hat{j} + \hat{k}$ and $2\hat{i} - \hat{j} + \hat{k}$.

4) Find the vector equation of the plane which bisects the segment joining A(2, 3, 6) and B(4, 3, -2) at right angles.

5) Find vector equation of the plane passing through A(-2, 7, 5) and parallel to vectors $4\hat{i} - \hat{j} + 3\hat{k}$ and $\hat{i} + \hat{j} + \hat{k}$.

6) Find the Cartesian and vector equation of the plane which makes intercepts 1, 1, 1 on the coordinate axes.

8) Find the distance of the point of intersection of the line $\frac{x-3}{1} = \frac{y-4}{2} = \frac{z-5}{2}$ and the plane $x + y + z = 17$ from the point (3, 4, 5) is given by

9) Show that $\frac{x+5}{1} = \frac{y-3}{7} = \frac{z+3}{3}$ and $x - y + 2z - 4 = 0 = 2x + y - 3z + 5$ are parallel

7. LINEAR PROGRAMMING PROBLEMS

I. MCQ (2 marks each)

1. The corner points of the feasible solutions are $(0,0)$ $(3,0)$ $(2,1)$ $(0,7/3)$ the maximum value of $Z = 4x+5y$ is
 - a) 12
 - b) 13
 - c) $35/3$
 - d) 0
2. The half plane represented by $4x+3y > 14$ contains the point
 - a) $(0,0)$
 - b) $(2,2)$
 - c) $(3,4)$
 - b) $(1,1)$
3. The feasible region is the set of point which satisfy
 - a) The object functions
 - b) All the given constraints
 - c) Some of the given constraints
 - d) Only one constraint
4. Objective function of LPP is
 - a) A constraint
 - b) A function to be maximized or minimized
 - c) A relation between the decision variable
 - d) Equation of straight line
5. The value of objective function is maximum under linear constraints
 - a) At the center of the feasible region
 - b) At $(0,0)$
 - c) At vertex of feasible region
 - d) At $(-1, -1)$
6. If a corner point of the feasible solutions are $(0,10)$ $(2,2)$ $(4,0)$ $(3,2)$ then the point of minimum $Z = 3x + 2y$ is
 - a) $(2,2)$
 - b) $(0,10)$
 - c) $(4,0)$
 - b) $(3,2)$
7. The point of which the maximum value of $z = x+y$ subject to constraints $x+2y \leq 70$, $2x+y \leq 90$, $x \geq 0$, $y \geq 0$ is obtained at
 - a) $(30,25)$
 - b) $(20,35)$
 - c) $(35,20)$
 - b) $(40,15)$
8. A solution set of the inequality $x \geq 0$
 - a) Half plane on the Left of y axis
 - b) Half plane on the right of y axis excluding the point on y-axis

- c) Half plane on the right of y axis including the point on y axis
 d) Half plane on the upword of x axis
9. Which value of x is in the solution set of inequality $-2X+Y \geq 17$
 a) - 8 b) -6 c) -4 b) 12
10. The graph of the inequality $3X- 4Y \leq 12, X \leq 1, X \geq 0, Y \geq 0$ lies in fully in
 a) I quadrant b) II quadrant c) III quadrant b) IV quadrant

II. Short Answers (2 marks)

- Solve $4x-18 \geq 0$ graphically using xy plane
- Sketch the graph of inequation $x \geq 5y$ in xoy co-ordinate system
- Find the graphical solution for the system of linear inequation $2x+y \leq 2, x-y \leq 1$
- Find the feasible solution of linear inequation $2x+3y \leq 12, 2x+y \leq 8,$
 $x \geq 0, y \geq 0$ by graphically
- Solve graphically $x \geq 0, y \geq 0$
- Find the solution set of inequalities $0 \leq x \leq 5, 0 \leq 2y \leq 7$
- Find the feasible solution of in equations $3x+2y \leq 18, 2x+ y \leq 10, X \geq 0,$
 $Y \geq 0$
- Draw the graph of inequalities $x \leq 6, y- 2 \leq 0, x \geq 0, y \geq 0$ and indicate the feasible region
- Check the ordered points (1, - 1), (2, - 1) is a solution of $2x+3y-6 \leq 0$
- Show the solution set of inequations $4x - 5y \leq 20$ graphically

III Short Answer type questions. (3 marks)

- A manufacturer produces bulbs and tubes. It takes one hour on machine M_1 and three hours of machine M_2 to produce one package of bulbs. While it takes two hours on machine M_1 and four hours of machine M_2 to produce one package of tubes. He earns a profit of Rs. 135 per package of bulbs and Rs. 550 per package of tubes. How manypackages of each item should be produced each day so as to maximize his profit, if he operates machine M_1 for at most 10 hours a day and machine M_2 for atleast 12 hours a day ? Form the LPP.

2) A company manufactures two types of fertilizers E and F. Each type of fertilizer requires two raw materials A and B. The number of units of A and B required to manufacture one unit of fertilizer E and F and availability of the raw materials A and B per day are given in the table. By selling one unit of E and one unit of F, the company gets a profit of Rs. 500 and Rs. 750 respectively. How many units of E and F should the company manufacture daily so as to maximize its profit ? Formulate the problem as LPP.

Raw Material	Fertilizer		Availability
	E	F	
A	2	3	40
B	1	4	70

3) A construction company uses blocks made up of cement and sand. The weight of a concrete block has to be at least 5 kg. Cement costs Rs. 20 per kg, while sand costs Rs. 6 per kg. Strength considerations dictate that the concrete block should contain minimum 4 kg of cement and not more than 2 kg of sand. Form the LPP for cost to be minimum.

III. Long Answers (4 marks)

1. Maximize $z = 5x + 2y$ subject to $3x + 5y \leq 15$, $5x + 2y \leq 10$, $x \geq 0$, $y \geq 0$
2. Maximize $z = 7x + 11y$ subject to $3x + 5y \leq 26$, $5x + 3y \leq 30$, $x \geq 0$, $y \geq 0$
3. Maximize $z = 10x + 25y$ subject to $x + y \leq 5$, $0 \leq x \leq 3$, $0 \leq y \leq 3$
4. Maximize $z = 3x + 5y$ subject to $x + 4y \leq 24$, $3x + y \leq 21$, $x + y \leq 9$, $x \geq 0$, $y \geq 0$ also find the maximum value of z
5. Minimize $z = 8x + 10y$ subjected to $2x + y \geq 7$, $2x + 3y \geq 15$, $y \geq 2$, $x \geq 0$, $y \geq 0$
6. Minimize $z = 7x + y$ subjected to $5x + y \geq 5$, $x + y \geq 3$, $x \geq 0$, $y \geq 0$
7. Minimize $z = 6x + 21y$ subject to $x + 2y \geq 3$, $x + 4y \geq 4$, $3x + y \geq 3$, $x \geq 0$, $y \geq 0$ show that the minimum value of z occurs at more than two points

8. minimize $z = 2x + 4y$ is subjected to $2x + y \geq 3$, $x + 2y \geq 6$, $x \geq 0$, $y \geq 0$ show that the minimum value of z occurs at more than two points
9. Maximize $z = -x + 2y$ subjected to constraints $x + y \geq 5$, $x \geq 3$, $x + 2y \leq 6$, $y \geq 0$ is this LPP solvable? Justify your answer
10. $x - y \leq 1$, $x - y \geq 0$, $x \geq 0$, $y \geq 0$ are the constant for the objective function $z = x + y$. It is solvable for finding optimum value of z ? Justify?
11. A co-operative society of farmers has 50 hectare of land to grow two crops A and B. The profit from crops A and B perpendicular hectare are estimated as Rs. 10,500 and RS. 9,000 respectively. To control weeds a liquid herbicide has to be used for crops A and B at rates of 20 liter and 10 liter per hectare. Further no more than 800 liter of herbicide should be used in order to protect fish and a wild life using the pond which collects draining from this land. How much land should be allocated to each crop so as to maximize the total profit of the society ?
12. A chemical company produces a chemical containing three basic elements A, B, C so that it has at least 16 liters of A, 24 liters of B and 18 liters of C. This chemical is made by mixing two compounds. Each unit of compound I contains 4 liters of A, 12 liters of B, 2 liters of C. and compound II coantains 2 liters of A, 2 liters of B and 6 liters of C. The cost per unit of compound I is Rs. 800/- and that of compound II is Rs. 640/- Formulate the problem as L.P.P. and solve it to minimize the cost.
13. A carpenter makes tables and chairs. Profit per table is Rs. 200 and that per chair is Rs. 100. He should make at least two chairs per table and the total number of tables and chairs should not exceed 30. Find the maximum profit.

1. DIFFERENTIATION

I. MCQ (2 Marks each)

- 1) If $y = \sec(\tan^{-1} x)$ then $\frac{dy}{dx}$ at $x = 1$ is _____
- (a) $\frac{1}{2}$ (b) 1 (c) $\frac{1}{\sqrt{2}}$ (d) $\sqrt{2}$
- 2) If $f(x) = \log_x(\log x)$ then $f'(e)$ is _____
- (a) 1 (b) e (c) $\frac{1}{e}$ (d) 0
- 3) If $y = 25^{\log_5 \sin x} + 16^{\log_4 \cos x}$ then $\frac{dy}{dx} =$ _____
- (a) 1 (b) 0 (c) 9 (d) $\cos x - \sin x$
- 4) If $f'(4) = 5, f(4) = 3, g'(6) = 7$ and $R(x) = g[3 + f(x)]$ then $R'(4) =$ _____
- (a) 35 (b) 12 (c) $\frac{7}{5}$ (d) 105
- 5) If $y = \tan^{-1}\left(\frac{2x}{1-x^2}\right), x \in (-1,1)$ then $\frac{dy}{dx} =$ _____.
- (a) $\frac{-2}{1+x^2}$ (b) 1 (c) $\frac{2}{1+x^2}$ (d) $\frac{1}{1+x^2}$
- 6) If g is the inverse of f and $f'(x) = \frac{1}{1+x^4}$ then $g'(x) =$ _____
- (a) $\frac{1}{1+[g(x)]^4}$ (b) $\frac{4x^3}{1+x^4}$ (c) $\frac{1}{1+[g(x)]^3}$ (d) $1+[g(x)]^4$
- 7) If $\sin^{-1}(x^3 + y^3) = a$ then $\frac{dy}{dx} =$ _____
- (a) $\frac{-x}{\cos a}$ (b) $\frac{-x^2}{y^2}$ (c) $\frac{y^2}{x^2}$ (d) $\frac{\sin a}{y}$
- 8) If $x = \cos^{-1}(t), y = \sqrt{1-t^2}$ then $\frac{dy}{dx} =$ _____
- (a) t (b) -t (c) $-\frac{1}{t}$ (d) $\frac{1}{t}$

9) If $x^2 + y^2 = 1$ then $\frac{d^2x}{dy^2} = \underline{\hspace{2cm}}$.

- (a) x^3 (b) y^3 (c) $-y^3$ (d) $-\frac{1}{x^3}$

10) If $x^2 + y^2 = t + \frac{1}{t}$ and $x^4 + y^4 = t^2 + \frac{1}{t^2}$ then $\frac{dy}{dx} = \underline{\hspace{2cm}}$

- (a) $\frac{x}{2y}$ (b) $-\frac{y}{x}$ (c) $-\frac{x}{2y}$ (d) $\frac{y}{x}$

11) If $x = at^4$ $y = 2at^2$ then $\frac{dy}{dx} = \underline{\hspace{2cm}}$

- (a) $\frac{1}{t}$ (b) $-\frac{1}{t}$ (c) $\frac{1}{t^2}$ (d) $-\frac{1}{t^2}$

II. Very Short answer questions (1 mark each)

1) Differentiate $y = \sqrt{x^2 + 5}$ w.r. to x

2) Differentiate $y = e^{\tan x}$ w.r. to x

3) If $y = \sin^{-1}(2^x)$, find $\frac{dy}{dx}$.

4) If $f(x)$ is odd and differentiable, then $f'(x)$ is

5) If $y = e^{1+\log x}$ then find $\frac{dy}{dx}$

6) Find the n th order derivative of the function $y = \log x$

7) Find the n th order derivative of the function $y = \cos x$

8) Find the n th order derivative of the function $y = \frac{1}{x}$

III. Short answer questions (2 mark each)

1) If $y = \log [\cos(x^5)]$ then find $\frac{dy}{dx}$

2) If $y = \sqrt{\tan \sqrt{x}}$, find $\frac{dy}{dx}$

3) Find the derivative of the inverse of function $y = 2x^3 - 6x$ and calculate its value at $x = -2$

4) Let $f(x) = x^5 + 2x - 3$ find $(f^{-1})'(-3)$

5) If $y = \cos^{-1} [\sin(4^x)]$, find $\frac{dy}{dx}$

- 6) If $y = \tan^{-1}\left(\sqrt{\frac{1+\cos x}{1-\cos x}}\right)$, find $\frac{dy}{dx}$
- 7) If $x = \sin \theta$, $y = \tan \theta$ then find $\frac{dy}{dx}$
- 8) Differentiate $\sin^2(\sin^{-1}(x^2))$ w.r. to x

IV. Short answer questions (3 mark each)

- 1) If $y = \log\left[\sqrt{\frac{1-\cos(3x/2)}{1+\cos(3x/2)}}\right]$, find $\frac{dy}{dx}$
- 2) If $y = \log\left[4^{2x}\left(\frac{x^2+5}{\sqrt{2x^3-4}}\right)^{3/2}\right]$, find $\frac{dy}{dx}$
- 3) Differentiate $\cot^{-1}\left(\frac{\cos x}{1+\sin x}\right)$ w.r. to x
- 4) Differentiate $\sin^{-1}\left(\frac{2\cos x+3\sin x}{\sqrt{13}}\right)$ w.r. to x
- 5) Differentiate $\tan^{-1}\left(\frac{8x}{1-15x^2}\right)$ w.r. to x
- 6) If $\log_5\left(\frac{x^4+y^4}{x^4-y^4}\right) = 2$, show that $\frac{dy}{dx} = \frac{12x^3}{13y^2}$
- 7) If $y = \sqrt{\cos x + \sqrt{\cos x + \sqrt{\cos x + \dots \infty}}}$, show that $\frac{dy}{dx} = \frac{\sin x}{1-2y}$
- 8) Find the derivative of $\cos^{-1}x$ w.r. to $\sqrt{1-x^2}$
- 9) If $x \sin(a+y) + \sin a \cos(a+y) = 0$ then show that $\frac{dy}{dx} = \frac{\sin^2(a+y)}{\sin a}$
- 10) If $y = 5^x \cdot x^5 \cdot x^x \cdot 5^5$, find $\frac{dy}{dx}$

