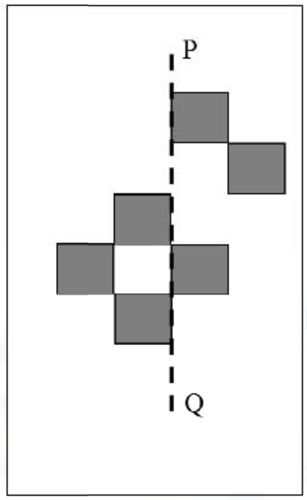
General Aptitude (GA)

Q.1 – Q.5 Multiple Choice Question (MCQ), carry ONE mark each (for each wrong answer: – 1/3).

Q.1	**The current population of a city is 11,02,500. If it has been increasing at the rate of 5% per annum, what was its population 2 years ago?**
(A)	9,92,500
(B)	9,95,006
(C)	10,00,000
(D)	12,51,506

Q.2	***p* and *q* are positive integers and** $\frac{p}{q}+\frac{q}{p}=3$, **then,** $\frac{p^2}{q^2}+\frac{q^2}{p^2}=$
(A)	3
(B)	7
(C)	9
(D)	11

Q.3	 **The least number of squares that must be added so that the line P-Q becomes the line of symmetry is ________**
(A)	4
(B)	3
(C)	6
(D)	7

Q.4	***Nostalgia* is to *anticipation* as _______ is to ________** **Which one of the following options maintains a similar logical relation in the above sentence?**
(A)	Present, past
(B)	Future, past
(C)	Past, future
(D)	Future, present

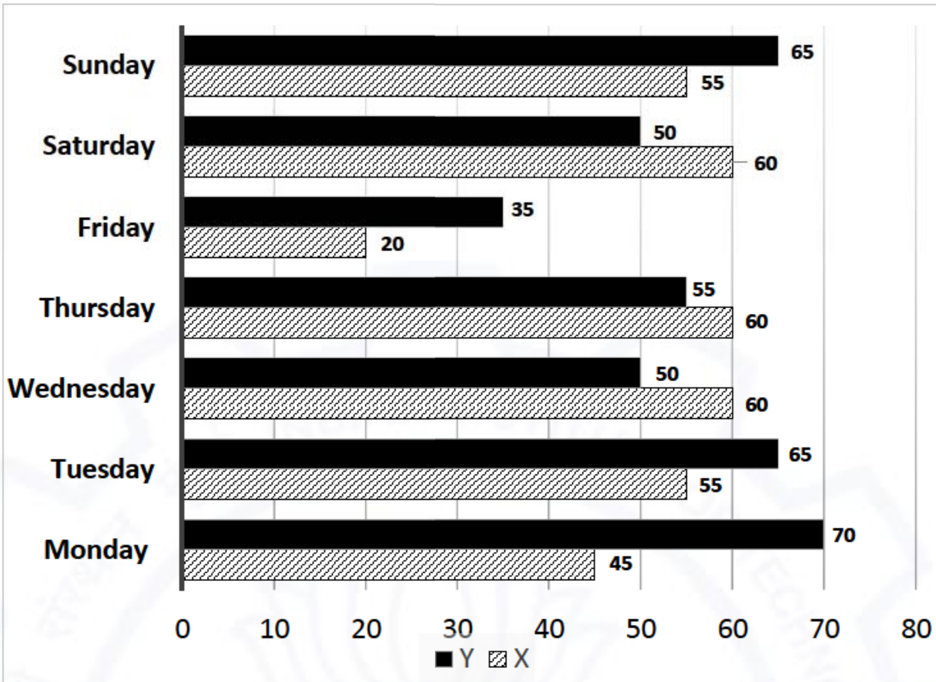
Q.5	**Consider the following sentences:** **(i) I woke up from sleep.** **(ii) I woked up from sleep.** **(iii) I was woken up from sleep.** **(iv) I was wokened up from sleep.** **Which of the above sentences are grammatically CORRECT?**
(A)	(i) and (ii)
(B)	(i) and (iii)
(C)	(ii) and (iii)
(D)	(i) and (iv)

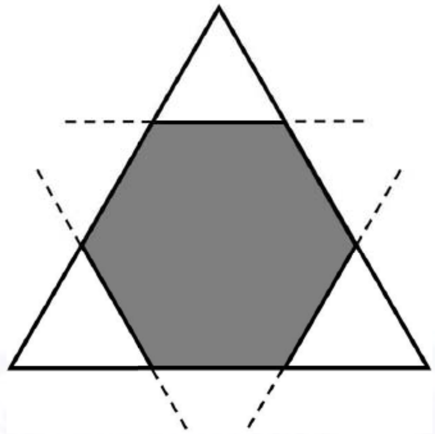
Q. 6 – Q. 10 Multiple Choice Question (MCQ), carry TWO marks each (for each wrong answer: – 2/3).

