Useful data	
$A\backslash B$	$\{a \in A : a \notin B\}$
$\mathbb{C}$	Set of all complex numbers
$\mathbb{C}^{m\times n}$	Set of all matrices of order $m \times n$ with complex entries
$\mathbb{C}^{\infty}(\Omega)$	Collection of all infinitely differentiable functions on the open domain Ω
i	$\sqrt{-1}$
I	Identity matrix of appropriate order
$L^2(\mathbb{R})$	$:= L^2(\mathbb{R}, dx)$
$L^2[a,b]$	$:= L^2([a,b], dx)$
$\mathbb{N}$	Set of all positive integers
$\mathbb{Q}$	Set of all rational numbers
$\mathbb{R}$	Set of all real numbers
$\mathbb{R}^{m\times n}$	Set of all matrices of order $m \times n$ with real entries
$\mathbb{S}^1$	$\{(x_1, x_2) \in \mathbb{R}^2 : x_1^2 + x_2^2 = 1\}$
$\mathbb{S}^2$	$\{(x_1, x_2, x_3) \in \mathbb{R}^3 : x_1^2 + x_2^2 + x_3^2 = 1\}$
$\mathbb{Z}$	Set of all integers

Q1) Suppose that the characteristic equation of $M \in C^{3\times 3}$ is

$$\lambda^3 + \alpha\lambda^2 + \beta\lambda - 1 = 0$$

where $\alpha, \beta \in C$ with $\alpha + \beta \neq 0$

Which of the following statements is TRUE?

(A) $M(I - \beta M) = M^{-1}(M + \alpha I)$

(B) $M(I + \beta M) = M^{-1}(M - \alpha I)$

(C) $M^{-1}(M^{-1} + \beta I) = M - \alpha I$

(D) $M^{-1}(M^{-1} - \beta I) = M + \alpha I$

Q2) Consider

P: Let $M \in R^{m\times n}$ with $m > n \geq 2$. If rank $(M) = n$, then the system of linear equations $Mx = 0$ has $x = 0$ as the only solution

Q: Let $E \in R^{n\times n}$, $n \geq 2$ be a non-zero matrix such that $E^3 = 0$. Then $I + E^2$ is a singular matrix.

Which of the following statements is TRUE?

(A) Both P and Q are TRUE

(B) Both P and Q are FALSE

(C) P is TRUE and Q is FALSE

(D) P is FALSE and Q is TRUE

Q3) The number of non-isomorphic abelian groups of order $2^2, 3^3, 5^4$ is__________

Q4) The number of subgroups of a cyclic group of order 12 is ________

Q5) The radius of convergence of the series

$$\sum_{n\geq 0} 3^{n+1} z^{2n}, \ z \in \mathbb{C}$$

is _______ (round off to TWO decimal places).

Q6) The number of zeros of the polynomial

$$2z^7 - 7z^5 + 2z^3 - z + 1$$

in the unit disc $\{z \in C : |z| < 1\}$ is _________

Q7) Consider the linear system of equations Ax = b with

$$A = \begin{pmatrix} 3 & 1 & 1 \\ 1 & 4 & 1 \\ 2 & 0 & 3 \end{pmatrix} \quad \text{and} \quad b = \begin{pmatrix} 2 \\ 3 \\ 4 \end{pmatrix}$$

Which of the following statements are TRUE?

(A) The Jacobi iterative matrix is $\begin{pmatrix} 0 & 1/4 & 1/3 \\ 1/3 & 0 & 1/3 \\ 2/3 & 0 & 0 \end{pmatrix}$

(B) The Jacobi iterative method converges for any initial vector

(C) The Gauss-Seidel iterative method converges for any initial vector

(D) The spectral radius of the Jacobi iterative matrix is less than 1

Q8) If P(x) is a polynomial of degree 5 and

$$\alpha = \sum_{i=0}^{6} P(x_i) \left(\prod_{j=0,\, j\neq i}^{6} (x_i - x_j)^{-1} \right)$$

where $x_0, x_1, \cdots, x_6$ are distinct points in the interval [2, 3], then the value of $\alpha^2 - \alpha + 1$ is ____

Q9) The maximum value of $f(x, y) = 49 - x^2 - y^2$ on the line $x + 3y = 10$ is _______

Q10) If the ordinary differential equation

$$x^2 \frac{d^2\phi}{dx^2} + x\frac{d\phi}{dx} + x^2\phi = 0, \ x > 0$$

has a solution of the form $\phi(x) = x^r \sum_{n=0}^{\infty} a_n x^n$, where a_n's are constants and $a_0 \neq 0$, then the value of $r^2 + 1$ is ______________.

