GATE 2024
IISc Bengaluru

GRADUATE APTITUDE TEST IN ENGINEERING 2024
अभियांत्रिकी स्नातक अभिक्षमता परीक्षा २०२४
ORGANISING INSTITUTE: INDIAN INSTITUTE OF SCIENCE, BENGALURU

(This subject-specific section will be preceded by General Aptitude section in GATE 2024).

Q.1 – Q.25 Carry ONE mark each.	
Q.1	Let b be the branching factor of a search tree. If the optimal goal is reached after d actions from the initial state, in the worst case, how many times will the initial state be expanded for iterative deepening depth first search (IDDFS) and iterative Deepening A* search (IDA*)?
(A)	IDDFS - d, IDA* -d.
(B)	IDDFS - d, IDA* -b^d.
(C)	IDDFS - b^d, IDA* -d.
(D)	IDDFS - b^d, IDA* -b^d.
Q.2	Given 3 literals $A, B,$ and C, the number of models for the sentence $A \vee \neg B \vee C$ is ________ .

Q.3	Which one of the following first order logic sentences matches closest with the sentence “All students are not equal”?
(A)	$\forall x\ \exists y[student(x) \wedge student(y)] \Rightarrow \neg Equal(x, y)$
(B)	$\forall x\ \forall y[student(x) \wedge student(y)] \Rightarrow \neg Equal(x, y)$
(C)	$\forall x\ \exists y[student(x) \wedge student(y) \wedge \neg Equal(x, y)]$
(D)	$\forall x\ \forall y[student(x) \wedge student(y) \wedge \neg Equal(x, y)]$
Q.4	The mean of the observations of the first 50 observations of a process is 12. If the 51st observation is 18, then, the mean of the first 51 observations of the process, rounded off to two decimal places is ______ .
(A)	12.01
(B)	12.12
(C)	12.36
(D)	18.18

Q.5	The value of $\lim_{x \to 2} \frac{\sqrt{x}-\sqrt{2}}{x-2}$ is
(A)	0
(B)	$\sqrt{2}$
(C)	$\frac{1}{2\sqrt{2}}$
(D)	$\frac{1}{\sqrt{2}}$
Q.6	Which among the following typically reduces overfitting in a supervised machine learning algorithm? i) Increase model complexity. ii) Reduce model complexity. iii) Increase the number of training points. iv) Reduce the number of training points.
(A)	i and ii
(B)	i, ii and iii
(C)	ii and iii
(D)	i, ii, iii and iv

Q.7	A fair coin is flipped twice, and it is given that at least one tail has been observed. The probability of getting two tails is
(A)	$\frac{1}{2}$
(B)	$\frac{1}{3}$
(C)	$\frac{2}{3}$
(D)	$\frac{1}{4}$
Q.8	Given *n* particles and *m (> n)* boxes, we place each particle in one of the boxes uniformly at random. What is the probability that no box receives more than one particle?
(A)	$\frac{n!}{(m-n)!\ m^n}$
(B)	$\frac{m!}{(m-n)!\ m^n}$
(C)	$\frac{1}{m^n}$
(D)	$\frac{(m-n)!}{m!}$

Q.9	For two events A and B, $B \subset A$, which one of the following is correct?
(A)	$P(B \mid A) \geq P(B)$
(B)	$P(B \mid A) \leq P(B)$
(C)	$P(A \mid B) < 1$
(D)	$P(A \mid B) = 0$
Q.10	X is a random variable with support $[-2,2] \cup [99.5,100.5]$. The PDF of X is such that it is equal to a constant c inside its support and 0 outside. The expected value of X is ________ .
Q. 11	A binary classification dataset contains only 5% of positive instances. Which one of the following experimental design and performance measures is most suited for measuring the generalizability of a classifier trained on this dataset?
(A)	fixed training and test sets, accuracy
(B)	fixed training and test sets, area under the ROC curve
(C)	stratified cross-validation, accuracy
(D)	stratified cross-validation, area under the ROC curve

Q.12	Increasing the regularisation coefficient value in ridge regression will typically i) Increase the bias of the resulting model. ii) Decrease the bias of the resulting model. iii) Increase the variance of the resulting model. iv) Decrease the variance of the resulting model. Which of the following statements are correct?
(A)	i and iii only
(B)	i and iv only
(C)	ii and iii only
(D)	ii and iv only