Q.6	**Given below are two statements and two conclusions.** **Statement 1: All purple are green.** **Statement 2: All black are green.** **Conclusion I: Some black are purple.** **Conclusion II: No black is purple.** **Based on the above statements and conclusions, which one of the following options is logically CORRECT?**
(A)	Only conclusion I is correct.
(B)	Only conclusion II is correct.
(C)	Either conclusion I or II is correct.
(D)	Both conclusion I and II are correct.

Q.7	**Computers are ubiquitous. They are used to improve efficiency in almost all fields from agriculture to space exploration. Artificial intelligence (AI) is currently a hot topic. AI enables computers to learn, given enough training data. For humans, sitting in front of a computer for long hours can lead to health issues.** **Which of the following can be deduced from the above passage?** (i) **Nowadays, computers are present in almost all places.** (ii) **Computers cannot be used for solving problems in engineering.** (iii) **For humans, there are both positive and negative effects of using computers.** (iv) **Artificial intelligence can be done without data.**
(A)	(ii) and (iii)
(B)	(ii) and (iv)
(C)	(i), (iii) and (iv)
(D)	(i) and (iii)

Q.8	**Consider a square sheet of side 1 unit. In the first step, it is cut along the main diagonal to get two triangles. In the next step, one of the cut triangles is revolved about its short edge to form a solid cone. The volume of the resulting cone, in cubic units, is ________**
(A)	$\frac{\pi}{3}$
(B)	$\frac{2\pi}{3}$
(C)	$\frac{3\pi}{2}$
(D)	3π

Q.9	 **The number of minutes spent by two students, X and Y, exercising every day in a given week are shown in the bar chart above.** **The number of days in the given week in which one of the students spent a minimum of 10% more than the other student, on a given day, is**
(A)	4
(B)	5
(C)	6
(D)	7

Q.10	 **Corners are cut from an equilateral triangle to produce a regular convex hexagon as shown in the figure above.** **The ratio of the area of the regular convex hexagon to the area of the original equilateral triangle is**
(A)	2 : 3
(B)	3 : 4
(C)	4 : 5
(D)	5 : 6

Statistics (ST)

Q.1 – Q.9 Multiple Choice Question (MCQ), carry ONE mark each (for each wrong answer: – 1/3).

Q.1	**Let X be a non-constant positive random variable such that $E(X) = 9$. Then which one of the following statements is true?**
(A)	$E\left(\frac{1}{X+1}\right) > 0.1$ and $P(X \geq 10) \leq 0.9$
(B)	$E\left(\frac{1}{X+1}\right) < 0.1$ and $P(X \geq 10) \leq 0.9$
(C)	$E\left(\frac{1}{X+1}\right) > 0.1$ and $P(X \geq 10) > 0.9$
(D)	$E\left(\frac{1}{X+1}\right) < 0.1$ and $P(X \geq 10) > 0.9$

Q.2	**Let $\{W(t)\}_{t\geq 0}$ be a standard Brownian motion. Then the variance of $W(1)W(2)$ equals**
(A)	1
(B)	2
(C)	3
(D)	4

Q.3	**Let $X_1, X_2, \ldots, X_n$ be a random sample of size $n\ (\geq 2)$ from a distribution having the probability density function** $$f(x;\theta) = \begin{cases} \frac{1}{\theta}\, e^{-\frac{x-\theta}{\theta}}, & x > \theta, \\ 0, & \text{otherwise,} \end{cases}$$ where $\theta \in (0, \infty)$. Then the method of moments estimator of θ equals
(A)	$\frac{1}{2n}\sum_{i=1}^{n} X_i$
(B)	$\frac{2}{n}\sum_{i=1}^{n} X_i$
(C)	$\frac{1}{n}\sum_{i=1}^{n} X_i$
(D)	$\frac{n}{\sum_{i=1}^{n} X_i}$

Q.4	**Let $\{x_1, x_2, \ldots, x_n\}$ be a realization of a random sample of size $n\ (\geq 2)$ from a $N(\mu, \sigma^2)$ distribution, where $-\infty < \mu < \infty$ and $\sigma > 0$. Which of the following statements is/are true?** P : 95% confidence interval of μ based on $\{x_1, x_2, \ldots, x_n\}$ is unique when σ is known. Q : 95% confidence interval of μ based on $\{x_1, x_2, \ldots, x_n\}$ is NOT unique when σ is unknown.
(A)	P only
(B)	Q only
(C)	Both P and Q
(D)	Neither P nor Q