V. Long answer questions (4 mark each)

- 1) If $y = e^{m \tan^{-1} x}$, show that $(1+x^2)\frac{d^2y}{dx^2} + (2x-m)\frac{dy}{dx} = 0$

2) If $x^7 \cdot y^5 = (x + y)^{12}$, show that $\frac{dy}{dx} = \frac{y}{x}$

3) Differentiate $\tan^{-1}\left(\frac{\sqrt{1+x^2}-1}{x}\right)$ w.r. to $\tan^{-1}\left(\frac{2x\sqrt{1-x^2}}{1-2x^2}\right)$

4) If $y = \sin^{-1}\left(\frac{a \cos x - b \sin x}{\sqrt{a^2 + b^2}}\right)$ then find $\frac{dy}{dx}$

5) If $y = \cos(m \cos^{-1} x)$ then show that $(1-x^2)\frac{d^2y}{dx^2} - x\frac{dy}{dx} + m^2y = 0$

6) Find the nth order derivative of the function $y = \sin(ax + b)$

7) Find the nth order derivative of the function $y = \log(ax + b)$

8) Find the nth order derivative of the function $y = \frac{1}{3x-5}$

:: Theorems ::

1) If $y = f(u)$ is a differentiable function of u and $u = g(x)$ is a differentiable function of x such that the composite function $y = f[g(x)]$ is a differentiable function of x

then $\frac{dy}{dx} = \frac{dy}{du} \cdot \frac{du}{dx}$ Hence find $\frac{dy}{dx}$ if $y = \sin^2 x$

2) Suppose $y = f(x)$ is a differentiable function of x on an interval I and y is one-

one, onto and $\frac{dy}{dx} \neq 0$ on I . Also if $f^{-1}(y)$ is differentiable on $f(I)$ then $\frac{dx}{dy} = \frac{1}{\frac{dy}{dx}}$

, $\frac{dy}{dx} \neq 0$

3) If $x = f(t)$ and $y = g(t)$ are differentiable functions of t so that y is a differentiable

function of x and it $\frac{dx}{dt} \neq 0$ then $\frac{dy}{dx} = \frac{\frac{dy}{dt}}{\frac{dx}{dt}}$.

2. APPLICATIONS OF DERIVATIVE

I. MCQ (2 Marks each)

- The slope of the tangent to the curve $x = 2 \sin^3 \theta$, $y = 3 \cos^3 \theta$ at $\theta = \frac{\pi}{4}$ is
(A) $\frac{3}{2}$ (B) $-\frac{3}{2}$ (C) $\frac{2}{3}$ (D) $-\frac{2}{3}$
- The slope of the normal to the curve $y = x^2 + 2e^x + 2$ at $(0,4)$ is
(A) 2 (B) -2 (C) $\frac{1}{2}$ (D) $-\frac{1}{2}$
- If the line $y = 4x - 5$ touches the curve $y^2 = ax^3 + b$ at the point $(2, 3)$ then $a + b$ is
(A) -5 (B) 2 (C) -7 (D) 9
- If the tangent at $(1, 1)$ on $y^2 = x(2-x)^2$ meets the curve again at P, then P is
(A) $(4,4)$ (B) $(-1, 2)$ (C) $(3, 6)$ (D) $(\frac{9}{4}, \frac{3}{8})$
- The displacement of a particle at time t is given by $s = 2t^3 - 5t^2 + 4t - 3$. The time when the acceleration is 14 ft/sec^2 , is
(A) 1 sec (B) 2 sec (C) 3 sec (D) 4 sec
- Let $f(x) = x^3 - 62 + 9x + 18$, then $f(x)$ is strictly decreasing in ...
(A) $(-\infty, 1)$ (B) $[3, \infty)$ (C) $(-\infty, 1) \cup [3, \infty)$ (D) $(1, 3)$
- A ladder 5 m in length is resting against vertical wall. The bottom of the ladder is pulled along the ground, away from the wall at the rate of 1.5 m/sec . The length of the higher point of the when foot of ladder is 4 m away from the wall decreases at the rate of ____
(A) 1 (B) 2 (C) 2.5 (D) 3
- The edge of a cube is decreasing at the rate of 0.6 cm/sec then the rate at which its volume is decreasing when the edge of the cube is 2 cm, is
(A) $1.2 \text{ cm}^3/\text{sec}$ (B) $3.6 \text{ cm}^3/\text{sec}$ (C) $4.8 \text{ cm}^3/\text{sec}$ (D) $7.2 \text{ cm}^3/\text{sec}$
- A particle moves along the curve $y = 4x^2 + 2$, then the point on the curve at which y - coordinate is changing 8 times as fast as the x - coordinate is
(A) $(2,18)$ (B) $(-1,6)$ (C) $(1,6)$ (D) $(0,2)$
- The function $f(x) = x \log x$ is minimum at $x =$
(A) e (B) $\frac{1}{e}$ (C) 1 (D) $-\frac{1}{e}$

II. Very Short answer questions (1 mark each)

1. Find the slope of tangent to the curve $y = 2x^3 - x^2 + 2$ at $(\frac{1}{2}, 2)$.
2. The displacement of a particle at time t is given by $s = 2t^3 - 5t^2 + 4t - 3$. Find the velocity when $t = 2$ sec.
3. Prove that function $f(x) = x - \frac{1}{x}$, $x \in \mathbb{R}$ and $x \neq 0$ is increasing function.
4. Show that $f(x) = x - \cos x$ is increasing for all x .
5. Show that the function $f(x) = x^3 + 10x + 7$ for $x \in \mathbb{R}$ is strictly increasing.

III. Short answer questions (2 mark each)

1. Find the slope of normal to the curve $3x^2 - y^2 = 8$ at the point $(2, 2)$.
2. Find the slope of tangent to the curve $x = \sin\theta$ and $y = \cos 2\theta$ at $\theta = \frac{\pi}{6}$.
3. Find the equation of normal to the curve $y = 2x^3 - x^2 + 2$ at $(\frac{1}{2}, 2)$
4. A car is moving in such a way that the distance it covers, is given by the equation $s = 4t^2 + 3t$, where s is in meters and t is in seconds. What would be the velocity and the acceleration of the car at time $t = 20$ second ?
5. A man of height 2 meters walks at a uniform speed of 6 km/hr away from a lamp post of 6 meters high. Find the rate at which the length of the shadow is increasing
6. Water is being poured at the rate of $36 \text{ m}^3/\text{sec}$ in to a cylindrical vessel of base radius 3 meters. Find the rate at which water level is rising.
7. Test whether the function $f(x) = x^3 + 6x^2 + 12x - 5$ is increasing or decreasing for all $x \in \mathbb{R}$.
8. Test whether the following function $f(x) = 2 - 3x + 3x^2 - x^3$, $x \in \mathbb{R}$ is increasing or decreasing.
9. Find the values of x for which the function $f(x) = 2x^3 - 6x^2 + 6x + 24$ is strictly increasing.
10. Find approximate value of $\sqrt[3]{27.027}$
11. Verify Rolle's theorem for the function $f(x) = x^2 - 5x + 9$, $x \in [1, 4]$
12. Verify LMVT for the function $f(x) = \log x$, $x \in [1, e]$

IV. Short answer questions (3 mark each)

1. Find the point on the curve $y = \sqrt{x - 3}$ where the tangent is perpendicular to the line $6x + 3y - 5 = 0$.
2. A spherical soap bubble is expanding so that its radius is increasing at the rate of 0.02 cm/sec. At what rate is the surface area is increasing, when its radius is 5 cm?
3. The surface area of a spherical balloon is increasing at the rate of 2 cm²/sec. At what rate the volume of the balloon is increasing when radius of the balloon is 6 cm?
4. A ladder 10 meter long is leaning against a vertical wall. If the bottom of the ladder is pulled horizontally away from the wall at the rate of 1.2 meters per seconds, find how fast the top of the ladder is sliding down the wall when the bottom is 6 meters away from the wall.
5. Find the values of x for which the function $f(x) = x^3 - 6x^2 - 36x + 7$ is strictly increasing.
6. Find the values of x , for which the function $f(x) = x^3 + 12x^2 + 36x + 6$ is monotonically decreasing.
7. The profit function $P(x)$ of a firm, selling x items per day is given by $P(x) = (150 - x)x - 1625$. Find the number of items the firm should manufacture to get maximum profit. Find the maximum profit.
8. Divide the number 30 in to two parts such that their product is maximum.
9. A wire of length 36meters is bent in the form of a rectangle. Find its dimensions if the area of the rectangle is maximum.
10. Find approximate value of $\tan^{-1}(1.001)$
11. Find approximate value of $\cos(89^\circ 30')$ given that $1^\circ = 0.0175^c$
12. Find approximate value of $e^{1.005}$, given that $e = 2.7183$
13. Find approximate value of $\log_{10}(1016)$, given that $\log_{10}e = 0.4343$
14. Find approximate value of $f(x) = x^3 + 5x^2 - 7x + 10$ at $x = 1.1$

V. Long answer questions (4 mark each)

1. Find points on the curve given by $y = x^3 - 6x^2 + x + 3$ where the tangents are parallel to the line $y = x + 5$.
2. The volume of the spherical ball is increasing at the rate of 4π cc/sec. Find the rate at which the radius and the surface area are changing when the volume is 288π cc.
3. The volume of a sphere increase at the rate of $20 \text{ cm}^3/\text{sec}$. Find the rate of change of its surface area when its radius is 5 cm.
4. A man of height 180 cm is moving away from a lamp post at the rate of 1.2 meters per second. If the height of the lamp post is 4.5 meters, find the rate at which
 - (i) his shadow is lengthening. (ii) the tip of the shadow is moving.
5. Find the values of x for which $f(x) = 2x^3 - 15x^2 - 144x - 7$ is
 - (a) Strictly increasing
 - (b) strictly decreasing
6. Find the local maximum and local minimum value of
$$f(x) = x^3 - 3x^2 - 24x + 5.$$
7. A wire of length 120cm is bent in the form of a rectangle. Find its dimensions if the area of the rectangle is maximum.
8. An open box is to be cut out of piece of square card board of side 18 cm by cutting of equal squares from the corners and turning up the sides. Find the maximum volume of the box.
9. A rectangular sheet of paper has it area 24 sq. Meters. The margin at the top & the bottom are 75cm each & the sides 50cm each. What are the dimensions of the paper if the area of the printed space is maximum?
10. A box with a square base is to have an open top. The surface area of the box is 192 sq. cm. What should be its dimensions in order that the volume is largest?
11. A wire of length l is cut into two parts. One part is bent into a circle and the other into a square. Show that the sum of the areas of the circle and the square is least, if the radius of the circle is half the side of the square.
12. Find c , if LMVT is applicable for $f(x) = (x - 3)(x - 6)(x - 9)$, $x \in [3, 5]$
13. Q.2 Verify LMVT for the function $f(x) = x + \frac{1}{x}$, $x \in [1, 3]$

3. INDEFINITE INTEGRATION

I. MCQ (2marks)

Q1) $\int \frac{\sqrt{\tan x}}{\sin x \cos x} dx = \dots$

a) $\frac{1}{2}\sqrt{\tan x} + c$

b) $2\sqrt{\tan x} + c$

c) $\sqrt{2} \tan x + c$

d) $\frac{1}{\sqrt{2\tan x}} + c$

Q2) $\int \frac{1}{\sqrt{x} + \sqrt{x^3}} dx = \dots$

a) $2\tan^{-1}(\sqrt{x}) + c$

b) $\tan^{-1}(2x - 1) + c$

c) $2\tan \sqrt{x} + c$

d) $2\tan^{-1}(x) + c$

Q3) $\int \frac{1}{\sqrt{(x-3)(x-2)}} dx = \dots$

a) $\frac{1}{2} \log[(2x - 1) + \sqrt{x^2 - x - 6}] + c$

b) $\tan^{-1}(2x - 1) + c$

c) $\log\left[\left(x - \frac{1}{2}\right) + \sqrt{x^2 - x - 6}\right] + c$

d) $\log\left[\left(x - \frac{1}{2}\right) + \sqrt{x^2 + x + 6}\right] + c$

Q4) $\int \frac{1}{\cos x - \sin x} dx = \dots$

a) $\frac{1}{\sqrt{2}} \log\left[\operatorname{cosec}\left(x + \frac{\pi}{4}\right) - \cot\left(x + \frac{\pi}{4}\right)\right] + c$

b) $\sqrt{2} \log\left[\operatorname{cosec}\left(x + \frac{\pi}{4}\right) + \cot\left(x + \frac{\pi}{4}\right)\right] + c$

c) $\frac{1}{\sqrt{2}} \log\left[\sec\left(x + \frac{\pi}{4}\right) + \tan\left(x + \frac{\pi}{4}\right)\right] + c$

d) $\sqrt{2} \log\left[\sec\left(x + \frac{\pi}{4}\right) - \tan\left(x + \frac{\pi}{4}\right)\right] + c$

Q5) $\int \frac{x^2}{\sqrt{1-x^6}} dx = \dots$

a) $-\sin^{-1}(x^3) + c$

b) $\cos^{-1}(x^3) + c$

c) $\sin^{-1}(x^3) + c$

d) $\frac{1}{3} \sin^{-1}(x^3) + c$

Q6) $\int \frac{e^x}{x} [x(\log x)^2 + 2 \log x] dx = \dots$

a) $e^x \log x + c$

b) $e^x (\log x)^2 + c$

c) $e^{2x} \log x + c$

d) $e^{2x} (\log x)^2 + c$

Q7) $\int \sqrt{x^2 + 2x + 5} dx = \dots$

a) $(x + 1)\sqrt{x^2 + 2x + 5} + \log[(x + 1) + \sqrt{x^2 + 2x + 5}] + c$

b) $(x + 2)\sqrt{x^2 + 2x + 5} + \log[(x + 2) + \sqrt{x^2 + 2x + 5}] + c$

c) $(\frac{x+2}{2})\sqrt{x^2 + 2x + 5} + \frac{1}{2}\log[(x + 2) + \sqrt{x^2 + 2x + 5}] + c$

d) $(\frac{x+1}{2})\sqrt{x^2 + 2x + 5} + 2\log[(x + 1) + \sqrt{x^2 + 2x + 5}] + c$