Q.13	A decision tree classifier learned from a fixed training set achieves 100% accuracy on the test set. Which of the following algorithms trained using the same training set is guaranteed to give a model with 100% accuracy? i) Logistic regression. ii) An SVM with a polynomial kernel. iii) k-Nearest neighbours. iv) Naïve Bayes classifier.
(A)	i only
(B)	i and ii only
(C)	i, ii, iii and iv
(D)	iv only

Q.14	Consider two relations *R(x, y)* and *S(x,z)*. Relation *R* has 100 records, and relation *S* has 200 records. What will be the number of attributes and records of the following query? SELECT * from *R* CROSS JOIN *S*;
(A)	3 attributes, 20000 records
(B)	4 attributes, 20000 records
(C)	3 attributes, 200 records
(D)	4 attributes, 200 records
Q.15	Consider two relations *R(x, y)* and S(y), and perform the following operation *R(x,y)* DIVIDE *S(Y)* If X is the relation returned by the above operation, which of the following option(s) is/are always TRUE?
(A)	$\|X\| \leq \|R\|$
(B)	$\|X\| \leq \|S\|$
(C)	$\|X\| \leq \|R\|$ AND $\|X\| \leq \|S\|$
(D)	$\|X\| > \|R\|$

Q.16	Which of the following statements is/are TRUE?
(A)	Every relation in BCNF is also in 3NF.
(B)	Every relation with two attributes is in BCNF.
(C)	No relation can be in both BCNF and 3NF.
(D)	Every relation in 3NF is also in BCNF.
Q.17	For matrix $H = \begin{bmatrix} 9 & -2 \\ -2 & 6 \end{bmatrix}$, one of the eigenvalues is 5. The other eigenvalue is
(A)	12
(B)	10
(C)	8
(D)	6

Q. 18	Two non-zero vectors **x** and **y** are perpendicular if
(A)	$\mathbf{x}^{T}\mathbf{y} = 0$
(B)	$\mathbf{x}^{T}\mathbf{y} > 0$
(C)	$\mathbf{x}^{T}\mathbf{y} < 0$
(D)	$\mathbf{x}^{T}\mathbf{y} \neq 0$
Q. 19	The function $f(x) = 1 + x + x^2$ has a
(A)	Saddle point at $x = -0.5$
(B)	Maxima at $x = -0.5$
(C)	Minima at $x = -0.5$
(D)	$f'(x) \neq 0$ at $x = -0.5$

Q. 20 What is the Pearson’s correlation coefficient between *x* and *y* for the data in the table below? (rounded off to the first decimal place)

x	y
–6	6.4
2	4.7
0.2	8
7	2
–4	3.4

(A) –0.5

(B) 0.5

(C) 0.3

(D) –0.3

Q.21 The worst-case running times of Insertion sort, Merge sort and Quick sort respectively are

(A) $\mathrm{O}(n \log n)$, $\mathrm{O}(n^2)$, $\mathrm{O}(n^2)$

(B) $\mathrm{O}(n^2)$, $\mathrm{O}(n \log n)$, $\mathrm{O}(n \log n)$

(C) $\mathrm{O}(n^2)$, $\mathrm{O}(n \log n)$, $\mathrm{O}(n^2)$

(D) $\mathrm{O}(n^2)$, $\mathrm{O}(n^2)$, $\mathrm{O}(n \log n)$

Q.22 Consider the following program.

```
def func(n):

  if n <= 1:
    return n
  else:
    return 3 * func(n - 3) - 3 * func(n - 2)
```

The running time of the above function is

(A) O(n)

(B) O(n^2)

(C) O($n \log n$)

(D) O(2^n)

Q.23 Which one of the following equations correctly describes the recurrence relation for the standard binary search algorithm on a sorted array of *n* numbers where *c* is a constant?

(A) $T(n) = 2T(n/2) + c$

(B) $T(n) = T(n/2)$

(C) $T(n) = T(n-1) + c$

(D) $T(n) = T(n/2) + c$

Q.24

Consider the following python program

```
def func(A, n, m):
  s = A[0]
  for i in range(1, n-1):
        m = m * s + A[i]
  return m
```

Let Z be an array of 10 elements with Z[i] = 2 for all i such that 0<=i<=9.
The value returned by func(Z,10,2) is ________.

Q.25

Two eigenvalues of 3 x 3 real matrix **X** are (1 + *i*) and 2.
The determinant of the matrix is _____.