Q.5	**Let $X_1, X_2, \dots, X_n$ be a random sample of size $n\,(\geq 2)$ from a $N(0, \sigma^2)$ distribution. For a given $\sigma > 0$, let f_σ denote the joint probability density function of $(X_1, X_2, \dots, X_n)$ and $S = \{f_\sigma : \sigma > 0\}$. Let $T_1 = \sum_{i=1}^{n} X_i^2$ and $T_2 = \left(\frac{1}{n}\sum_{i=1}^{n} X_i\right)^2$. For any positive integer ν and any $\alpha \in (0,1)$, let $\chi^2_{\nu,\alpha}$ denote the $(1-\alpha)$-th quantile of the central chi-square distribution with ν degrees of freedom. Consider testing $H_0: \sigma = 1$ against $H_1: \sigma > 1$ at level α. Then which one of the following statements is true?**
(A)	S has a monotone likelihood ratio in T_1 and H_0 is rejected if $T_1 > \chi^2_{n,\alpha}$
(B)	S has a monotone likelihood ratio in T_1 and H_0 is rejected if $T_1 > \chi^2_{n,1-\alpha}$
(C)	S has a monotone likelihood ratio in T_2 and H_0 is rejected if $T_2 > \chi^2_{n,\alpha}$
(D)	S has a monotone likelihood ratio in T_2 and H_0 is rejected if $T_2 > \chi^2_{n,1-\alpha}$

Q.6	**Let X and Y be two random variables such that $p_{11}+p_{10}+p_{01}+p_{00}=1$, where $p_{ij}=P(X=i,Y=j)$, $i,j=0,1$. Suppose that a realization of a random sample of size 60 from the joint distribution of (X,Y) gives $n_{11}=10,\ n_{10}=20,\ n_{01}=20$ and $n_{00}=10$, where n_{ij} denotes the frequency of (i,j) for $i,j=0,1$. If the chi-square test of independence is used to test** **$H_0: p_{ij}=p_{i\cdot}p_{\cdot j}$ for $i,j=0,1$ against $H_1: p_{ij}\neq p_{i\cdot}p_{\cdot j}$ for at least one pair (i,j),** **where $p_{i\cdot}=p_{i0}+p_{i1}$ and $p_{\cdot j}=p_{0j}+p_{1j}$, then which one of the following statements is true?**
(A)	Under H_0, the test statistic follows central chi-square distribution with one degree of freedom and the observed value of the test statistic is $\frac{20}{3}$
(B)	Under H_0, the test statistic follows central chi-square distribution with three degrees of freedom and the observed value of the test statistic is $\frac{20}{3}$
(C)	Under H_0, the test statistic follows central chi-square distribution with one degree of freedom and the observed value of the test statistic is $\frac{16}{3}$
(D)	Under H_0, the test statistic follows central chi-square distribution with three degrees of freedom and the observed value of the test statistic is $\frac{16}{3}$

Q.7	**Let the joint distribution of (X,Y) be bivariate normal with mean vector $\begin{pmatrix}0\\0\end{pmatrix}$ and variance-covariance matrix $\begin{pmatrix}1 & \rho\\ \rho & 1\end{pmatrix}$, where $-1<\rho<1$. Let $\Phi_\rho(0,0)=P(X\leq 0,Y\leq 0)$. Then the Kendall's τ coefficient between X and Y equals**
(A)	$4\Phi_\rho(0{,}0)-1$
(B)	$4\Phi_\rho(0{,}0)$
(C)	$4\Phi_\rho(0{,}0)+1$
(D)	$\Phi_\rho(0{,}0)$

Q.8	**Consider the simple linear regression model** $$Y_i = \beta_0 + \beta_1 x_i + \epsilon_i,\ \ i = 1, 2, \ldots, n\ \ (n \geq 3),$$ **where β_0 and β_1 are unknown parameters and ϵ_i's are independent and identically distributed random variables with mean zero and finite variance $\sigma^2 > 0$. Suppose that $\widehat{\beta_0}$ and $\widehat{\beta_1}$ are the ordinary least squares estimators of β_0 and β_1, respectively. Define $\bar{x} = \frac{1}{n}\sum_{i=1}^{n} x_i$, $S_1 = \sum_{i=1}^{n}(x_i - \bar{x})^2$ and $S_2 = \sum_{i=1}^{n} y_i(x_i - \bar{x})$, where y_i is the observed value of Y_i, $i = 1, 2, \ldots, n$. Then for a real constant c, the variance of $\widehat{\beta_0} + c$ is**
(A)	$\sigma^2\left(\frac{1}{n} + \frac{\bar{x}^2}{S_2}\right)$
(B)	$\sigma^2\left(\frac{1}{n} + \frac{\bar{x}^2}{S_1}\right)$
(C)	$\frac{\sigma^2}{n}$
(D)	$\sigma^2\left(\frac{1}{n} + \frac{\bar{x}^2}{S_2}\right) + c^2$