Q8) If $f(x) = \frac{\sin^{-1} x}{\sqrt{1-x^2}}$, $g(x) = e^{\sin^{-1} x}$, then $\int f(x)g(x)dx = \dots$

a) $e^{\sin^{-1} x}(\sin^{-1} x - 1) + c$

b) $e^{\sin^{-1} x}(1 - \sin^{-1} x) + c$

c) $e^{\sin^{-1} x}(\sin^{-1} x + 1) + c$

d) $-e^{\sin^{-1} x}(\sin^{-1} x + 1) + c$

Q9) $\int \frac{\sin^m x}{\cos^{m+2} x} dx = \dots$

a) $(m + 2) \tan^{m+1} x + c$ b) $\frac{\tan^m x}{m} + c$

c) $(m + 1) \tan^{m+1} x + c$ d) $\frac{\tan^{m+1} x}{m+1} + c$

Q10) $\int \frac{2}{\sqrt{x}-\sqrt{x+3}} dx = \dots$

a) $-\frac{2}{3} \left[x^{\frac{3}{2}} + (x + 3)^{\frac{3}{2}} \right] + c$ b) $\frac{2}{3} \left[x^{\frac{3}{2}} - (x + 3)^{\frac{3}{2}} \right] + c$

c) $\frac{4}{9} \left[x^{\frac{3}{2}} - (x + 3)^{\frac{3}{2}} \right] + c$ d) $-\frac{4}{9} \left[x^{\frac{3}{2}} + (x + 3)^{\frac{3}{2}} \right] + c$

Q11) $\int \cos \sqrt{x} dx = \dots$

a) $2[\sqrt{x} \sin \sqrt{x} + \cos \sqrt{x}] + c$ b) $\sqrt{x} \sin \sqrt{x} + \cos \sqrt{x} + c$

c) $2[\sqrt{x} \cos \sqrt{x} + \sin \sqrt{x}] + c$ d) $\frac{1}{2}[\sqrt{x} \sin \sqrt{x} - \cos \sqrt{x}] + c$

Q12) $\int \frac{2(\cos^2 x - \sin^2 x)}{\cos^2 x + \sin^2 x} dx = \dots$

a) $\sin 2x + c$

b) $\cos 2x + c$

c) $\tan 2x + c$

d) $2\sin 2x + c$

$$\text{Q13)} \int \frac{\log x}{(\log ex)^2} dx = \dots$$

$$\text{a)} x(1 + \log x) + c$$

$$\text{b)} \frac{x}{1 + \log x} + c$$

$$\text{c)} \frac{1}{1 + \log x} + c$$

$$\text{d)} \frac{1}{1 - \log x} + c$$

$$\text{Q14)} \text{If } \int \frac{1}{x+x^5} dx = f(x) + c, \text{ then } \int \frac{x^4}{x+x^5} dx = \dots$$

$$\text{a)} f(x) - \log x + c$$

$$\text{b)} f(x) + \log x + c$$

$$\text{c)} \log x - f(x) + c$$

$$\text{d)} \frac{1}{5} x^5 f(x) + c$$

$$\text{Q15)} \int \frac{e^x(x-1)}{x^2} dx = \dots$$

$$\text{a)} xe^{-x} + c$$

$$\text{b)} \frac{e^x}{x^2} + c$$

$$\text{c)} \left(x - \frac{1}{x}\right) e^x + c$$

$$\text{d)} \frac{e^x}{x} + c$$

II. Very Short Answers (1 mark)

$$\text{Q1)} \int \sqrt{1 + \sin 2x} dx$$

$$\text{Q2)} \int \frac{\sin 4x}{\cos 2x} dx$$

$$\text{Q3)} \int \frac{e^{3x}}{e^{3x}+1} dx$$

$$\text{Q4)} \int \frac{1}{\sqrt{3x^2+8}} dx$$

$$\text{Q5)} \int \frac{\log x}{x} dx$$

$$\text{Q6)} \int (2 + \cot x - \operatorname{cosec}^2 x) e^x dx$$

$$\text{Q7)} \int e^x \left[\frac{(x+3)}{(x+4)^2} \right] dx$$

$$\text{Q8)} \int \frac{e^{2x} + e^{-2x}}{e^x} dx$$

$$\text{Q9)} \int x^x (1 + \log x) dx$$

$$\text{Q10)} \int \frac{1}{x \sin^2(\log x)} dx$$

$$\text{Q11)} \int \sqrt{x} \sec(x)^{\frac{3}{2}} \tan(x)^{\frac{3}{2}} dx$$

$$\text{Q12)} \int \frac{\cos 2x}{\sin^2 x} dx$$

$$\text{Q13)} \int \frac{\cos 2x}{(\sin x + \cos x)^2} dx$$

$$\text{Q14)} \int \cot^2 x dx$$

$$\text{Q15)} \int \frac{x}{x+2} dx$$

III. Short Answers (2 marks)

$$\text{Q1)} \int \frac{\sin x}{1 + \sin x} dx$$

$$\text{Q2)} \int \frac{(2x-7)}{\sqrt{4x-1}} dx$$

$$\text{Q3)} \int \frac{1}{4x+5x^{-11}} dx$$

$$\text{Q4)} \int e^{3 \log x} (x^4 + 1)^{-1} dx$$

$$\text{Q5)} \int \frac{\sin(x-a)}{\cos(x+b)} dx$$

$$\text{Q6)} \int \cos^7 x dx$$

$$\text{Q7)} \int \frac{x^7}{(1+x^4)^2} dx$$

$$\text{Q8)} \int \frac{1}{\sqrt{2x^2-5}} dx$$

$$\text{Q9)} \int \frac{\log(\log x)}{x} dx$$

$$\text{Q10)} \int x^2 \sqrt{a^2 - x^6} dx$$

$$\text{Q11)} \int \sqrt{4^x(4^x + 4)} dx$$

$$\text{Q12)} \int [\operatorname{cosec}(\log x)][1 - \cot(\log x)] dx$$

$$\text{Q13)} \int \frac{\cos 2x}{\sin^2 x \cos^2 x} dx$$

$$\text{Q14)} \int \sin 4x \cos 3x dx$$

$$\text{Q15)} \int \frac{e^x \log(\sin e^x)}{\tan e^x} dx$$

IV. Short answers (3 Marks)

$$Q1) \int \frac{1}{x(x^3-1)} dx$$

$$Q2) \text{If } f'(x) = x - \frac{3}{x^3}, f(1) = \frac{11}{2} \text{ find } f(x)$$

$$Q3) \int \frac{(x^2+2)}{x^2+1} a^{x+\tan^{-1}x} dx$$

$$Q4) \int \frac{7+4x+5x^2}{(2x+3)^{\frac{3}{2}}} dx$$

$$Q5) \int \sqrt{\frac{9+x}{9-x}} dx$$

$$Q6) \int \frac{1}{4x^2-20x+17} dx$$

$$Q7) \int \frac{\sin x}{\sin 3x} dx$$

$$Q8) \int \frac{1}{2+\cos x - \sin x} dx$$

$$Q9) \int \sec^3 x dx$$

$$Q10) \int \sin(\log x) dx$$

$$Q11) \int \sec^2 x \sqrt{\tan^2 x + \tan x - 7} dx$$

$$Q12) \int e^{\sin^{-1}x} \left[\frac{x+\sqrt{1-x^2}}{\sqrt{1-x^2}} \right] dx$$

$$Q13) \int e^x \frac{(1+x^2)}{(1+x)^2} dx$$

$$Q14) \int \frac{x^2+x-1}{x^2+x-6} dx$$

$$Q15) \int \frac{6x^3+5x^2-7}{3x^2-2x-7} dx$$

V. Long answers (4 Marks)

$$Q1) \int \frac{dx}{2+3\tan x}$$

$$Q2) \int \sqrt{\tan x} + \sqrt{\cot x} dx$$

$$Q3) \int \frac{3x+4}{\sqrt{2x^2+2x+1}} dx$$

$$\text{Q4)} \int \sqrt{\frac{e^{3x}-e^{2x}}{e^x+1}} dx$$

$$\text{Q5)} \int x^3 \tan^{-1} x dx$$

$$\text{Q6)} \int x \sin 2x \cos 5x dx$$

$$\text{Q7)} \int \frac{x+\sin x}{1-\cos x} dx$$

$$\text{Q8)} \int \frac{x^2}{(x^2+1)(x^2-2)(x^2+3)} dx$$

$$\text{Q9)} \int \frac{dx}{x^3-1}$$

$$\text{Q10)} \int \frac{5e^x}{(e^x+1)(e^{2x}+9)} dx$$

$$\text{Q11)} \int \frac{1}{\sin x(3+2\cos x)} dx$$

$$\text{Q12)} \int x \cos^3 x dx$$

$$\text{Q13)} \int \frac{\sin 2x}{3\sin^4 x - 4\sin^2 x + 1} dx$$

$$\text{Q14)} \int \frac{3e^{2x}+5}{4e^{2x}-5} dx$$

$$\text{Q15)} \int \frac{(2\log x+3)}{x(3\log x+2)[(\log x)^2+1]} dx$$

Q16) Integrate the functions w. r. t. x

i) $(3x-2)\sqrt{x^2+x+1}$

ii) $x\sqrt{1+x-x^2}$

iii) $(x-5)\sqrt{x^2-1}$

iv) $(x+1)\sqrt{x^2+x+1}$

v) $(2x+3)\sqrt{3x^2+2x-5}$

4. DEFINITE INTEGRATION

I. MCQ (2 marks each)

(1) $\int_1^9 \frac{x+1}{\sqrt{x}} dx$

(A) $\frac{80}{3}$ (B) $\frac{64}{3}$ (C) $\frac{17}{3}$ (D) $\frac{15}{3}$

(2) $\int_0^1 \sqrt{\frac{1-x}{1+x}} dx =$

(A) $\frac{\pi}{2} - 1$ (B) $\frac{\pi}{2} + 1$ (C) $\frac{\pi}{2} - 2$ (D) $\frac{\pi}{2} + 2$

(3) $\int_1^2 \frac{e^{\frac{1}{x}}}{x^2} dx =$

(A) $2\sqrt{e}(1 + \sqrt{e})$ (B) $\sqrt{e}(1 - \sqrt{e})$ (C) $\sqrt{e}(\sqrt{e} - 1)$ (D) $\sqrt{e}(1 + \sqrt{e})$

(4) $\int_0^{\frac{\pi}{4}} \sqrt{1 + \sin 2x} dx =$

(A) $\frac{1}{\sqrt{2}}$ (B) $\sqrt{2}+1$ (C) $2\sqrt{2}$ (D) 1

(5) If $\int_0^1 \frac{dx}{\sqrt{1+x}-\sqrt{x}} = \frac{k}{3}$ then $k = \dots$

(A) $\sqrt{2}(2\sqrt{2} - 2)$ (B) $\frac{\sqrt{2}}{3}(2 - 2\sqrt{2})$ (C) $\frac{2\sqrt{2}-2}{3}$ (D) $4\sqrt{2}$

(6) $\int_{\frac{\pi}{5}}^{\frac{3\pi}{10}} \frac{\sin x}{\sin x + \cos x} dx =$

(A) $\frac{\pi}{10}$ (B) $\frac{\pi}{20}$ (C) $\frac{\pi}{6}$ (D) $\frac{\pi}{12}$

(7) $\int_0^1 \frac{x^2-2}{x^2+1} dx =$

(A) $1 - \frac{3\pi}{4}$ (B) $2 - \frac{3\pi}{4}$ (C) $1 + \frac{3\pi}{4}$ (D) $2 + \frac{3\pi}{4}$

(8) Let $I_1 = \int_e^{e^2} \frac{1}{\log x} dx$ and $I_2 = \int_1^2 \frac{e^x}{x} dx$ then

(A) $I_1 = \frac{1}{3}I_2$ (B) $I_1 + I_2 = 0$ (C) $I_1 = 2I_2$ (D) $I_1 = I_2$

$$(9) \int_0^4 \frac{1}{\sqrt{4x-x^2}} dx =$$

- (A) 0 (B) 2π (C) π (D) 4π

$$(10) \int_0^{\frac{\pi}{2}} \log(\tan x) dx =$$

- (A) $\frac{\pi}{8}(\log 2)$ (B) 0 (C) $-\frac{\pi}{8}(\log 2)$ (D) $\frac{\pi}{2}(\log 2)$

II. Very Short Answers (1 mark)

$$(11) \text{ Evaluate: } \int_{\frac{\pi}{6}}^{\frac{\pi}{3}} \cos x dx.$$

$$(12) \text{ Evaluate: } \int_{-\frac{\pi}{4}}^{\frac{\pi}{4}} x^3 \sin^4 x dx$$

$$(13) \text{ Evaluate: } \int_0^1 \frac{1}{1+x^2} dx$$

$$(14) \text{ Evaluate: } \int_0^{\frac{\pi}{4}} \sec^2 x dx$$

$$(15) \text{ Evaluate: } \int_0^1 |x| dx$$

$$(16) \text{ Evaluate: } \int_0^1 \frac{1}{\sqrt{1-x^2}} dx$$

$$(17) \text{ Evaluate: } \int_1^2 \frac{x}{1+x^2} dx$$

$$(18) \text{ Evaluate: } \int_0^1 \frac{e^x}{\sqrt{e^x-1}} dx$$

$$(19) \text{ Evaluate: } \int_0^{\frac{\pi}{2}} \frac{\sin 2x}{1+\sin^2 x} dx$$

$$(20) \text{ Evaluate: } \int_0^1 (x+1)^2 dx$$

III. Short Answers (2 marks)

$$(21) \text{ Evaluate: } \int_{\frac{\pi}{6}}^{\frac{\pi}{3}} \sin^2 x dx$$

$$(22) \text{ Evaluate: } \int_0^{\pi/2} \sqrt{1-\cos 4x} dx$$

$$(23) \text{ Evaluate: } \int_0^{\frac{\pi}{2}} \cos^3 x dx$$

(24) Evaluate: $\int_0^{\pi} \cos^2 x \cdot dx$

(25) Evaluate: $\int_0^{\frac{\pi}{4}} \frac{\tan^3 x}{1+\cos 2x} dx$

(26) Evaluate: $\int_0^{\frac{\pi}{4}} \frac{\cos x}{4 - \sin^2 x} dx$

(27) Evaluate: $\int_1^3 \frac{\cos(\log x)}{x} dx$

(28) Evaluate: $\int_0^{\frac{\pi}{2}} \frac{\sin^2 x}{(1+\cos x)^2} dx$

(29) Evaluate: $\int_0^9 \frac{\sqrt{x}}{\sqrt{x}+\sqrt{9-x}} dx$

IV. Short Answers (3 marks)

(30) Prove that : $\int_a^b f(x)dx = \int_a^c f(x)dx + \int_c^b f(x)dx$ where $a < c < b$

(31) Prove that: $\int_a^b f(x) dx = \int_a^b f(a+b-x) dx$

(32) Prove that: $\int_0^a f(x) dx = \int_0^a f(a-x) dx$. Hence find $\int_0^{\frac{\pi}{2}} \sin^2 x dx$.