Q.26 – Q.55 Carry TWO marks each.

Q.26

Given the following relation instances

X	Y	Z
1	4	2
1	5	3
1	4	3
1	5	2
3	2	1

Which of the following conditions is/are TRUE?

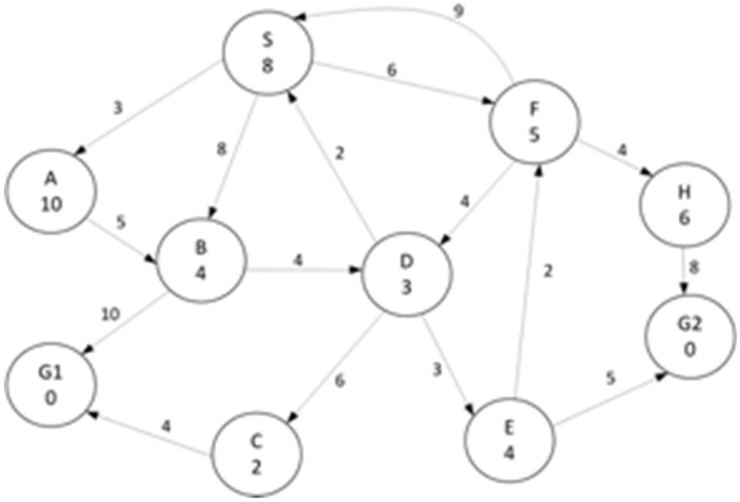
(A) XY → Z and Z → Y

(B) YZ → X and X → Y

(C) Y → X and X → Y

(D) XZ → Y and Y → X

Q.27 Consider the search space depicted in the figure below.



S is the initial state. G1 and G2 are two states that satisfy the goal test. The cost of traversing from one state to another is depicted by the numerical value close to the edge connecting the two states. The estimated cost to the goal is reported inside the states. Use alphabetical order of nodes to break ties. Which goal state is reached if you perform A* (graph) search? What is the largest value that the heuristic function can take for node A while still being admissible?

(A) G1 and 16

(B) G1 and 15

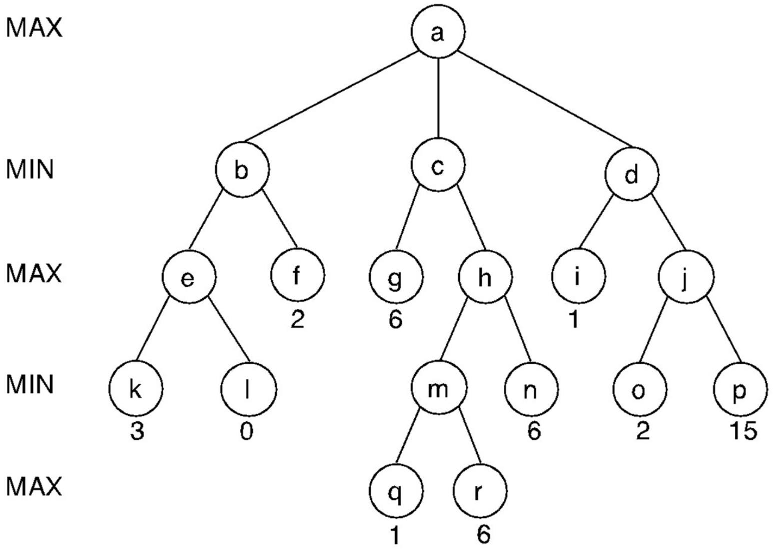
(C) G2 and 16

(D) G2 and 15

Q.28	Given a K-class dataset containing N points, where sample points are described using D discrete features with each feature capable of taking V values, how many parameters need to be estimated for Naïve Bayes Classifier?
(A)	$V^D K$
(B)	K^{V^D}
(C)	$VDK + K$
(D)	$K(V + D)$
Q.29	A maximum margin linear SVM (SVM1) is learned for a binary classification task. Another maximum margin linear SVM (SVM2) is trained for the same task using the same training set but with one of the non-support vectors of SVM1 removed. Which one of the following statements is TRUE?
(A)	Margin of SVM1 > Margin of SVM2
(B)	Margin of SVM2 > Margin of SVM1
(C)	Margin of SVM1 = Margin of SVM2
(D)	No conclusion can be drawn between the margins of SVM1 and SVM2

Q.30	For perfectly spherical 2D data centered at the origin, which **all** of the following pairs of vectors are possible pairs of principal components? i) (1, 0) and (0, 1) ii) (0, -1) and (-1, 0) iii) (1, 1) and (1, 0)
(A)	i only
(B)	i and iii only
(C)	i, ii, and iii only
(D)	i and ii only

Q.31 Consider the game tree shown below.