Q.9	**Let $\underline{X_1}, \underline{X_2}, \underline{X_3}, \underline{Y_1}, \underline{Y_2}, \underline{Y_3}$ and $\underline{Y_4}$ be independent random vectors such that $\underline{X_i}$ follows $N_4(\underline{0}, \Sigma_1)$ distribution for $i = 1, 2, 3$, and $\underline{Y_j}$ follows $N_4(\underline{0}, \Sigma_2)$ distribution for $j = 1, 2, 3, 4$, where Σ_1 and Σ_2 are positive definite matrices. Further, let** $$Z = \Sigma_1^{-1/2} X X^T \Sigma_1^{-1/2} + \Sigma_2^{-1/2} Y Y^T \Sigma_2^{-1/2},$$ **where $X = [\underline{X_1}\ \underline{X_2}\ \underline{X_3}]$ is a 4×3 matrix, $Y = [\underline{Y_1}\ \underline{Y_2}\ \underline{Y_3}\ \underline{Y_4}]$ is a 4×4 matrix and X^T and Y^T denote transposes of X and Y, respectively. If $W_m(n, \Sigma)$ denotes a Wishart distribution of order m with n degrees of freedom and variance-covariance matrix Σ and I_n denotes the $n \times n$ identity matrix, then which one of the following statements is true?**
(A)	Z follows $W_4(7, I_4)$ distribution
(B)	Z follows $W_4(4, I_4)$ distribution
(C)	Z follows $W_7(4, I_7)$ distribution
(D)	Z follows $W_7(7, I_7)$ distribution

Q.10 – Q.25 Numerical Answer Type (NAT), carry ONE mark each (no negative marks).

Q.10	$\lim_{n\to\infty}\left(2^n + n2^n \sin^2\frac{n}{2}\right)^{\frac{1}{2n-n\cos\frac{1}{n}}}$ equals ________

Q.11	Let $$I = 4\int_{\frac{1}{\sqrt{2}}}^{1}\int_{\sqrt{1-x^2}}^{x}\frac{1}{\sqrt{x^2+y^2}}\,dy\,dx$$ Then the value of $e^{I+\pi}$ equals ________ (round off to 2 decimal places).

Q.12	Let $A = \begin{bmatrix}0 & 0 & 1\\ 1 & 0 & 0\\ 0 & 1 & 0\end{bmatrix}$ and I_3 be the 3×3 identity matrix. Then the nullity of $5A(I_3 + A + A^2)$ equals ________

Q.13	Let A be the 2×2 real matrix having eigenvalues 1 and -1, with corresponding eigenvectors $\begin{bmatrix}\frac{\sqrt{3}}{2}\\ \frac{1}{2}\end{bmatrix}$ and $\begin{bmatrix}\frac{-1}{2}\\ \frac{\sqrt{3}}{2}\end{bmatrix}$, respectively. If $A^{2021} = \begin{bmatrix}a & b\\ c & d\end{bmatrix}$, then $a+b+c+d$ equals ________ (round off to 2 decimal places).

Q.14	Let A and B be two events such that $P(B) = \frac{3}{4}$ and $P(A\cup B^c) = \frac{1}{2}$. If A and B are independent, then $P(A)$ equals ________ (round off to 2 decimal places).

Q.15	A fair die is rolled twice independently. Let X and Y denote the outcomes of the first and second roll, respectively. Then $E(X+Y \mid (X-Y)^2=1)$ equals ________

Q.16	Let X be a random variable having distribution function $$F(x)=\begin{cases}0, & x<1,\\ \frac{a}{2}, & 1\le x<2,\\ \frac{c}{6}, & 2\le x<3,\\ 1, & x\ge 3,\end{cases}$$ where a and c are appropriate constants. Let $A_n=\left[1+\frac{1}{n},\ 3-\frac{1}{n}\right], n\ge 1$, and $A=\cup_{i=1}^{\infty}A_i$. If $P(X\le 1)=\frac{1}{2}$ and $E(X)=\frac{5}{3}$, then $P(X\in A)$ equals ________ (round off to 2 decimal places).