(33) Evaluate: $\int_0^{\pi/2} \frac{\sin^4 x}{\sin^4 x + \cos^4 x} dx$

(34) Evaluate: $\int_3^8 \frac{(11-x)^2}{x^2+(11-x)^2} dx$

(35) Evaluate: $\int_{-1}^1 |5x - 3| dx$

(36) Evaluate: $\int_{-4}^2 \frac{1}{x^2+4x+13} dx$

(37) Evaluate: $\int_0^1 \frac{1}{\sqrt{3+2x-x^2}} dx$

(38) Evaluate: $\int_0^1 x \cdot \tan^{-1} x dx$

(39) Evaluate: $\int_0^{\frac{1}{\sqrt{2}}} \frac{\sin^{-1} x}{(1-x^2)^{\frac{3}{2}}} dx$

(40) Evaluate: $\int_0^{\frac{\pi}{4}} \sec^4 x dx$

(41) Evaluate: $\int_0^{\frac{\pi}{2}} \frac{1}{5+4\cos x} dx$

(42) Evaluate: $\int_0^{\frac{\pi}{2}} \frac{\cos x}{(1+\sin x)(2+\sin x)} dx$

(43) Evaluate: $\int_{-1}^1 \frac{1}{a^2 e^x + b^2 e^{-x}} dx$

(44) Evaluate: $\int_0^a \frac{1}{x + \sqrt{a^2 - x^2}} dx$

(45) Evaluate: $\int_0^3 x^2(3-x)^{\frac{5}{2}} dx$

(46) Evaluate: $\int_0^1 t^2 \sqrt{1-t} dt$

V. Long answers (4 Marks)

(47) Prove that: $\int_0^{2a} f(x) dx = \int_0^a f(x) dx + \int_0^a f(2a-x) dx$

(48) Prove that: $\int_{-a}^a f(x) dx = 2 \int_0^a f(x) dx$, if $f(x)$ is even function.

$= 0$, if $f(x)$ is odd function

(49) Evaluate: $\int_0^{1/2} \frac{1}{(1-2x^2)\sqrt{1-x^2}} dx$

(50) Evaluate: $\int_0^{\frac{\pi}{4}} \frac{\sec^2 x}{3\tan^2 x + 4\tan x + 1} dx$

(51) Evaluate: $\int_{\frac{1}{\sqrt{2}}}^1 \frac{(e^{\cos^{-1} x})(\sin^{-1} x)}{\sqrt{1-x^2}} dx$

(52) Evaluate: $\int_0^1 \frac{\log(x+1)}{x^2+1} dx$

(53) Evaluate: $\int_0^{\pi} x \cdot \sin x \cdot \cos^2 x \cdot dx$

(54) Evaluate: $\int_0^{\frac{\pi}{2}} x \sin x dx$

(55) Evaluate: $\int_{-1}^1 \frac{1+x^3}{9-x^2} dx$

(56) Evaluate: $\int_0^1 \left(\frac{1}{1+x^2}\right) \sin^{-1}\left(\frac{2x}{1+x^2}\right) dx$

(57) Evaluate: $\int_0^{\frac{\pi}{4}} \frac{\cos 2x}{1+\cos 2x+\sin 2x} dx$

(58) Evaluate: $\int_0^{\pi/4} \log(1 + \tan x) dx$

(59) Evaluate: $\int_0^{\pi} \frac{1}{3+2\sin x + \cos x} dx$

60) Evaluate integral using limit of sum

i) $\int_0^3 (x + 4) dx$

ii) $\int_0^2 (2x + 1) dx$

iii) $\int_0^1 2^x dx$

iv) $\int_0^4 (x - x^2) dx$

v) $\int_0^4 (8x^3) dx$

5. APPLICATION OF DEFINITE INTEGRATION

I. MCQ (2 marks each)

Ex.1: The area of the region bounded by the curve $y = \sin x$, X-axis and the lines $x = 0, x = 4\pi$ is sq. units.

- (A) 2 (B) 4 (C) 8 (D) 16

Ex.2: The area of the region bounded by the ellipse $x^2/64 + y^2/100 = 1$, is sq. units.

- (A) 64π (B) 80π (C) $\pi/80$ (D) 100π

Ex.3: The area bounded by the parabola $y^2 = x$ along the X-axis & the lines $x=0, x=2$ is sq. units.

- (A) $4/3$ (B) $(4\sqrt{2})/3$ (C) $2/3$ (D) $(2\sqrt{2})/3$

Ex.4: The area bounded by the curve $y^2 = x^2$, and the line $x = 8$ is

- (A) 16 sq. units (B) 64 sq. units (C) 32 sq. units (D) 4 sq. units

Ex.5: The area of the region included between the parabolas $y^2 = 16x$ and $x^2 = 16y$, is given by sq. units

- (A) 256 (B) $16/3$ (C) $256/3$ (D) $64/3$

Ex.6: The area enclosed between the two parabolas $y^2 = 20x$ and $y = 2x$ is sq. units.

- (A) $20/3$ (B) $40/3$ (C) $10/3$ (D) $50/3$

Ex.7: The area bounded by the parabola $y^2 = 32x$ the X-axis and the latus rectum is sq. units

- (A) $512/3$ (B) $512/5$ (C) 512 (D) $64/3$

Ex.8: The area bounded by the ellipse $x^2/4 + y^2/25 = 1$ & and the line $x/2 + y/5 = 1$ is sq. units

- (A) $5(\pi-2)$ (B) $(5/2)(\pi-2)$ (C) $(5/3)(\pi-2)$ (D) $(5/4)(\pi-2)$

Ex.9: The area of triangle ΔABC whose vertices are $A(1,1), B(2,1)$ & $C(3,3)$ issq. units.

- (A) 1 (B) 2 (C) 3 (D) 4

Ex.10: The area enclosed by the line $2x + 3y = 6$ along X-axis & the lines $x = 0, x = 3$ is sq. units.

- A) 1 (B) 2 (C) 3 (D) 4

II. Very Short Answers (1 mark)

Ex. 1 : Find the area bounded by the curve $y^2 = 36x$, the line $x = 2$
in first quadrant .

Ex.2: Find the area bounded by the curve $y = \sin x$, the lines $x = 0$ and $x = \pi/2$.

Ex.3: Find the area enclosed between $y = \cos x$ and X-axis between the
lines $x = \pi/2$ & $x \leq 3\pi/2$

Ex.4: Find the area of the region bounded by the parabola $y^2 = 32x$ and
its Latus rectum in first quadrant .

Ex.5: Find the area of the region bounded by the curve $y = x^2$, the
X-axis and the given lines $x = 0$, $x = 3$

Ex.6: Find the area of the region bounded by the curve $y^2 = 8x$, the
X-axis and the given lines $x = 1, x = 3, y \geq 0$

Ex.7: Find the area of the region bounded by the curve $x^2 = 12y$, the
Y-axis and the given lines $y = 2, y = 4, x \geq 0$

Ex.8: Find the area of the ellipse $x^2 / 1 + y^2 / 4 = 1$, in first quadrant

Ex.9: Find the area of sector bounded by the circle $x^2 + y^2 = 25$,
in the first quadrant.

Ex.10: Using integration, find the area of the region bounded by the line
 $2y + x = 8$, X-axis and the lines $x=2$ & $x = 4$.

III. Short Answers (3 marks)

Ex.1: Find the area enclosed between the X-axis and the curve $y = \sin x$
for values of x between 0 to 2π .

Ex.2: Find the area of the region bounded by the parabola $x^2 = 4y$ and
The X-axis & the line $x = 1, x = 4$.

Ex.3: Find the area of the region bounded by the parabola $y^2 = 16x$ and
the line $x = 4$.

Ex.4: Find the area of the region bounded by the curves
 $x^2 = 8y, y = 2, y = 4$ and the Y-axis, lying in the first quadrant.

Ex.5: Find the area of the region bounded by the curve $y = \sin x$,
the X-axis and the given lines $x = -\pi$, $x = \pi$

Ex.6: Find the area of the ellipse $x^2/36 + y^2/64 = 1$, using integration.

IV. Long answers (4 Marks)

Ex.1 : Find the area of the region bounded by the curves $y^2 = 4ax$
and $x^2 = 4ay$.

Ex.2: Find the area of the region lying between the parabolas
 $4y^2 = 9x$ and $3x^2 = 16y$

Ex.3: Find the area of the sector bounded by the circle $x^2 + y^2 = 16$,
& the line $y = x$ in the first quadrant.

Ex.4: Find the area of the region included between $y = x^2 + 5$ and
the line $y = x + 7$

Ex.5: Find the area enclosed between the circle $x^2 + y^2 = 9$,
along X – axis and the line $x = y$, lying in the first quadrant.

Ex.6: Find the area enclosed between the circle $x^2 + y^2 = 1$ and
the line $x + y = 1$, lying in the first quadrant.

Ex.7: Find the area of the region bounded by the curve
 $(y - 1)^2 = 4(x + 1)$ and the line $y = (x - 1)$.

6. DIFFERENTIAL EQUATIONS

II. Select & write the correct alternative from the given option for each question (2 Marks)

1. Solution of the equation $x \frac{dy}{dx} = y \log y$ is

- a) $y = ae^x$ b) $y = be^{2x}$ c) $y = be^{-2x}$ d) $y = e^{ax}$

2. Bacterial increases at the rate proportional to the number present. If original number M doubles in 3 hours, then number of bacteria will be 4M in

- a) 4 hours b) 6 hours c) 8 hours d) 10 hours

3. The general solution of $\frac{dy}{dx} = e^{-x}$ is

a) $y = e^x + c$ b) $y = e^{-x} + c$

c) $y = -e^{-x} + c$ d) $y = e^x + c$

4. The differential equation of $y = Ae^{5x} + Be^{-5x}$ is

a) $\frac{d^2y}{dx^2} = 25y$ b) $\frac{d^2y}{dx^2} = -25y$

c) $\frac{d^2y}{dx^2} = 5y$ d) $y \frac{d^2y}{dx^2} = -5y$

5. Differential equation of the function $c + 4yx = 0$ is

a) $xy + \frac{dy}{dx} = 0$ b) $x \frac{dy}{dx} + y = 0$

c) $\frac{dy}{dx} - 4xy = 0$ d) $x \frac{dy}{dx} + 1 = 0$

6. General solution of $y - x \frac{dy}{dx} = 0$

a) $3 \log x + \frac{7}{y} = c$ b) $2 \log x + \frac{3}{y} = c$

c) $\log x - \log y = \log c$ d) $3 \log y + \frac{2}{x} = c$

7. The order and degree of $\left(\frac{dy}{dx}\right)^3 - \frac{d^3y}{dx^3} + ye^x$ is

- a) 3, 1 b) 1, 3 c) 3, 3 d) 1, 1

8. The order and degree of $\left(1 + \left(\frac{dy}{dx}\right)^3\right)^{\frac{2}{3}} = 8 \frac{d^3y}{dx^3}$ are respectively

- a) 3 , 1 b) 1 , 3 c) 3 , 3 d) 1 , 1

9. The solution of $\frac{dy}{dx}=1$ is

- a) $x + y = c$ b) $xy = c$ c) $x^2 + y^2 = c$ d) $y - x = c$

10. The solution of $\frac{dy}{dx} + \frac{x^2}{y^2} = 0$ is

- a) $x^3 + y^3 = 7$ b) $x^2 + y^2 = c$ c) $x^3 + y^3 = c$ d) $x + y = c$

10. The integrating factor of Linear differential equation $x\sin x \frac{dy}{dx} + (x\cos x + \sin x)y = \sin x$ is ----

- A) $-x\sin x$ B) $x\sin x$ C) $e^{x\sin x}$ D) $e^{-x\sin x}$

11. If $\sec x + \tan x$ is I.F. of $\frac{dy}{dx} + Py = Q$ then Value of P =-----

- (A) $\tan x$ (B) $-\tan x$ (C) $\sec x$ (D) $-\sec x$
1 (B) 2 (C) 3 (D) 4

II Answer the following 1 marks

1. Find the differential equation of family of lines making equal intercept on coordinate axes.

2. Find the general solution of $\frac{dy}{dx} = \frac{1+y^2}{1+x^2}$

3. Form the differential equation of family of standard circle

4. State the degree of differential equation $e \frac{dy}{dx} + \frac{dy}{dx} = x$

5. Write the general Solution of a differential equation $\frac{dy}{dx} + Py = Q$ Where P&Q are functions of x or Constants

6. . Write the general Solution of a differential equation $\frac{dx}{dy} + Px = Q$ Where P&Q are functions of y or Constants