Q11) The partial differential equation

$$7\frac{\partial^2 u}{\partial x^2} + 16\frac{\partial^2 u}{\partial x \partial y} + 4\frac{\partial^2 u}{\partial y^2} = 0$$

is transformed to

$$A\frac{\partial^2 u}{\partial \xi^2} + B\frac{\partial^2 u}{\partial \xi \partial \eta} + C\frac{\partial^2 u}{\partial \eta^2} = 0,$$

using $\xi = y - 2x$ and $\eta = 7y - 2x$.

Then, the value of $\frac{1}{12^3}(B^2 - 4AC)$ is ______________.

Q12) Let R[X] denote the ring of polynomials in X with real coefficients. Then, the quotient ring R[X] / $(X^4 + 4)$ is

(A) a field

(B) an integral domain, but not a field

(C) not an integral domain, but has 0 as the only nilpotent element

(D) a ring which contains non-zero nilpotent elements

Q13) Consider the following conditions on two proper non-zero ideals J_1 and J_2 of a non-zero commutative ring R

P: For any $r_1, r_2 \in R$, there exists a unique $r \in R$ such that $r - r_1 \in J_1$ and $r - r_2 \in J_2$.

Q: $J_1 + J_2 = R$

Then, which of the following statements is TRUE?

(A) P implies Q but Q does not imply P

(B) Q implies P but P does not imply Q

(C) P implies Q and Q implies P

(D) P does not imply Q and Q does not imply P

Q14) P :

Suppose that $\sum_{n=0}^{\infty} a_n x^n$ converges at $x = -3$ and diverges at $x = 6$. Then $\sum_{n=0}^{\infty} (-1)^n a_n$ converges.

Q :

The interval of convergence of the series $\sum_{n=2}^{\infty} \frac{(-1)^n x^n}{4^n \log_e n}$ is $[-4, 4]$.

Which of the following statements is TRUE?

(A) P is true and Q is true

(B) P is false and Q is false

(C) P is true and Q is false

(D) P is false and Q is true

Q15) Let

$$f_n(x) = \frac{x^2}{x^2 + (1 - nx)^2}, \; x \in [0, 1], \; n = 1, 2, 3, \cdots$$

Then, which of the following statements is TRUE?

(A) $\{f_n\}$ is not equicontinuous on [0, 1]

(B) $\{f_n\}$ is uniformly convergent on [0, 1]

(C) $\{f_n\}$ is equicontinuous on [0, 1]

(D) $\{f_n\}$ is uniformly bounded and has a subsequence converging uniformly on [0, 1]

Q16) Let X, Y and Z be Banach spaces. Suppose that T : X → Y is linear and S : Y → Z is linear, bounded and injective. In addition, if S ∘ T : X → Z is bounded, then, which of the following statements is TRUE?

(A) T is surjective

(B) T is bounded but not continuous

(C) T is bounded

(D) T is not bounded

Q17) The first derivative of a function $f \in C^\infty(-3, 3)$ is approximated by an interpolating polynomial of degree 2, using the data

$$(-1, f(-1)), \; (0, f(0)) \text{ and } (2, f(2)).$$

It is found that

$$f'(0) \approx -\frac{2}{3} f(-1) + \alpha f(0) + \beta f(2)$$

Then, the value of 1 / αβ is

(A) 3

(B) 6

(C) 9

(D) 12

Q18) Let u(x, t) be the solution of the wave equation

$$\frac{\partial^2 u}{\partial t^2} - \frac{\partial^2 u}{\partial x^2} = 0,\ 0 < x < \pi,\ t > 0,$$

with the initial conditions

$$u(x, 0) = \sin x + \sin 2x + \sin 3x,\ \frac{\partial u}{\partial t}(x, 0) = 0,\ 0 < x < \pi$$

and the boundary conditions u(0, t) = u(π, t) = 0, t ≥ 0. Then, the value of u (π / 2 , π) is
(A) −1/2
(B) 0
(C) 1/2
(D) 1