Q.17	If the marginal probability density function of the k^{th} order statistic of a random sample of size 8 from a uniform distribution on $[0,2]$ is $$f(x)=\begin{cases}\frac{7}{32}x^6(2-x), & 0<x<2,\\ 0, & \text{otherwise},\end{cases}$$ then k equals ________

Q.18	For $\alpha>0$, let $\left\{X_n^{(\alpha)}\right\}_{n\ge 1}$ be a sequence of independent random variables such that $$P\left(X_n^{(\alpha)}=1\right)=\frac{1}{n^{2\alpha}}=1-P\left(X_n^{(\alpha)}=0\right).$$ Let $S=\{\alpha>0:\ X_n^{(\alpha)}$ converges to 0 almost surely as $n\to\infty\}$. Then the infimum of S equals ________ (round off to 2 decimal places).

Q.19	**Let $\{X_n\}_{n\geq 1}$ be a sequence of independent and identically distributed random variables each having uniform distribution on $[0,2]$. For $n \geq 1$, let** $$Z_n = -\log_e\left(\prod_{i=1}^{n}(2-X_i)\right)^{\frac{1}{n}}.$$ **Then, as $n \to \infty$, the sequence $\{Z_n\}_{n\geq 1}$ converges almost surely to _________ (round off to 2 decimal places).**

Q.20	**Let $\{X_n\}_{n\geq 0}$ be a time-homogeneous discrete time Markov chain with state space $\{0,1\}$ and transition probability matrix** $$\begin{bmatrix} 0.25 & 0.75 \\ 0.75 & 0.25 \end{bmatrix}.$$ **If $P(X_0 = 0) = P(X_0 = 1) = 0.5$, then** $$\sum_{k=1}^{100} E\left[(X_{2k})^{2k}\right]$$ **equals __________**

Q.21	**Let $\{0,2\}$ be a realization of a random sample of size 2 from a binomial distribution with parameters 2 and p, where $p \in (0,1)$. To test $H_0: p = \frac{1}{2}$ against $H_1: p \neq \frac{1}{2}$, the observed value of the likelihood ratio test statistic equals __________ (round off to 2 decimal places).**

Q.22	**Let X be a random variable having the probability density function** $$f(x) = \begin{cases} \frac{3}{13}(1-x)(9-x), & 0 < x < 1, \\ 0, & \text{otherwise}. \end{cases}$$ **Then $\frac{4}{3}E[X(X^2 - 15X + 27)]$ equals __________ (round off to 2 decimal places).**

Q.23	Let (Y, X_1, X_2) be a random vector with mean vector $\begin{pmatrix}5\\2\\0\end{pmatrix}$ and variance-covariance matrix $\begin{bmatrix}10 & 0.5 & -0.5\\0.5 & 7 & 1.5\\-0.5 & 1.5 & 2\end{bmatrix}$. Then the value of the multiple correlation coefficient between Y and its best linear predictor on X_1 and X_2 equals __________ (round off to 2 decimal places).

Q.24	Let $\underline{X_1}$, $\underline{X_2}$ and $\underline{X_3}$ be a random sample from a bivariate normal distribution with unknown mean vector $\underline{\mu}$ and unknown variance-covariance matrix Σ, which is a positive definite matrix. The p-value corresponding to the likelihood ratio test for testing $H_0: \underline{\mu} = \underline{0}$ against $H_1: \underline{\mu} \neq \underline{0}$ based on the realization $\left\{\begin{pmatrix}1\\2\end{pmatrix}, \begin{pmatrix}4\\-2\end{pmatrix}, \begin{pmatrix}-5\\0\end{pmatrix}\right\}$ of the random sample equals __________ (round off to 2 decimal places).

Q.25	Let $Y_i = \alpha + \beta x_i + \epsilon_i$, $i = 1, 2, 3$, where x_i's are fixed covariates, α and β are unknown parameters and ϵ_i's are independent and identically distributed random variables with mean zero and finite variance. Let $\widehat{\alpha}$ and $\widehat{\beta}$ be the ordinary least squares estimators of α and β, respectively. Given the following observations

y_i	8.62	26.86	54.02
x_i	3.29	21.53	48.69

the value of $\widehat{\alpha} + \widehat{\beta}$ equals __________ (round off to 2 decimal places).

Q.26 – Q.43 Multiple Choice Question (MCQ), carry TWO mark each (for each wrong answer: – 2/3).