7. Find I.F. of Linear differential equation $\frac{dy}{dx} + y = e^{-x}$

III. Attempt the following questions (2 marks)

1. Form the differential equation of $y = (c_1 + c_2)e^x$

2. Solve the differential equation

$$\sec^2 y \tan x dy + \sec^2 x \tan y dx = 0$$

3. Solve the differential equation $\frac{dy}{dx} = e^{(x+y)} + x^2 e^y$

4. Find the differential equation of family of all ellipse whose major axis is twice the minor axis.

IV. Attempt the following questions (3 marks)

1. Solve the differential equation $\frac{dy}{dx} + y = e^{-x}$

2. Solve the differential equation $x \frac{dy}{dx} + 2y = x^2 \log x$

3. Solve $\frac{dy}{dx} = \frac{x+y+1}{x+y-1}$ when $x = \frac{2}{3}, y = \frac{1}{3}$

4. Solve the differential equation $x dx + 2y dy = 0$

5. Solve the differential equation $(x^2 - yx^2)dy + (y^2 + xy^2)dx = 0$

6. Solve the following differential equation

$$\frac{dy}{dx} = x^2 y + y$$

7. Find the differential equation by eliminating arbitrary constants from the relation $x^2 + y^2 = 2ax$

8. Find the differential equation by eliminating arbitrary constants from the relation $y = (c_1 + c_2 x)e^x$

9. Verify $y = \log x + c$ is the solution of differential equation

$$x \frac{d^2 y}{dx^2} + \frac{dy}{dx} = 0$$

10. Solve : $\frac{dy}{dx} + \frac{2}{x}y = x^2$

11. Find I.F. of a differential equation

$$\frac{dx}{dy} + \left(\frac{1}{1+y^2}\right)x = \frac{\tan^{-1} y}{1+y^2}$$

12. Find I.F. of a differential equation $\frac{dy}{dx} + \left(\cot x + \frac{1}{x}\right)y = \frac{1}{x}$

13. Find I.F. of a differential equation $\frac{dy}{dx} + \frac{y}{x} = x^3 - 3$

V Attempt the following questions (4 marks)

1. For the differential equation, find the particular solution
 $(x - y^2x)dx - (y + x^2y)dy = 0$ when $x = 2, y = 0$

2. Solve the differential equation $\frac{dy}{dx} + \frac{x-2y}{2x-y} = 0$

3. Find the differential equation from the relation $x^2 + 4y^2 = 4b^2$

4. If the population of a town increases at a rate proportional to the

population at that time. If the population increases from 40 thousands to 60 thousands in 40 years, what will be the population in another 20 years?.

$$\text{(Given } \sqrt{\frac{3}{2}} = 1.2247)$$

5. The rate of growth of bacteria is proportional to the number present. If initially, there were 1000 bacteria and the number doubles in 1 hours, find the number of bacteria after $5/2$ hours. (Given $\sqrt{2} = 1.414$)

6. Solve the following differential equation

$$yx \frac{dy}{dx} = x^2 + 2y^2$$

7. Solve the following differential equation

$$y \log y \frac{dy}{dx} = \log y - x$$

8. For the differential equation, find the particular solution

$$\frac{dy}{dx} = (4x + y + 1), \text{ when } y = 1, x = 0$$

9. Solve the following differential equation

$$y^2 dx + (xy + x^2)dy = 0$$

10. Solve the following differential equation

$$x^2 \frac{dy}{dx} = x^2 + xy - y^2$$

7. PROBABILITY DISTRIBUTIONS

I. MCQ (2 marks each)

(i) Let the p.m.f. of a random variable X be

$$P(x) = \frac{3-x}{10}, \text{ for } x = -1, 0, 1, 2$$

$$= 0, \text{ otherwise}$$

Then $E(X)$ is

- (A) 1 (B) 2 (C) 0 (D) -1

(ii) c.d.f of a discrete random variable X is

- (A) an identity function. (B) a step function.
(B) an even function. (D) an odd function.

(iii) If X denotes the number on the uppermost face of cubic die when

it is tossed, then $E(x)$ is

- (A) $\frac{2}{7}$ (B) $\frac{7}{2}$ (C) 1 (D) $\frac{1}{2}$

(iv) A random variable X has the following probability distribution

X	2	3	4
P(x)	0.3	0.4	0.3

Then the variance of this distribution is

- (A) 0.6 (B) 0.7

(C) 0.77

(D) 0.66

(v) For the random variable X , if $V(X) = 4$, $E(X) = 3$, then $E(X^2)$ is ...

(A) 9

(B) 13

(c) 12

(D) 7

(vi) If a d.r.v. X takes values $0, 1, 2, 3, \dots$ with probability

$P(X = x) = k(x + 1) \times 5^{-x}$, where k is a constant then $P(X = 0) = \dots$

(A) $\frac{7}{25}$

(B) $\frac{16}{25}$

(C) $\frac{18}{25}$

(D) $\frac{19}{25}$

(vii) The p.m.f of a d.r.v. X is $P(X = x) = \begin{cases} \frac{\binom{5}{x}}{2^5}, & \text{for } x = 0, 1, 2, 3, 4, 5 \\ 0, & \text{otherwise} \end{cases}$

If $a = P(X \leq 2)$ and $b = P(X \geq 3)$ then

(A) $a < b$

(B) $a > b$

(C) $a = b$

(D) $a + b = 2$

(viii) If the p.m.f of a d.r.v. X is $P(X = x) = \begin{cases} \frac{x}{n(n+1)}, & \text{for } x = 1, 2, 3, \dots, n \\ 0, & \text{otherwise} \end{cases}$ then

$E(X) = \dots$

(A) $n + \frac{1}{2}$

(B) $\frac{n}{3} + \frac{1}{6}$

(C) $\frac{n}{2} + \frac{1}{5}$

(D) $n + \frac{1}{3}$

(ix) If the p.m.f of a d.r.v. X is $P(X = x) = \begin{cases} \frac{c}{x^3}, & \text{for } x = 1, 2, 3 \\ 0, & \text{otherwise} \end{cases}$

then $E(X) = \dots$

(A) $\frac{343}{297}$

(B) $\frac{294}{251}$

(C) $\frac{297}{294}$

(D) $\frac{294}{297}$

(x) If a d.r.v. X has the following probability distribution :

X	-2	-1	0	1	2	3
$P(X=x)$	0.1	k	0.2	$2k$	0.3	k

then $P(X = -1)$ is ...

- (A) $\frac{1}{10}$ (B) $\frac{2}{10}$ (C) $\frac{3}{10}$ (D) $\frac{4}{10}$

(xi) If a d.r.v. X has the following probability distribution:

X	1	2	3	4	5	6	7
$P(X=x)$	k	$2k$	$2k$	$3k$	k^2	$2k^2$	$7k^2+k$

then $k = \dots$

- (A) $\frac{1}{7}$ (B) $\frac{1}{8}$ (C) $\frac{1}{9}$ (D) $\frac{1}{10}$

xii) The *p. d. f.* of random variable X is $f(x) = \begin{cases} -kx, & 0 < x < 2 \\ 0, & \text{otherwise} \end{cases}$, then $k =$

- (A) -2 (B) 2 (C) $-\frac{1}{2}$ (D) $\frac{1}{2}$

xiii) If X is *c. r. v.* and $F(x)$ denote *c. d. f.* of X , then which of the following is not true?

- (A) $P(a < X < b) = F(b) - F(a)$ (B) $P(X < a) = F(a)$
 (C) $P(X > a) = 1 - F(a)$ (D) $P(X > b) = F(b)$

xiv) If X is *c. r. v.* with *p. d. f.* $f(x) = \begin{cases} \frac{1}{16}(4-x), & -2 < x < 2 \\ 0, & \text{otherwise} \end{cases}$,

then $P\left(X < -\frac{1}{2} \text{ or } X > \frac{1}{2}\right) =$

- (A) $\frac{1}{4}$ (B) $\frac{3}{4}$ (C) $\frac{1}{16}$ (D) $\frac{3}{16}$

xv) If the cumulative distribution function of X is $F(x) = \frac{1}{2}(3x^2 - x^3)$, $0 \leq x \leq 1$ then $P(X > 1)$ is

$(A) \frac{1}{4}$

$(B) \frac{1}{2}$

$(C) 1$

$(D) 0$

xvi) The *p. d. f.* of random variable X is $f(x) = \begin{cases} \frac{1}{2}, & 0 < x < 2 \\ 0, & \text{otherwise} \end{cases}$

If $a = P\left(X < \frac{1}{2}\right)$ and $b = P\left(X > \frac{3}{2}\right)$ then

$(A) a = b$

$(B) a = 3b$

$(C) a = 2b$

$(D) b = 2a$

II. Very Short Answers (1 mark)

1. Let X represent the difference between number of heads and number of tails obtained when a coin is tossed 6 times. What are possible values of X ?
2. An urn contains 5 red and 2 black balls. Two balls are drawn at random. X denotes number of black balls drawn. What are possible values of X ?
3. State which of the following are not probability mass function of random variable. Give reasons of your answers.

(i)

X	0	1	2
P (X)	0.4	0.4	0.2

(ii)

X	0	1	2	3	4
P (X)	0.1	0.5	0.2	-0.1	0.2

(iii)

X	0	1	2
P (X)	0.1	0.6	0.3

(iv)

Y	-1	0	1
P(Y)	0.6	0.1	0.2

4. Find mean for the following probability distribution

X	0	1	2	3
P(X=x)	$\frac{1}{6}$	$\frac{1}{3}$	$\frac{1}{3}$	$\frac{1}{6}$

5. State

whether the following is not the probability mass function of random variable. Give reasons for your answer.

X	3	2	1	0	-1
P(X=x)	0.3	0.2	0.4	0	0.05

6. If X is

c. r. v. and $F(x)$ denote *c. d. f.* of X and $F(0) = 0.3$, $F(3) = 0.7$ then find $P(0 \leq X \leq 3)$

III. Short Answers (2 marks)

(1) Find the expected value and variance of r.v. X whose p.m.f. is given below.

X	1	2	3
P(X=x)	$\frac{1}{5}$	$\frac{2}{5}$	$\frac{2}{5}$

(2) Find the probability distribution of number of heads in two tosses of a coin .

(3) The probability distribution of X is as follows:

X	0	1	2	3	4
$P(X=x)$	0.1	k	$2k$	$2k$	k

Find k and $P[X < 2]$.

(4) The following probability distribution of r.v. X

X	-3	-2	-1	0	1	2	3
$P(X=x)$	0.05	0.1	0.15	0.20	0.25	0.15	0.1

Find the probability that (i) X is positive. (ii) X is odd

(5) In the p.m.f. of r.v. X

X	1	2	3	4	5
$P(X=x)$	$\frac{1}{20}$	$\frac{3}{20}$	a	$2a$	$\frac{1}{20}$

Find a and obtain c.d.f. of X .

IV. Short answers (3 Marks)

- (1) Find the probability distribution of the number of successes in tosses of a die, where a success is defined as number greater than 4 appears on at least one die.
- (2) A coin is biased so that the head is 3 times as likely to occur as a tail. If the coin is tossed twice, find the probability distribution of tails..

(3) A random variable X has the following probability distribution:

X	0	1	2	3	4	5	6	7
P(X)	0	k	2k	2k	3k	k ²	2k ²	7k ² +k

Determine : (i) k ii) $P(X < 3)$ iii) $P(X > 4)$

(4) Find the probability distribution of the number of doublets in three throws of a pair of dice.

(5) Find the mean and variance of the number randomly selected from 1 to 15.

(6) Let the p.m.f. of r.v. X be $f(x) = \begin{cases} \frac{3-x}{10}, & \text{for } x = -1, 0, 1, 2 \\ 0, & \text{otherwise} \end{cases}$

Calculate $E(X)$ and $Var(X)$.

(7) Find the probability distribution of the number of successes in two tosses of a die, where a success is defined as six appears on at least one die.

(8) If the p.d.f. of random variable X is $f(x) = \begin{cases} \frac{k}{\sqrt{x}}, & 0 < x < 4 \\ 0, & \text{otherwise} \end{cases}$

then find the cumulative distribution function of a random variable X.

(9) If the p.d.f. of random variable X is $f(x) = \begin{cases} \frac{k}{x^2+1}, & 0 < x < \infty \\ 0, & \text{otherwise} \end{cases}$

then find k and cumulative distribution function of a random variable X.

(10) If X is c.r.v. with p.d.f. $f(x) = \begin{cases} 4x^3, & 0 < x < 1 \\ 0, & \text{otherwise} \end{cases}$,

then find $P\left(X \leq \frac{3}{4} / X > \frac{1}{4}\right)$

(11) Find k , if the *p.d.f.* of random variable X is $f(x) = \begin{cases} ke^{-3x}, & 0 \leq x < \infty \\ 0 & , \text{ otherwise} \end{cases}$

$$\text{and } P(0 < X < k) = \frac{1}{2}.$$

(12) If X is *c.r.v.* with *p.d.f.* $f(x) = \begin{cases} kx, & 0 \leq x < 1 \\ k & , 1 < x < 2 \\ (3-x), & 2 \leq x \leq 3 \\ 0 & , \text{ otherwise} \end{cases}$,

then find k .

V. Long answers (4 Marks)

1. Let a pair of dice be thrown and the random variable X be the sum of the numbers that appear on the two dice. Find the mean or expectation of X and variance of X .
2. Two cards are drawn simultaneously (or successively without replacement) from a well shuffled pack of 52 cards. Find the mean, variance and standard deviation of the number of kings drawn.
3. Two numbers are selected at random (without replacement) from the first six positive integers. Let X denote the larger of the two numbers obtained. Find $E(X)$.
4. In a meeting, 70% of the members favor and 30% oppose a certain proposal. A member is selected at random and we take $X=0$ if he opposed, and $X=1$ if he is in favor. Find $E(X)$ and $\text{Var}(X)$.

5. The following is the c.d.f. of r.v. X

X	-3	-2	-1	0	1	2	3	4
$F(x)$	0.1	0.3	0.5	0.65	0.75	0.85	0.9	1

Find p.m.f. of X . (i) $P(-1 \leq X \leq 2)$ (ii) $P(X \leq 3 / X > 0)$.