The value below each node is the output of the utility function. The subtrees rooted at which of these nodes will be pruned because of alpha-beta pruning?

(A) m and j

(B) r and j

(C) h and p

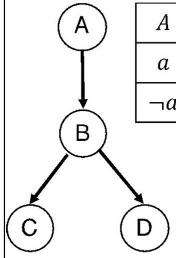
(D) no nodes are pruned

Q.32

Consider the Bayes Net containing four Boolean random variables $(A, B, C\ , D)$, with the following convention:

$A = True \Rightarrow A = a, and\ A = False \Rightarrow A = \neg a$

and similarly for the other variables. The conditional probability tables for the nodes in the network are also indicated in the figure. The following samples were generated through likelihood weighting:



A	$P(A)$
a	1/5
$\neg a$	4/5

A	B	$P(B\|A)$
a	b	1/5
$\neg a$	b	1/2
a	$\neg b$	4/5
$\neg a$	$\neg b$	1/2

B	C	$P(C\|B)$
b	c	1/4
$\neg b$	c	3/4
b	$\neg c$	2/5
$\neg b$	$\neg c$	3/5

B	D	$P(D\|B)$
b	d	1/2
$\neg b$	d	4/5
b	$\neg d$	1/2
$\neg b$	$\neg d$	1/5

$s_1: (\neg a, \neg b, \neg c, \neg d); s_2: (\neg a, b, \neg c, \neg d);\ s_3: (\neg a, \neg\ b, \neg c, d); s_4: (\neg a, b, \neg c, d)$

Estimate the likelihood weight of each sample and thereby estimate $P(b|\neg a, \neg c)$

(A) $s_1: 0.48, s_2: 0.32, s_3: 0.48, s_4: 0.32, P(b|\neg a, \neg c) = 0.4$

(B) $s_1: 0.48, s_2: 0.32, s_3: 0.48, s_4: 0.32, P(b|\neg a, \neg c) = 0.64$

(C) $s_1: 0.32, s_2: 0.48, s_3: 0.48, s_4: 0.32, P(b|\neg a, \neg c) = 0.64$

(D) $s_1: 0.48, s_2: 0.32, s_3: 0.32, s_4: 0.32, P(b|\neg a, \neg c) = 0.4$

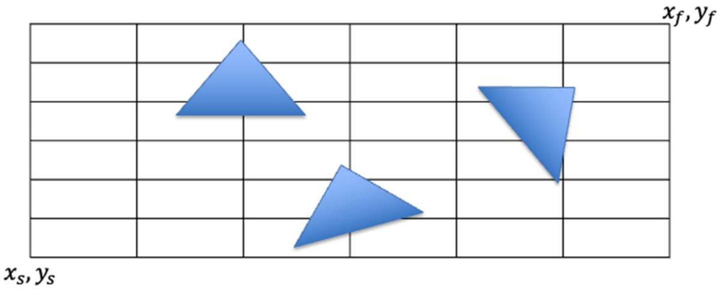
Q.33	X is a uniformly distributed random variable from 0 to 1 $f(x) = \begin{cases} 1, & 0 \leq x \leq 1 \\ 0, & otherwise \end{cases}$ The variance of X is
(A)	$\frac{1}{2}$
(B)	$\frac{1}{3}$
(C)	$\frac{1}{4}$
(D)	$\frac{1}{12}$
Q.34	The function $f(x) = 1 + 2x + 3x^2 + \cdots + 2026x^{2025}$. Which of the following statement is true?
(A)	*f(x)* has a global minimum.
(B)	*f(x)* has a global maximum.
(C)	*f(x)* does not have a global minimum.
(D)	*f(x)* is not continuous.