Q.26	**Let $f:\mathbb{R}\to\mathbb{R}$ be defined by** $$f(x)=\begin{cases} x^3\sin x, & x=0 \text{ or } x \text{ is irrational},\\ \dfrac{1}{q^3}, & x=\dfrac{p}{q}, p\in\mathbb{Z}\setminus\{0\}, q\in\mathbb{N} \text{ and } \gcd(p,q)=1,\end{cases}$$ **where $\mathbb{R}$ denotes the set of all real numbers, $\mathbb{Z}$ denotes the set of all integers, $\mathbb{N}$ denotes the set of all positive integers and $\gcd(p,q)$ denotes the greatest common divisor of p and q. Then which one of the following statements is true?**
(A)	f is not continuous at 0
(B)	f is not differentiable at 0
(C)	f is differentiable at 0 and the derivative of f at 0 equals 0
(D)	f is differentiable at 0 and the derivative of f at 0 equals 1

Q.27	**Let $f:[0,\infty)\to\mathbb{R}$ be a function, where $\mathbb{R}$ denotes the set of all real numbers. Then which one of the following statements is true?**
(A)	If f is bounded and continuous, then f is uniformly continuous
(B)	If f is uniformly continuous, then $\lim_{x\to\infty} f(x)$ exists
(C)	If f is uniformly continuous, then the function $g(x)=f(x)\sin x$ is also uniformly continuous
(D)	If f is continuous and $\lim_{x\to\infty} f(x)$ is finite, then f is uniformly continuous

Q.28	**Let $f:\mathbb{R}\to\mathbb{R}$ be a differentiable function such that $f(0)=0$ and $f'(x)+2f(x)>0$ for all $x\in\mathbb{R}$, where f' denotes the derivative of f and $\mathbb{R}$ denotes the set of all real numbers. Then which one of the following statements is true?**
(A)	$f(x)>0$, for all $x>0$ and $f(x)<0$, for all $x<0$
(B)	$f(x)<0$, for all $x\neq 0$
(C)	$f(x)>0$, for all $x\neq 0$
(D)	$f(x)<0$, for all $x>0$ and $f(x)>0$, for all $x<0$

Q.29	**Let M be the collection of all 3×3 real symmetric positive definite matrices. Consider the set $$S=\left\{A\in M : A^{50}-\frac{1}{4}A^{48}=0\right\},$$ where 0 denotes the 3×3 zero matrix. Then the number of elements in S equals**
(A)	0
(B)	1
(C)	8
(D)	∞

Q.30	**Let A be a 3×3 real matrix such that I_3+A is invertible and let $B=(I_3+A)^{-1}(I_3-A)$, where I_3 denotes the 3×3 identity matrix. Then which one of the following statements is true?**
(A)	If B is orthogonal, then A is invertible
(B)	If B is orthogonal, then all the eigenvalues of A are real
(C)	If B is skew-symmetric, then A is orthogonal
(D)	If B is skew-symmetric, then the determinant of A equals -1

Q.31	**Let X be a random variable having Poisson distribution such that $E(X^2) = 110$. Then which one of the following statements is NOT true?**
(A)	$E(X^n) = 10\, E[(X+1)^{n-1}]$, for all $n = 1, 2, 3, \ldots$
(B)	$P(X \text{ is even}) = \frac{1}{4}(1 + e^{-20})$
(C)	$P(X = k) < P(X = k+1)$, for $k = 0, 1, \ldots, 8$
(D)	$P(X = k) > P(X = k+1)$, for $k = 10, 11, \ldots$

Q.32	**Let X be a random variable having uniform distribution on $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$. Then which one of the following statements is NOT true?**
(A)	$Y = \cot X$ follows standard Cauchy distribution
(B)	$Y = \tan X$ follows standard Cauchy distribution
(C)	$Y = -\log_e\left(\frac{1}{2} + \frac{X}{\pi}\right)$ has moment generating function $M(t) = \frac{1}{1-t}$, $t < 1$
(D)	$Y = -2\log_e\left(\frac{1}{2} + \frac{X}{\pi}\right)$ follows central chi-square distribution with one degree of freedom