6. A player tosses two coins he wins Rs.10 if 2 heads appears , Rs. 5 if 1 head appears and Rs.2 if no head appears. Find the expected winning amount and variance of winning amount.

7. From a lot of 30 bulbs which include 6 defectives, a sample of 4 bulbs is drawn at random with replacement. Find the probability distribution of the number of defective bulbs.

8. Let X denote the sum of the number obtained when two fair dice are rolled. Find the standard deviation of X .

8. BINOMIAL DISTRIBUTION

Q.1) Multiple Choice Questions (M.C.Q)

2m

1. In a Binomial distribution with $n = 4$, if $2 P(X=3) = 3 P(X=2)$, then value of p is

- a) $\frac{9}{13}$ b) $\frac{4}{13}$ c) $\frac{6}{13}$ d) $\frac{7}{13}$

2. Given that $X \sim B(n, P)$. If $n = 10$, $P = 0.4$ then $E(X) = \dots\dots\dots$

- a) 1 b) 2 c) 3 d) 4

3. For $X \sim B(n, P)$, If $v(x) = 2.4$ and $P = 0.4$ then $n = \dots\dots\dots$

- a) 10 b) 20 c) 30 d) 40

4. Given that $X \sim B(n = 10, P)$, $E(X) = 8$ then value of $P = \dots\dots\dots$

- a) 0.4 b) 0.8 c) 0.6 d) 0.7

5. Bernoulli distribution is a particular case of binomial distribution if $n = \dots\dots\dots$

- a) 4 b) 10 c) 2 d) 1

II. Very Short Answers (1 mark)

i) If $X \sim B(n, P)$, $E(X) = 12$, $V(X) = 4$ then find n

ii) For Bernoulli Distribution, state formula for $E(X)$ and $V(X)$

iii) For $X \sim B(n, P)$ and $P(X=x) = {}^n C_x (1/2)^x (1/2)^{3-x}$ then state values of n and P

iv) State the formula for p.m.f of Binomial Distribution.

v) A die is thrown. If X denotes the number of positive divisor of the outcomes then find the range of random variable X .

III. Short Answers (2 marks)

i) A r.v $X \sim B(n, P)$. If the value of mean and variance of X are 18 and 12 respectively then find total number of positive value of X .

ii) Given $X \sim B(n, P)$, If $E(X) = 6$, $V(X) = 4.2$ find n and P

iii) If $X \sim B(n, P)$ with $n = 10$, $P = 0.4$ then find $E(X^2)$

iv) If $X \sim B(6, P)$ and $2. P(X=3) = P(X=2)$ then find P .

v) Let the p.m.f of r.v.x be $p(x) = {}^4 C_x \left(\frac{5}{9}\right)^x \left(\frac{4}{9}\right)^{4-x}$, $x=0,1,2,4$ Find $E(X)$ and $\text{Var}(X)$

IV. Short answers (3 Marks)

i) A Fair coin is tossed 5 times , find the probability that a) coin shows exactly three times head b) no head.

ii) The probability that certain kind of component will survive a check test is 0.6. Find the probability that exactly 2 of the next 4 tested components survive.

iii) Find the probability of guessing correctly at least nine out of ten answers in a "true" or "false" objective test.

iv) A fair coin is tossed 8 times. Find the probability that it shows heads i) exactly 5 times ii) at least once.

v) The Probability that a person who undergoes a kidney operation will be recovered is 0.5. find the probability that out of 6 patients who undergo similar operation a) none will recover b) half of them recover.

Answer Key

Part I

1. MATHEMATICAL LOGIC

Q1) i) c ii) c iii) c iv) b v) b vi) c vii) d viii) b ix) a x) b

Q2)

i) $10 + 20 \neq 30$ ii) not defined iii) $(p \wedge \sim q)$

iv) F v) T vi) F

Q3)

iii) a) F b) F ii) a) $p \wedge q$ b) $p \vee q$

iii) converse: If function is continuous then it is differentiable.

Contrapositive: If function is not continuous then it is not differentiable.

Q 4) i) An angle is a right angle and it is not of measure 90° , or an angle is of measure 90° and it is not a right angle.

ii) a) $p \leftrightarrow \sim q$ b) $p \rightarrow q$ c) $\sim p \wedge q$

iii) a) $\forall n \in \mathbb{N}, n^2 \geq 1$ is a true statement

b) $\exists x \in \mathbb{N}$ such that $3x - 4 < 9$ is a true statement

c) $\exists y \in \mathbb{N}$ such that $y + 4 > 6$ is a true statement

iv) Contingency

v) proof

vi) a) 13 is a prime number or India is a democratic country

b) $(p \vee \sim q) \wedge (\sim p \vee q) \equiv (p \wedge q) \vee \sim (p \vee q)$

vii) Converse: If they do not drive the car, then it snows

Inverse: If it does not snow, then they drive the car

Contrapositive: If they drive the car, then it does not snow

Q 5 i) Contradiction

ii) Proof

iii) Proof

iv) $[(p \wedge \sim q) \vee (q \wedge \sim p)] \wedge (\sim q \wedge r)$

v)

P	q	r	$\sim r$	$P \wedge q$	$(p \wedge q) \vee \sim r$
T	T	T	F	T	T
T	T	F	T	T	T
T	F	T	F	F	F
T	F	F	T	F	T
F	T	T	F	F	F
F	T	F	T	F	T
F	F	T	F	F	F
F	F	F	T	F	T

2. MATRICES

I. MCQ (2 marks)

1. b) $\begin{bmatrix} 1 & -1 & 0 \\ -2 & 3 & -4 \\ -2 & 3 & -3 \end{bmatrix}$

2. c) $\text{adj}(A)$

3. d) $(-1, 0, 2)$

4. a) Singular matrix

5. c) 2

6. d) $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

7. a) $A^2 = I$

8. b) $\begin{bmatrix} \cos 10\alpha & \sin 10\alpha \\ -\sin 10\alpha & \cos 10\alpha \end{bmatrix}$

9. b) -1

10. d) $\frac{1}{40}$

11. b) 1, -2, 3

12. a) $x = 1, y = 2, z = 3$

13. b) 3^9

14. a) no solution

15. d) 5

II. Very Short Answers (1 mark)

1. $a_{31}A_{31} + a_{32}A_{32} + a_{33}A_{33} = -1$

2. $|A| = 10$

3. $\alpha = -2$

4. $(B^{-1}A^{-1})^{-1} = \begin{bmatrix} 2 & -2 \\ 2 & 3 \end{bmatrix}$

5. $A^{-1} = \begin{bmatrix} \cos\theta & -\sin\theta \\ -\sin\theta & -\cos\theta \end{bmatrix}$

6. $|A|^{-1} = \frac{1}{ad-bc}$

7. $B = \begin{bmatrix} 2 & -1 \\ -5 & 3 \end{bmatrix}$

8. $A^2(\alpha) = A(2\alpha) = \begin{bmatrix} \cos 2\alpha & \sin 2\alpha \\ -\sin 2\alpha & \cos 2\alpha \end{bmatrix}$

9. Order of AB is 3×3

$$10. (A+I)(A-I) = \begin{bmatrix} -5 & -4 \\ 8 & -9 \end{bmatrix}$$

$$11. A^2 = \begin{bmatrix} 2 & -1 & 1 \\ -2 & 3 & -2 \\ -4 & 4 & -3 \end{bmatrix} = A$$

$$12. A^2 = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix} = 0$$

$$13. B'(AB) = 0$$

III. Short Answers (2 marks)

$$1. f(A) = A^2 - 2A - 3I = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix} = 0$$

$$2. B'A' = \begin{bmatrix} 3 \\ 1 \\ -2 \end{bmatrix} \begin{bmatrix} -1 & 2 & 3 \end{bmatrix} = \begin{bmatrix} -3 & 6 & 9 \\ -1 & 2 & 3 \\ 2 & -4 & -6 \end{bmatrix}$$

$$3. |adjA| = |A|^{n-1} = 5^{3-1} = 5^2 = 25$$

$$4. A'B' = \begin{bmatrix} 66 & 55 \\ 55 & 66 \end{bmatrix}$$

$$5. A^{-1}B^{-1} = \frac{1}{2} \begin{bmatrix} 3 & -7 \\ -1 & 3 \end{bmatrix}$$

$$6. X = AB = \begin{bmatrix} 2 \\ 2 \end{bmatrix}$$

$$7. X = A^{-1} = \begin{bmatrix} 3 & -17 \\ -1 & 6 \end{bmatrix}$$

$$8. A^{-1} = \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix}$$

$$9. \text{i) } A^{-1} = \frac{-1}{19} \begin{bmatrix} -2 & -3 \\ -3 & 5 \end{bmatrix} \quad \text{ii) } A^{-1} = \begin{bmatrix} 2 & 3 \\ 1 & 2 \end{bmatrix}$$

$$10. adjA = \begin{bmatrix} 4 & -5 \\ -3 & 6 \end{bmatrix}$$

$$11. A \sim \begin{bmatrix} 1 & 2 & 4 \\ 0 & -7 & -7 \\ 0 & 0 & -2 \end{bmatrix} \text{ (Operations } R_2 - 3R_1, R_3 - 2R_1)$$

IV. Short answers (3 Marks)

$$1. A^2 = I, \text{ Hence } A^{-1} = A = \begin{bmatrix} 0 & 4 & 3 \\ 1 & -3 & -3 \\ -1 & 4 & 4 \end{bmatrix}$$

$$2. AB = \begin{bmatrix} 2 & 1 & 0 \\ 8 & 7 & 2 \\ -1 & 1 & 1 \end{bmatrix} |AB| = 0, \text{ Hence } (AB)^{-1} \text{ does not exist.}$$

$$3. \text{adj}(A) = A = \begin{bmatrix} -4 & -3 & -3 \\ 1 & 0 & 1 \\ 4 & 4 & 3 \end{bmatrix}$$

$$4. x = 4, y = -3$$

$$5. A \sim \begin{bmatrix} 13 & -2 & 5 \\ -1 & 2 & -1 \end{bmatrix}$$

$$6. ₹ 1300$$

$$7. A^{-1} = \frac{1}{6} \begin{bmatrix} -1 & 5 \\ 2 & -4 \end{bmatrix} = \frac{1}{6} (A - 5I)$$

$$8. \text{adj}(A) = [A_{ij}]^T_{3 \times 3} = \begin{bmatrix} 0 & -1 & 1 \\ -8 & 3 & -7 \\ 4 & -2 & 2 \end{bmatrix}$$

$$9. X = \begin{bmatrix} -5 & 2 & 23 \\ 2 & -3 & -18 \\ 1 & 2 & 6 \end{bmatrix}$$

$$10. A^{-1} = \begin{bmatrix} \sec\theta & -\tan\theta & 0 \\ -\tan\theta & \sec\theta & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$11. A \sim \begin{bmatrix} 1 & 2 & 4 \\ 0 & -7 & -7 \\ 0 & 0 & -2 \end{bmatrix}$$

$$12. X = \frac{1}{6} \begin{bmatrix} 4 & 4 & 2 \\ 11 & 8 & -5 \\ 10 & 10 & 2 \end{bmatrix}$$

V. Long answers (4 Marks)

$$1. A^{-1} = \frac{1}{5} \begin{bmatrix} -2 & 0 & 3 \\ -1 & 1 & 0 \\ 2 & 1 & -2 \end{bmatrix}$$

$$2. A^{-1} = \frac{1}{3} \begin{bmatrix} 3 & 0 & 0 \\ -3 & 1 & 0 \\ 9 & 2 & -3 \end{bmatrix}$$

$$3. x = \frac{5}{2}, y = -\frac{3}{2}, z = -2$$

4. The required numbers are 1, -2 and 3.

$$5. A^{-1} = \frac{1}{6} \begin{bmatrix} 4 & -2 & 2 \\ -3 & 0 & 3 \\ 2 & 2 & -2 \end{bmatrix}$$

$$6. A^{-1} = \begin{bmatrix} \frac{1}{x} & 0 & 0 \\ 0 & \frac{1}{y} & 0 \\ 0 & 0 & \frac{1}{z} \end{bmatrix} \quad \text{hence} \quad A^{-1} = \begin{bmatrix} \frac{1}{2} & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -1 \end{bmatrix}$$

$$7. A^{-1} = \begin{bmatrix} \cos \theta & \sin \theta & 0 \\ -\sin \theta & \cos \theta & 0 \\ 0 & 0 & 1 \end{bmatrix}.$$

$$8. X = \begin{bmatrix} 2 & 3 \\ 1 & 1 \end{bmatrix},$$

$$9. A(\text{adj } A) = (\text{adj } A)A = \begin{bmatrix} 11 & 0 & 0 \\ 0 & 11 & 0 \\ 0 & 0 & 11 \end{bmatrix}.$$

$$10. AB = \begin{bmatrix} 11 & 3 \\ 7 & 2 \end{bmatrix}, \quad (AB)^{-1} = \begin{bmatrix} 2 & -3 \\ -1 & 11 \end{bmatrix}$$

$$11. x = \frac{1}{3}, \quad y = \frac{2}{3}, \quad z = 1$$

12. The cost is ₹ 5 per dozen for pencils, ₹ 8 per dozen for pens and ₹ 8 per dozen for erasers.

3. TRIGONOMETRIC FUNCTIONS

Type- I:

1) b) $\frac{\pi}{6}, \frac{11\pi}{6}$

6) a) $1:\sqrt{3}:2$

2) c) an isosceles triangle

7) b) $\frac{\pi}{3}$

3) b) $\frac{\sqrt{3}}{2}$

8) c) $\left(-\frac{3}{4\sqrt{2}}, \frac{3}{4\sqrt{2}}\right)$

4) d) $\theta = n\pi \pm \frac{\pi}{4}, n \in Z$

9) b) $\frac{\pi}{6}$

5) c) $\frac{2\pi}{3}$

10) a) $\frac{1}{5}$

Type- II:

1) 0

8) $\frac{\pi}{6}, \frac{5\pi}{6}$

4) $\left(2, \frac{\pi}{3}\right)$

9) $x = n\pi \pm \frac{\pi}{4}, n \in Z$

6) $\frac{4}{5}$

10) $(2, 2\sqrt{3})$

7) $\angle B = \frac{\pi}{2}$

Type- III:

$$2) \quad x = \frac{2n\pi}{5} + \frac{\pi}{10}, n \in Z \text{ or}$$

$$x = \frac{2n\pi}{3} - \frac{\pi}{6}, n \in Z$$

$$4) \quad x = \frac{n\pi}{3} + (-1)^n \frac{7\pi}{18}, n \in Z$$

$$5) \quad \frac{\pi}{2}$$

$$6) \quad \cos B = \frac{33}{65}$$

$$8) \quad \frac{2\pi}{3}, \frac{5\pi}{3}$$

$$9) \quad -1$$

4. PAIR OF LINES

I. MCQ

1-C : $x^2 - y^2 = 0$

2-C: $x - \sqrt{3}y = 0$ and $\sqrt{3}x + y = 0$

3-B: real and coincident lines

4-B: $k = 6$

5-B: $9m^2 - 3m - 2 = 0$

6-A: $5x^2 + 4xy - 3y^2 = 0$

7-B. : $\frac{\pi}{4}$

8- B: $\frac{11}{2}$

Q2

1. $xy + 3x - 2y + 6 = 0$

2. $(\sec\alpha - \tan\alpha)x + y = 0$: $(\sec\alpha + \tan\alpha)x - y = 0$

3. $k = -2$

4. 60°

5. $h = \pm \frac{7}{2}$

Q3

1. $(2x - 3y + 5)(3x + y - 9) = 0$, $6x^2 - 7xy - 3y^2 - 3x + 32y - 45 = 0$

2. $3x^2 + 2xy - 5y^2 = 0$

3. $25a + 16b - 40h = 0$

4. 30°

Q4

1. $2x^2 - xy - 6y^2 = 0$

3. 45°

6. LINES AND PLANE

I. MCQ

1)C (2) A (3) B (4)B (5) D (6) A (7) B (8) B (9) D

II. Very Short answer questions (1 marks)

1) $3x + 2y + 5z - 22 = 0$ (2) $(2, 3, 1)$ (3) $\vec{r} = (\hat{j} + 2\hat{k}) + \lambda(\hat{i} + 2\hat{j} + 3\hat{k})$

4) Lies on the line

$$5) \frac{x-1}{2} = \frac{y-2}{3} = \frac{z-3}{7}$$

6) $\vec{r} = (4\hat{i} - \hat{j} + 2\hat{k}) + \mu(-2\hat{i} - \hat{j} + \hat{k})$ (7) $\frac{x-3}{-2} = \frac{y-2}{1}; z = 1$

III. Short answer questions (2 marks)

(1) 4, 5, 7 (2) $\frac{3}{5}, 0, \frac{4}{5}$ (3) $2x - y + 2z = 18$ (4) $\vec{r} = \frac{3\hat{i} + 4\hat{j} + 12\hat{k}}{13} = \frac{8}{13}$

(5) $x-1 = y-2 = z-3$ (6) 1 (7) 45^0

IV. Short answer questions (3 marks)

1) $\frac{x-2}{-3} = \frac{y-1}{2} = \frac{z+3}{1}$ (2) $\vec{r} = (-\hat{i} - \hat{j} + 2\hat{k}) + \lambda(3\hat{i} + 2\hat{j} + \hat{k})$

3) $\frac{x+1}{3} = \frac{y+1}{2} = \frac{z-2}{1}$ (4) $z = 6$ (5) $(\frac{18}{7}, \frac{54}{7}, \frac{-27}{7})$ (6) $\vec{r} \cdot (2\hat{i} - \hat{j} + 2\hat{k}) = 18$

7) $y - z + 1 = 0$ (8) 60^0 (9) $\sqrt{2}$ (10) $6x + 8y + 7z = 148$ (11) $m = \frac{70}{11}$

V. Long answer questions (4 marks)

1) $(-11, -4, 5)$ (2) $\frac{x+2}{3} = \frac{y-3}{-2} = \frac{z-4}{-2}$ (3) $\frac{x-1}{2} + \frac{y-2}{1} = \frac{z-3}{-3}$

4) $\vec{r} \cdot (\hat{i} - 4\hat{k}) = -5$ (5) $\vec{r} \cdot (4\hat{i} + \hat{j} - 5\hat{k}) = -26$

6) $\vec{r} \cdot (\hat{i} + \hat{j} + \hat{k}) = 1$

7. LINEAR PROGRAMMING PROBLEM

Q. 1

1. b) 13
2. c) (3,4)
3. b) All the given constraint
4. b) A function to be maximized or minimized
5. c) At vertex of the feasible region
6. a) (2,2)
7. d) (40,15)
8. c) Half plane on the right of y axis including the point on the y axis
9. a) - 8
- 10.d) Fourth quadrant

Q.3

- 1) z is maximum at a point $a = (2,0)$, $b = \left(\frac{20}{19}, \frac{45}{19}\right)$ is 10. z occurs at every point lying on the line segment of ab. hence there are infinite many numbers of optimal solutions
- 2) z has a maximum value 59 when $x = 4.5$, $y = 2.5$
- 3) z has a maximum value 95 when $x = 2$, $y = 3$ 4
- 4) z has maximum value 37 when $x = 4$, $y = 5$
- 5) z has minimum value 52 when $x = 1.5$, $y = 4$
- 6) z has a minimum value 5 when $x = 0$ $y = 5$
- 7) z has a minimum value 22.5 when $x = 2$, $y = 0.5$
- 8) z has a minimum value 12 at vertices $c = (6,0)$ and $b = (0,3)$
z has a minimum value 12 at every point at the segment of bc that is minimum value of z occurs at more than two points
- 9) Shaded region of the graph is unbounded and hence lpp has unbounded solution
- 10) This lpp has unbounded solution hence optimal solution does not exist

1 : DIFFERENTIATION

MCQ :

- 1) c 2) c 3) b 4) a 5) c 6) d
7) b 8) a 9) d 10) b 11) c

1 Mark :

- 1) $\frac{x}{\sqrt{x^2+5}}$ 2) $e^{\tan x} \cdot \sec^2 x$ 3) $\frac{2^x \log 2}{\sqrt{1-(2^x)^2}}$
4) even function 5) e OR $\frac{e^{1+\log x}}{x}$

2 Mark :

- 1) $-5x^4 \tan(x^5)$ 2) $\frac{\sec^2 \sqrt{x}}{4\sqrt{x} \cdot \sqrt{\tan \sqrt{x}}}$ 3) $\frac{1}{18}$ 4) $\frac{1}{2}$
5) $-4^x \log 4$ 6) $-\frac{1}{2}$ 7) $\sec^3 \theta$

3 Mark :

- 1) $\frac{3}{2} \operatorname{cosec}\left(\frac{3x}{2}\right)$ 2) $2 \log^4 + \frac{3x}{x^2+5} - \frac{9x^2}{2(2x^3-4)}$ 3) $\frac{1}{2}$
4) 1 5) $\frac{3}{1+9x^2} + \frac{5}{1+25x^2}$ 6) $\frac{1}{x}$ 8) $\frac{1}{x}$
10) $5^x \cdot x^5 \cdot x^x \cdot 5^5 \left[\log 5 + \frac{5}{x} + 1 + \log x \right]$

4 Mark :

- 3) $\frac{\sqrt{1-x^2}}{4(1+x^2)}$ 4) -1

2. APPLICATION OF DERIVATIVE

1. (B) $-\frac{3}{2}$ 2. (D) $-\frac{1}{2}$ 3. (A) -5 4. (D) $\left(\frac{9}{4}, \frac{3}{8}\right)$
5. (B) 2 sec 6. (D) (1, 3) 7. (C) 2.5 8. (D) 7.2

cm³/sec

9. (C) (1,6) 10. (B) $\frac{1}{e}$

Solve the following

1 Marks

1. $\frac{1}{2}$ 2. 8 unit/sec

2 Marks

1. $-\frac{1}{3}$ 2. -2 3. $2x + y - 3 = 0$ 4. 163m/s
5. 3 km/hr
6. $\frac{4}{\pi}$ m/sec 7. Increasing 8. Decreasing. 9. $R - \{1\}$

3 Marks

1. (4,1) 2. 0.8π cm²/sec 3. 6 cm³/sec
4. 0.9 m/sec
5. $x \in (-\infty, -2) \cup (6, \infty)$ 6. $x \in (-6, -2)$ 7. 75, 4000
8. 15,15
9. 9 cms, 9 cms

4 Marks

1. (0,3) and (4 - 25) 2. $\frac{1}{36}$ cm/sec, $\frac{4\pi}{3}$ cm²/sec 3. 8
cm²/sec

4.(i) 0.8 m/ sec (ii) 2 m/ sec

5.(i) $x \in (-\infty, -3) \cup (8, \infty)$ (ii) $x \in$

$(-3, 8)$

6. Max Value = 33 at $x = -2$, Min value = -75 at $x = 4$

7. 30 cms ,

30cms

8. 432 cc

9. 4 cms, 6cms

10. 8 cms, 8 cms, 4 cms

3. INDEFINITE INTEGRATION

Type-I]

1. b) $2\sqrt{\tan x} + c$

2. a) $2\tan^{-1}(\sqrt{x}) + c$

3. c) $\log\left[\left(x - \frac{1}{2}\right) + \sqrt{x^2 - x - 6}\right] + c$

4. c) $\frac{1}{\sqrt{2}}\log\left[\sec\left(x + \frac{\pi}{4}\right) + \tan\left(x + \frac{\pi}{4}\right)\right] + c$

5. d) $\frac{1}{3}\sin^{-1}(x^3) + c$

6. b) $e^x(\log x)^2 + c$

7. d) $\left(\frac{x+1}{2}\right)\sqrt{x^2 + 2x + 5} + 2\log[(x + 1) + \sqrt{x^2 + 2x + 5}] + c$

8. a) $e^{\sin^{-1}x}(\sin^{-1}x - 1) + c$

9. d) $\frac{\tan^{m+1}x}{m+1} + c$

10. d) $-\frac{4}{9}\left[x^{\frac{3}{2}} + (x + 3)^{\frac{3}{2}}\right] + c$

11. a) $2[\sqrt{x}\sin\sqrt{x} + \cos\sqrt{x}] + c$

12. a) $\sin 2x + c$

13. b) $\frac{x}{1+\log x} + c$

14. c) $\log x - f(x) + c$

15. d) $\frac{e^x}{x} + c$

Type-II]

1. $\sin x - \cos x + c$

2. $-\cos 2x + c$

3. $\frac{1}{3}\log(e^{3x} + 1) + c$

4. $\frac{1}{\sqrt{3}}\log[\sqrt{3}x + \sqrt{3x^2 + 8}] + c$

5. $\frac{(\log x)^2}{2} + c$

6. $e^x(2 + \cot x) + c$

$$7. \frac{e^x}{x+4} + c$$

$$8. e^x - \frac{1}{3e^{3x}} + c$$

$$9. x^x + c$$

$$10. -\cot(\log x) + c$$

$$11. \frac{2}{3} \sec\left(x^{\frac{3}{2}}\right) + c$$

$$12. -\cot x - 2x + c$$

$$13. \log[\cos x + \sin x] + c$$

$$14. -\cot x - x + c$$

$$15. x - 2 \log(x + 2) + c$$

Type-III]

$$1. \sec x - \tan x + c$$

$$2. \frac{1}{12} (4x - 1)^{\frac{3}{2}} - \frac{13}{4} \sqrt{4x - 1} + c$$

$$3. \frac{1}{48} \log(4x^{12} + 5) + c$$

$$4. \frac{1}{2} \log(x^4 + 1) + c$$

$$5. [\cos(a + b) \log[\sec(x + b)] - x \sin(a + b)] + c$$

$$6. \sin x - \sin^3 x + \frac{3}{5} \sin^5 x - \frac{1}{7} \sin^7 x + c$$

$$7. \frac{1}{4} \left[\log(1 + x^4) + \frac{1}{1+x^4} \right] + c$$

$$8. \frac{1}{\sqrt{2}} \log \left(x + \sqrt{x^2 - \frac{5}{2}} \right) + c$$

$$9. (\log x)[\log(\log x) - 1] + c$$

$$10. \frac{1}{6} \left[x^3 \sqrt{a^2 - x^6} + a^2 \sin^{-1} \left(\frac{x^3}{a} \right) \right] + c$$

$$11. \frac{1}{\log 2} \left[\frac{2^x}{2} \sqrt{4^x + 4} + 2 \log(2^x + \sqrt{4^x + 4}) \right] + c$$

$$12. x \operatorname{cosec}(\log x) + c$$

$$13. -\cot x - \tan x + c$$

$$14. -\frac{1}{14} \cos 7x - \frac{1}{2} \cos x + c$$

$$15. [\log \sin(e^x)]^2 + c$$

Type-IV]

$$1. \frac{1}{3} \log \left(\frac{x^3-1}{x^3} \right) + c$$

$$2. f(x) = \frac{x^2}{2} + \frac{3}{2x^2} + \frac{7}{2}$$

$$3. \frac{a^{x+\tan^{-1}x}}{\log a} + c$$

$$4. \frac{5}{12} (2x+3)^{\frac{3}{2}} - \frac{11}{2} \sqrt{2x+3} - \frac{49}{4} \frac{1}{\sqrt{2x+3}} + c$$

$$5. 9 \sin^{-1} \left(\frac{x}{9} \right) - \sqrt{81-x^2} + c$$

$$6. \frac{1}{8\sqrt{2}} \log \left(\frac{2x-5-2\sqrt{2}}{2x-5+2\sqrt{2}} \right) + c$$

$$7. \frac{1}{2\sqrt{3}} \log \left(\frac{\sqrt{3}+\tan x}{\sqrt{3}-\tan x} \right) + c$$

$$8. \sqrt{2} \tan^{-1} \left[\frac{\tan\left(\frac{x}{2}\right)-1}{\sqrt{2}} \right] + c$$

$$9. \frac{1}{2} [\sec x \tan x + \log(\sec x + \tan x)] + c$$

$$10. \frac{x}{2} [\sin(\log x)] - \cos(\log x) + c$$

$$11. \left(\frac{2\tan x+1}{4} \right) \sqrt{\tan^2 x + \tan x - 7} - \frac{29}{8} \log \left[\left(\tan x + \frac{1}{2} \right) + \sqrt{\tan^2 x + \tan x - 7} \right] + c$$

$$12. x e^{\sin^{-1}x} + c$$

$$13. e^x \left(\frac{x-1}{x+1} \right) + c$$

$$14. x + \log \left(\frac{x-2}{x+3} \right) + c$$

$$15. x^2 + 3x + \log(x-1) + \frac{5}{3} \log(3x+1) + c$$

Type-V]

$$1. \frac{2x}{13} + \frac{3}{13} \log(2\cos x + 3\sin x) + c$$

$$2. \sqrt{2} \tan^{-1} \left(\frac{\tan x - 1}{\sqrt{2} \tan x} \right) + c$$

$$3. \frac{3}{2} \sqrt{2x^2 + 2x + 1} + \frac{5}{2\sqrt{2}} \log \left[\left(x + \frac{1}{2} \right) + \sqrt{x^2 + x + \frac{1}{2}} \right] + c$$