Q.35	Consider the following statements: (P): A smooth twice differentiable function can have a global minimum. (Q): All smooth twice differentiable functions have a global minimum.
(A)	P and Q are true.
(B)	P is true and Q is false.
(C)	P is false and Q is true.
(D)	P and Q are false.
Q.36	Consider the following joint distribution of random variables *X* and *Y*: $f(x,y) = \begin{cases} \frac{x(1+3y^2)}{4}, & 0 \leq x \leq 2, 0 \leq y \leq 1 \\ 0, & otherwise \end{cases}$ Which of the following is the conditional pdf of *X*|*Y*?
(A)	$\frac{x}{4}$
(B)	$\frac{y}{4}$
(C)	$\frac{x}{2}$
(D)	$\frac{y}{2}$

Q.37	For the matrix $H = \begin{bmatrix} 3 & -1 \\ -1 & 3 \end{bmatrix}$, one of the eigenvectors is $[-1\ -1\,]^T$. The other eigenvector is
(A)	$[1\ -1\,]^T$
(B)	$[1\ \ 1\,]^T$
(C)	$[1\ \ 0\,]^T$
(D)	$[0\ \ 1\,]^T$
Q.38	The following statements are made regarding a matrix $\mathbf{A}_{m\times n}$. P. The column space is orthogonal to the row space. Q. The column space is orthogonal to the left null space. R. The row space is orthogonal to the null space. T. The null space is orthogonal to the left null space. Which of the following statements are true?
(A)	P and Q only
(B)	P and R only
(C)	Q and R only
(D)	P and T only

Q.39	Consider a matrix $\begin{bmatrix} 0 & 1 & 0 \\ a & 2 & d \\ b & 3 & c \end{bmatrix}$. The matrix cannot have rank
(A)	0
(B)	1
(C)	2
(D)	3
Q.40	A file with 100,000 records is indexed with B+ tree. If the size of a memory block is 2K bytes, the size of a key is 4 bytes, the size of a pointer is 4 bytes, the minimum possible height of the B+ tree index is ______. Height is always greater than or equal to 1. Hints: No records are stored in the nodes, only keys are stored. The sizes of the pointers are same, irrespective of whether they point to a node or a record.
Q.41	Consider a schema R(A, B, C, D, E, F) and functional dependencies A → B, C → D, and E → F. The number of superkeys of this schema is _____.

Q.42	Given the dataset: (1, 1), (3, 3), (4, 4), (5, 5), (6, 6), (9, 9), (0, 3), (3, 0) and assuming the initial centroids for ($K = 3$ – means clustering) to be $C_1 = (3,3), C_2 = (5,5)\ and\ C_3 = (6,6)$. One iteration of the Expectation Maximization Algorithm for K-means clustering, will update C_3 to (a, a). The value of a is ____ .
Q.43	Consider a Multi-Layer Perceptron (MLP) model with one hidden layer and one output layer. The hidden layer has 10 neurons, and the output layer has 3 neurons. The input to the MLP is a 5-dimensional vector. Each neuron is connected to every neuron in the previous layer, and a bias term is included for each neuron. The activation function used is the sigmoid function. The total number of trainable parameters in this MLP model is ______.
Q.44	A company is manufacturing a product at the rate of P units per day. The cost per unit in Rs is $C = 230 + 0.1P + 9000/P$. The selling prices per unit is Rs. 300. The production level minimizing the cost per unit and the total profit per day, respectively, are
(A)	290, 3000
(B)	150, 2500
(C)	290, 2500
(D)	150, 3000

Q.45	A class contains 60% of students who are incapable of changing their opinions about anything, and 40% of students are changing their minds at random, with probability 0.3, between subsequent votes on the same issue. Then, the probability of a student randomly chosen voted twice in the same way is _______.
Q.46	Consider the grid world shown in the figure below.  An agent is planning to move from the starting location (x_s, y_s) to the final location (x_f, y_f). The obstacles along the path are triangular in form. Consider the following heuristic functions to conduct A* search. $-$ h_c assumes the obstacles are the smallest circles circumscribing the triangles. $-$ h_r assumes the obstacles are smallest rectangles circumscribing the triangles. $-$ $h_c{}'$ assumes the obstacles are largest circles inscribed in the triangles. $-$ $h_r{}'$ assumes the obstacles are largest rectangles inscribed the triangles. Which of the following statement(s) is(are) true?
(A)	h_c is an admissible heuristic
(B)	h_r is an admissible heuristic
(C)	$h_c{}'$ is an admissible heuristic
(D)	$h_r{}'$ is an admissible heuristic