Q.33	**Let $\Omega = \{1, 2, 3, \ldots\}$ represent the collection of all possible outcomes of a random experiment with probabilities $P(\{n\}) = \alpha_n$ for $n \in \Omega$. Then which one of the following statements is NOT true?**
(A)	$\lim_{n\to\infty} \alpha_n = 0$
(B)	$\sum_{n=1}^{\infty} \sqrt{\alpha_n}$ converges
(C)	For any positive integer k, there exist k disjoint events $A_1, A_2, \ldots, A_k$ such that $P(\cup_{i=1}^{k} A_i) < 0.001$
(D)	There exists a sequence $\{A_i\}_{i\geq 1}$ of strictly increasing events such that $P(\cup_{i=1}^{\infty} A_i) < 0.001$

Q.34	**Let (X, Y) have the joint probability density function** $$f_{X,Y}(x,y) = \begin{cases} \frac{4}{(x+y)^3}, & x>1, y>1, \\ 0, & \text{otherwise.} \end{cases}$$ **Then which one of the following statements is NOT true?**
(A)	The probability density function of $X+Y$ is $$f_{X+Y}(z) = \begin{cases} \frac{4}{z^3}(z-2), & z>2, \\ 0, & \text{otherwise.} \end{cases}$$
(B)	$P(X+Y>4) = \frac{3}{4}$
(C)	$E(X+Y) = 4\log_e 2$
(D)	$E(Y \mid X=2) = 4$

Q.35	**Let X_1, X_2 and X_3 be three uncorrelated random variables with common variance $\sigma^2 < \infty$. Let $Y_1 = 2X_1 + X_2 + X_3$, $Y_2 = X_1 + 2X_2 + X_3$ and $Y_3 = X_1 + X_2 + 2X_3$. Then which of the following statements is/are true?** **P : The sum of eigenvalues of the variance covariance matrix of (Y_1, Y_2, Y_3) is $18\sigma^2$.** **Q : The correlation coefficient between Y_1 and Y_2 equals that between Y_2 and Y_3.**
(A)	P only
(B)	Q only
(C)	Both P and Q
(D)	Neither P nor Q

Q.36	**Let $\{X_n\}_{n\geq 0}$ be a time-homogeneous discrete time Markov chain with either finite or countable state space S. Then which one of the following statements is true?**
(A)	There is at least one recurrent state
(B)	If there is an absorbing state, then there exists at least one stationary distribution
(C)	If all the states are positive recurrent, then there exists a unique stationary distribution
(D)	If $\{X_n\}_{n\geq 0}$ is irreducible, $S = \{1, 2\}$ and $[\pi_1 \ \pi_2]$ is a stationary distribution, then $\lim_{n\to\infty} P(X_n = i \mid X_0 = i) = \pi_i$ for $i = 1, 2$

Q.37	**Let customers arrive at a departmental store according to a Poisson process with rate 10. Further, suppose that each arriving customer is either a male or a female with probability $\frac{1}{2}$ each, independent of all other arrivals. Let $N(t)$ denote the total number of customers who have arrived by time t. Then which one of the following statements is NOT true?**
(A)	If S_2 denotes the time of arrival of the second female customer, then $P(S_2 \leq 1) = 25\int_0^1 se^{-5s}ds$
(B)	If $M(t)$ denotes the number of male customers who have arrived by time t, then $P\left(M\left(\frac{1}{3}\right) = 0 \,\middle|\, M(1) = 1\right) = \frac{1}{3}$
(C)	$E\left[\left(N(t)\right)^2\right] = 100t^2 + 10t$
(D)	$E[N(t)N(2t)] = 200t^2 + 10t$

Q.38	**Let $X_{(1)} < X_{(2)} < X_{(3)} < X_{(4)} < X_{(5)}$ be the order statistics corresponding to a random sample of size 5 from a uniform distribution on $[0, \theta]$, where $\theta \in (0, \infty)$. Then which of the following statements is/are true?** **P : $3X_{(2)}$ is an unbiased estimator of θ.** **Q : The variance of $E[2X_{(3)} \mid X_{(5)}]$ is less than or equal to the variance of $2X_{(3)}$.**
(A)	P only
(B)	Q only
(C)	Both P and Q
(D)	Neither P nor Q

Q.39	**Let $X_1, X_2, \ldots, X_n$ be a random sample of size $n\ (\geq 2)$ from a distribution having the probability density function** $$f(x;\theta) = \begin{cases} \frac{1}{\theta} e^{-\frac{x}{\theta}}, & x > 0, \\ 0, & \text{otherwise}, \end{cases}$$ **where $\theta \in (0, \infty)$. Let $X_{(1)} = \min\{X_1, X_2, \ldots, X_n\}$ and $T = \sum_{i=1}^{n} X_i$. Then $E(X_{(1)} \mid T)$ equals**
(A)	$\frac{T}{n^2}$
(B)	$\frac{T}{n}$
(C)	$\frac{(n+1)T}{2n}$
(D)	$\frac{(n+1)^2 T}{4n^2}$