4. $\sqrt{e^{2x} - 1} - \log(e^x + \sqrt{e^{2x} - 1}) + c$
5. $\frac{1}{4}(\tan^{-1}x)(x^4 - 1) - \frac{x}{12}(x^2 - 3) + c$
6. $-\frac{x\cos 7x}{14} + \frac{x\cos 3x}{6} + \frac{\sin 7x}{98} - \frac{\sin 3x}{18} + c$
7. $-x\cot\left(\frac{x}{2}\right) + 4\log\left[\sin\left(\frac{x}{2}\right)\right] + c$
8. $\frac{1}{6}\tan^{-1}x + \frac{1}{15\sqrt{2}}\log\left(\frac{x-\sqrt{2}}{x+\sqrt{2}}\right) - \frac{\sqrt{3}}{10}\tan^{-1}\left(\frac{x}{\sqrt{3}}\right) + c$
9. $\frac{1}{3}\log(x-1) - \frac{1}{6}\log(x^2+x+1) - \frac{1}{\sqrt{3}}\tan^{-1}\left(\frac{2x+1}{\sqrt{3}}\right) + c$
10. $\frac{1}{2}\log(e^x+1) - \frac{1}{4}\log(e^{2x}+9) + \frac{1}{6}\tan^{-1}\left(\frac{e^x}{3}\right) + c$
11. $\frac{1}{10}\log(1-\cos x) - \frac{1}{2}\log(1+\cos x) + \frac{2}{5}\log(3+2\cos x) + c$
12. $\frac{1}{4}\left[\frac{x}{3}\sin 3x + \frac{1}{9}\cos 3x + 3x\sin x + 3\cos x\right] + c$
13. $\frac{1}{2}\log\left(\frac{3\sin^2 x - 3}{3\sin^2 x - 1}\right) + c$
14. $\frac{7}{8}\log(4e^{2x} - 5) - x + c$
15. $\frac{5}{13}\log(3\log x + 2) - \frac{5}{26}\log[(\log x)^2 + 1] + \frac{12}{13}\tan^{-1}(\log x) + c$

4. DEFINITE INTEGRATION

1	B	2	A
3	C	4	D
5	D	6	B
7	A	8	D
9	C	10	B
11	$\frac{\sqrt{3} - 1}{2}$	12	0
13	$\frac{\pi}{4}$	14	1
15	$\frac{1}{2}$	16	$\frac{\pi}{2}$
17	$\frac{1}{2} \log\left(\frac{5}{2}\right)$	18	$2\sqrt{e-1}$
19	$\log 2$	20	$\frac{7}{3}$
21	$\frac{\pi}{12}$	22	$\sqrt{2}$
23	$\frac{2}{3}$	24	$\frac{\pi}{2}$

----	----	25	$\frac{1}{8}$
26	$\frac{1}{4} \log \left(\frac{2\sqrt{2} + 1}{2\sqrt{2} - 1} \right)$	27	$\sin(\log 3)$
28	$\frac{4 - \pi}{2}$	29	$\frac{9}{2}$
30	Theorem	31	Theorem
32	$\frac{\pi}{4}$	33	$\frac{\pi}{4}$
34	$\frac{5}{2}$	35	$\frac{34}{5}$
36	$\frac{1}{3} \left[\tan^{-1} \frac{4}{3} + \tan^{-1} \frac{2}{3} \right]$	37	$\frac{\pi}{6}$
38	$\frac{\pi}{4} - \frac{1}{2}$	39	$\frac{\pi}{4} - \frac{1}{2} \log 2$
40	$\frac{4}{3}$	41	$\frac{2}{3} \tan^{-1} \left(\frac{1}{3} \right)$
42	$\log \left(\frac{4}{3} \right)$	43	$\frac{1}{ab} \left[\tan^{-1} \frac{ae}{b} - \tan^{-1} \frac{a}{be} \right]$

44	$\frac{\pi}{4}$	45	$\frac{16}{77}(3)^{\frac{7}{2}}$
46	$\frac{16}{105}$	47	Theorem
48	Theorem	49	$\frac{1}{2}\log(2 + \sqrt{3})$
50	$\frac{1}{2}\log 2$	51	$e^{\frac{\pi}{4}}\left(\frac{\pi}{4} + 1\right) - \left(\frac{\pi}{2} + 1\right)$
52	$\frac{\pi}{8}\log 2$	53	$\frac{\pi}{3}$
54	1	55	$\frac{1}{3}\log 2$
56	$\frac{\pi^2}{16}$	57	$\frac{1}{2}\left[\frac{\pi}{4} - \log 2\right]$
58	$\frac{\pi}{8}(\log 2)$	59	$\frac{\pi}{4}$

5. APPLICATION OF DEFINITE INTEGRATION

Two mark questions:-

Ex. 1 : The required area $A = 8\sqrt{2}$ sq. units

Ex.2: Required area = 1 Sq. units.

Ex.3: Required area = 3 sq.units.

Ex.4: Required area = $128/3$ sq.units.

Ex.5: Required area = 9 Sq.units.

Ex.6: Required area = $(4\sqrt{2}/3)[3\sqrt{3}-1]$ Sq.units.

Ex.7: Required area = $(8/\sqrt{3})[4-\sqrt{2}]$ Sq.units.

Ex.8: Required area = $\pi/2$ sq.units.

Ex.9: Required area = $25\pi/4$ sq.units.

Ex.10: Required area = 5 sq.units.

Three marks questions:-

Ex.1: Required area = 4 sq.units.

Ex.2: Required area = $21/4$ sq.units.

Ex.3: Required area = $128/3$ sq.units.

Ex.4: Required area = $(16\sqrt{2}/3)(4-\sqrt{2})$ sq.units.

Ex.5: Required area = 4 Sq.units

Ex.6: Required area = 48π sq.units.

Four marks questions:-

Ex.1 : Required area $A = (16/3) a^2$ sq. units.

Ex.2: Required area = 4 sq.units.

Ex.3: Required area = 2π sq.units.

Ex.4: Required area = 1 sq.units.

Ex.5: Required area = $(9\pi/8)$ sq.units.

Ex.6: Required area = $(\pi/4 - 1/2)$ sq.units.

Ex.7: Required area = $64/3$ sq.units.

6. DIFFERENTIAL EQUATIONS

I. Select & write the correct alternative from the given option for each question (2 Marks)

Ques. No	Answer	Ques. No	Answer
1	d) $y = e^{ax}$	6	c) $\log x - \log y = \log c$
2	b) 6 hours	7	a) 3, 1
3	c) e^x	8	c) 3, 3
4	a) $ye^{-x} = x + c$	9	d) $y - x = c$
5	b) $x \frac{dy}{dx} + y = 0$	10	c) $x^3 + y^3 = c$

II. Attempt the following questions (1 marks)

- | | |
|------------------------------|------------------------------------|
| 1. $1 + \frac{dy}{dx} = 0$ | 2. $\tan^{-1} x - \tan^{-1} y = c$ |
| 3. $x + y \frac{dy}{dx} = 0$ | 4. Not defined |

III. Attempt the following questions (2 marks)

- | | |
|---------------------------------------|-------------------------------|
| 1. $\frac{d^2y}{dx^2} - 2y + 1 = 0$ | 2. $\tan x \tan y = c$ |
| 3. $e^{-y} + e^x + \frac{x^3}{3} = c$ | 4. $x + 4y \frac{dy}{dx} = 0$ |

IV. Attempt the following questions (3 marks)

Ques. No	Answer
1	$ye^x = x + c$

2	$x^2y = \frac{x^4}{16}(\log x - 1) + c$
3	$\log x + y = y - x + \frac{1}{3}$
4	$x^2 + 2y^2 = c$
5	$\log x - \log y = \frac{1}{x} + \frac{1}{y} + c$
6	$\log y = \frac{x^3}{3} + x + c$
7	$2xy \frac{dy}{dx} = y^2 - x^2$
8	$\frac{d^2y}{dx^2} - \frac{2dy}{dx} + y = 0$
9	Verified
10	$5x^2y = x^5 + c$

V. Attempt the following questions (4 marks)

Ques. No	Answer
1	$(1 - x^2)(1 - y^2) = 5$
2	$\log \left(\frac{x + y}{x - y} \right) - \frac{1}{2} \log(x^2 - y^2) + 2 \log x = \log c$
3	$x + 4y \frac{dy}{dx} = 0$
4	73482
5	45248

6	$x^2 + y^2 = c^2 x^4$
7	$2xlogy = (logy)^2 + c$
8	$log \left \frac{4x + y + 5}{6} \right = x + c$
9	$logx + \frac{1}{4} log \left \frac{2y^2 + xy}{x^2} \right + \frac{3}{4} log \left \frac{2y}{x + 2y} \right = c$
10	$\frac{x + y}{x - y} = cx^2$

7. PROBABILITY DISTRIBUTIONS

Q.1.MCQs

(i) (C) 0

(ii) (B) a step function

(iii) (B) $\frac{7}{2}$

(iv) (A) 0.6

(v) (B) 13

(vi) (B) $\frac{16}{25}$

(vii) (C) $a=b$

(viii) (B) $\frac{n}{3} + \frac{1}{6}$

(ix) (B) $\frac{294}{251}$

(x) (A) $\frac{1}{10}$

(xi) (D) $\frac{1}{10}$

Q.2

(1) $X = \{-6, -4, -2, 0, 2, 4, 6\}$

(2) $X = \{0, 1, 2\}$

(3) (i) $P(x)$ can be regarded as p.m.f.of the random variable X.

(ii) $P(x)$ cannot be regarded as p.m.f.of the random variable X.

(iii) $P(x)$ can be regarded as p.m.f.of the random variable X.

(iv) $P(x)$ cannot be regarded as p.m.f.of the random variable X.

(4) mean = $3/2$

(5) $P(x)$ cannot be regarded as p.m.f.of the random variable X.

Section-B

(1) $E(X) = 11/5$ and $V(X) = 14/25$

(2)

X	0	1	2
P(X=x)	0.25	0.5	0.25

(3) $k = 0.15$. and $P[X < 2] = 0.25$

(4) (i) $P(X \text{ is positive}) = 0.50$

(ii) $P(X \text{ is odd}) = 0.55$

(5) $a = \frac{1}{4}$ and c.d.f.is

X	1	2	3	4	5
F(X=x)	$\frac{1}{20}$	$\frac{1}{5}$	$\frac{9}{20}$	$\frac{19}{20}$	1

Section -C

(1)

X	0	1	2
P(X=x)	$\frac{4}{9}$	$\frac{4}{9}$	$\frac{1}{9}$

(2)

X	0	1	2
P(X=x)	$\frac{9}{16}$	$\frac{3}{8}$	$\frac{1}{16}$

(3) k $= \frac{1}{10}$, P (X < 3) = $\frac{3}{10}$, P

(X > 4) = $\frac{1}{5}$

(4)

X	0	1	2	3
P(X=x)	$\frac{125}{216}$	$\frac{75}{216}$	$\frac{15}{216}$	$\frac{1}{216}$

(5) Mean = 8, Var(X)=18.67

(6) E(X)= 0 and Var(X)=1

(7)

X	0	1	2
P(X=x)	$\frac{25}{36}$	$\frac{10}{36}$	$\frac{1}{36}$

Section -D

(1) E(X) = 7 , Var(X) = 5.83

(2) E(X) = $\frac{34}{221}$, Var(X) = 0.1392, S.D.(X) = 0.3730

(3) E(X) = 4.67

(4) E(X) = 0.7 , Var(X) = 0.21

(5) p.m.f. of r.v. X is

X	-3	-2	-1	0	1	2	3	4
P(x)	0.1	0.2	0.2	0.15	0.10	0.10	0.05	0.10

(i) $P(-1 \leq X \leq 2) = 0.55$, (ii) $P(X \leq 3/X > 0) = 0.25$.

(6) Expected winning amount = Rs.5.5 and

variance of winning amount = Rs.8.

(7) The required probability distribution is

X	0	1	2	3	4
P(x)	$\frac{256}{625}$	$\frac{256}{625}$	$\frac{96}{625}$	$\frac{16}{625}$	$\frac{1}{625}$

(8) Standard deviation of $X = 2.41$

8. BINOMIAL DISTRIBUTION

Q1) 1) a) $\frac{9}{13}$ 2) d) 4 3) a) 40 4) b) 0.8 5) d) 1

Q2) i) $n = 18$, ii) $E(X) = P$, $V(x) = pq$ iii) $n = 8$, $P = 1/2$,

iv) $P(X=x) = {}^n C_x p^x q^{n-x}$ for $x = 0, 1, 2, \dots$ $0 < p < 1$ & $q = 1 - p$

v) range = $\{ 1, 2, 3, 4 \}$

Q.3) i) 55 ii) $n = 20$, $p = 0.3$ iii) 18.4 iv) $p = \frac{3}{11}$ v) $E(X) = 2.22$, $V(X) = 0.9876$

Q.4) i) $\frac{5}{16}, \frac{1}{32}$ ii) 0.3456 iii) $\frac{11}{1024}$ iv) $\frac{7}{32}, \frac{255}{256}$ v) $\frac{1}{64}, \frac{5}{16}$