Q.47	Let {O1, O2, O3, O4} represent the possible outcomes of a random experiment, with Pr({O1}) = Pr({O2}) = Pr({O3}) = Pr({O4}). Consider the following events: P = {O1,O2}, Q = {O2,O3}, R = {O3,O4}, S = {O1,O2,O3}. Then, which of the following statements are true?
(A)	P and Q are independent
(B)	P and Q are not independent
(C)	R and S are independent
(D)	Q and S are not independent
Q.48	Consider the matrix **X** whose eigenvalues are 1, -1 and 3. The Trace of $\mathbf{X}^3 - 3\mathbf{X}^2$ is _______.
Q.49	What is the output of the following python program?

```
i = 1
j = 1
for i in range(1, 11):
  if i % 3 != 0:
    j += 2
    continue
  if j % 3 == 0:
    break
print(i + j)
```

(A)	3
(B)	5
(C)	12
(D)	15

Q.50	Assume that S is a stack and Q1 and Q2 are two Queues which support the Enqueue and Dequeue operations. Consider the following pseudo code for implementing the Pop and Push operation on S. [Note: Swap(x,y) exchanges the two queues x and y.]

```
Push(S,x)
    A(Q2,x)
    while(Q1 not empty)
        B(Q2, C(Q1));
    Swap(Q1, Q2)

Pop(S)
    return(D(Q1))
```

Which of the following options for the functions A, B, C, and D correspond to correctly implementing the Push and Pop operations on the stack S?

(A)	A,B - Enqueue C,D - Dequeue
(B)	A,C - Enqueue B, D - Dequeue
(C)	A,C - Dequeue B,D- Enqueue
(D)	A,D - Enqueue B,C - Dequeue

Q.51

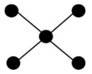
Consider the following python program.

```
   import math
    def fun(a, b, d):
     n1 = 0
     n2 = 0
     flag = 1
     for i in range(d):
       n1 = n1 + (a[i] * a[i])
       n2 = n2 + (b[i] * b[i])
     for i in range(d):
       a[i] = a[i]/math.sqrt(n1)
       b[i] = b[i]/math.sqrt(n2)
     for i in range(d):
       if a[i] != b[i]:
         flag = 0
         break
return flag
```

For which of the following inputs does the above algorithm produce 1 as an output?

(P) a = [1,2,3,4]; b = [3,4,5,6], d = 4
(Q) a = [1,2,3,4]; b = [2,4,6,8], d = 4
(R) a = [1,2,3,4]; b = [10,20,30,40], d = 4
(S) a = [1,2,3,4]; b = [1.1,2.1,3.1,4.1], d = 4

(A)	P, Q, R, S
(B)	Q, R, S only
(C)	Q, R only
(D)	R, S only

Q.52	Consider the following undirected graph on 5 nodes  Assume you are performing breadth first search on this graph using a queue data structure. How many unique breadth first orderings are possible?
(A)	9
(B)	24
(C)	48
(D)	120
Q.53	Let S^2 be the variance of a random sample of size $n > 1$ from a normal population with an unknown mean μ and an unknown finite variance σ^2. Consider the following statements: (I) S^2 is an unbiased estimator of σ^2, and S is an unbiased estimator of σ. (II) $(n-1/n)\ S^2$ is a maximum likelihood estimator of σ^2, and $\sqrt{\frac{n-1}{n}}S$ is a maximum likelihood estimator of σ. Which of the above statements is true?
(A)	(I) only
(B)	(II) only
(C)	Both (I) and (II)
(D)	Neither (I) nor (II)

Q.54 The value of the real variable $x \geq 0$, which maximizes the function $f(x) = x^e e^{-x}$ is_____. (rounded off to two decimal places)

Q.55 Consider the following relational schema:

employee(empId,empName,empDept)

customer(custId,custName,salesRepId,rating)

salesRepId is a foreign key referring to empId of the employee relation. Assume that each employee makes a sale to at least one customer. What does the following query return?

```
SELECT empName

FROM employee E

WHERE NOT EXISTS (SELECT custId

                                        FROM customer C

                                        WHERE C.salesRepId = E.empId

                                        AND C.rating <> ’GOOD’);
```

(A) Names of all the employees with at least one of their customers having a ‘GOOD’ rating.

(B) Names of all the employees with at most one of their customers having a ‘GOOD’ rating.

(C) Names of all the employees with none of their customers having a ‘GOOD’ rating.

(D) Names of all the employees with all their customers having a ‘GOOD’ rating.

END OF THE QUESTION PAPER