Q19) Let T : $R^2 \rightarrow R^2$ be a linear transformation defined by

T((1, 2)) = (1, 0) and T((2, 1)) = (1, 1)

For p, q ∈ R, let T^{-1} ((p, q)) = (x, y)
Which of the following statements is TRUE?
(A) x = p − q; y = 2p − q
(B) x = p + q; y = 2p − q
(C) x = p + q; y = 2p + q
(D) x = p − q; y = 2p + q

Q20) Let K denote the subset of C consisting of elements algebraic over Q. Then, which of the following statements are TRUE?
(A) No element of C\K is algebraic over Q
(B) K is an algebraically closed field
(C) For any bijective ring homomorphism f : C → C, we have f(K) = K
(D) There is no bijection between K and Q

Q21) Let X = (R, T), where T is the smallest topology on R in which all the singleton sets are closed. Then, which of the following statements are TRUE?
(A) [0, 1) is compact in X
(B) X is not first countable
(C) X is second countable
(D) X is first countable

Q22) Consider (Z, T), where T is the topology generated by sets of the form

$$A_{m,n} = \{m + nk \mid k \in Z\}$$

for m, n $\in$ Z and n $\neq$ 0. Then, which of the following statements are TRUE?

(A) (Z, T) is connected

(B) Each $A_{m,n}$ is a closed subset of (Z, T)

(C) (Z, T) is Hausdorff

(D) (Z, T) is metrizable

Q23) Three companies C_1, C_2 and C_3 submit bids for three jobs J_1, J_2 and J_3. The costs involved per unit are given in the table below:

	J_1	J_2	J_3
C_1	10	12	8
C_2	9	15	10
C_3	15	10	9

Then, the cost of the optimal assignment is _________ .

Q24) The surface area of the paraboloid $z = x^2 + y^2$ between the planes z = 0 and z = 1 is (round off to ONE decimal place).

Q25) The rate of change of f(x, y, z) = x+x cos z −y sin z +y at P_0 in the direction from P_0(2, −1, 0) to P_1(0, 1, 2) is _______.

Q26) If the Laplace equation

$$\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0,\ 1 < x < 2,\ 1 < y < 2$$

with the boundary conditions

$$\frac{\partial u}{\partial x}(1, y) = y,\ \frac{\partial u}{\partial x}(2, y) = 5,\ 1 < y < 2$$

and

$$\frac{\partial u}{\partial y}(x, 1) = \frac{\alpha x^2}{7},\ \frac{\partial u}{\partial y}(x, 2) = x,\ 1 < x < 2$$

has a solution, then the constant α is _______ .

Q27) Let u(x, y) be the solution of the first order partial differential equation

$$x\frac{\partial u}{\partial x} + (x^2 + y)\frac{\partial u}{\partial y} = u, \text{ for all } x, y \in \mathbb{R}$$

satisfying u(2, y) = y − 4, y ∈ R. Then, the value of u(1, 2) is _______ .

Q28) The optimal value for the linear programming problem
Maximize: $6x_1 + 5x_2$
subject to: $3x_1 + 2x_2 \leq 12$
$-x_1 + x_2 \leq 1$
$x_1, x_2 \geq 0$
is _______

Q29) A certain product is manufactured by plants P_1, P_2 and P_3 whose capacities are 15, 25 and 10 units, respectively. The product is shipped to markets M_1, M_2, M_3 and M_4, whose requirements are 10, 10, 10 and 20, respectively. The transportation costs per unit are given in the table below.

	M_1	M_2	M_3	M_4	
P_1	1	3	1	3	15
P_2	2	2	4	1	25
P_3	2	1	1	2	10
	10	10	10	20	

Then the cost corresponding to the starting basic solution by the Northwest-corner method is ________ .

Q30) Let M be a 3 × 3 real matrix such that $M^2 = 2M + 3I$. If the determinant of M is −9, then the trace of M equals .

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