Q.40	**Let $X_1, X_2, \dots, X_n$ be a random sample of size $n\,(\geq 2)$ from a uniform distribution on $[-\theta, \theta]$, where $\theta \in (0, \infty)$. Let $X_{(1)} = \min\{X_1, X_2, \dots, X_n\}$ and $X_{(n)} = \max\{X_1, X_2, \dots, X_n\}$. Then which of the following statements is/are true?** **P : $(X_{(1)}, X_{(n)})$ is a complete statistic.** **Q : $X_{(n)} - X_{(1)}$ is an ancillary statistic.**
(A)	P only
(B)	Q only
(C)	Both P and Q
(D)	Neither P nor Q

Q.41	**Let $\{X_n\}_{n\geq 1}$ be a sequence of independent and identically distributed random variables having common distribution function $F(\cdot)$. Let $a < b$ be two real numbers such that $F(x) = 0$ for all $x \leq a$, $0 < F(x) < 1$ for all $a < x < b$ and $F(x) = 1$ for all $x \geq b$. Let $S_n(x)$ be the empirical distribution function at x based on $X_1, X_2, \dots, X_n$, $n \geq 1$. Then which one of the following statements is NOT true?**
(A)	$P\left[\lim_{n\to\infty} \sup_{-\infty<x<\infty} \lvert S_n(x) - F(x)\rvert = 0\right] = 1$
(B)	For fixed $x \in (a, b)$ and $t \in (-\infty, \infty)$, $$\lim_{n\to\infty} P\left[\frac{\sqrt{n}\ \lvert S_n(x) - F(x)\rvert}{\sqrt{S_n(x)(1 - S_n(x))}} \leq t\right] = P(Z \leq t),$$ where Z is the standard normal random variable
(C)	The covariance between $S_n(x)$ and $S_n(y)$ equals $\frac{1}{n}F(x)(1 - F(y))$ for all $n \geq 2$ and for fixed $-\infty < x, y < \infty$
(D)	If $Y_n = \sup_{-\infty<x<\infty} \left(S_n(x) - F(x)\right)^2$, then $\{4n\,Y_n\}_{n\geq 1}$ converges in distribution to a central chi-square random variable with 2 degrees of freedom

Q.42	**Let the joint distribution of random variables X_1, X_2, X_3 and X_4 be $N_4(\underline{\mu}, \Sigma)$, where** $$\underline{\mu} = \begin{pmatrix} 1 \\ 0 \\ 0 \\ 1 \end{pmatrix} \quad \text{and} \quad \Sigma = \begin{bmatrix} 1 & 0.2 & 0 & 0 \\ 0.2 & 2 & 0 & 0 \\ 0 & 0 & 2 & 0.2 \\ 0 & 0 & 0.2 & 1 \end{bmatrix}.$$ **Then which one of the following statements is true?**
(A)	$\frac{5}{17}[(X_1 + X_2)^2 + (X_3 + X_4 - 1)^2]$ follows a central chi-square distribution with 2 degrees of freedom
(B)	$\frac{1}{3}[(X_1 + X_3 - 1)^2 + (X_2 + X_4 - 1)^2]$ follows a central chi-square distribution with 2 degrees of freedom
(C)	$E\left[\sqrt{\left\lvert\frac{X_1+X_2-1}{X_3+X_4-1}\right\rvert}\right]$ is NOT finite
(D)	$E\left[\left\lvert\frac{X_1+X_2+X_3+X_4-2}{X_1+X_2-X_3-X_4}\right\rvert\right]$ is NOT finite

Q.43	**Let $\underline{Y}$ follow $N_8(\underline{0}, I_8)$ distribution, where I_8 is the 8×8 identity matrix. Let $\underline{Y}^T\Sigma_1\underline{Y}$ and $\underline{Y}^T\Sigma_2\underline{Y}$ be independent and follow central chi-square distributions with 3 and 4 degrees of freedom, respectively, where Σ_1 and Σ_2 are 8×8 matrices and $\underline{Y}^T$ denotes transpose of $\underline{Y}$. Then which of the following statements is/are true?** **P : Σ_1 and Σ_2 are idempotent.** **Q : $\Sigma_1\Sigma_2 = 0$, where 0 is the 8×8 zero matrix.**
(A)	P only
(B)	Q only
(C)	Both P and Q
(D)	Neither P nor Q

Q.44 – Q.55 Numerical Answer Type (NAT), carry TWO mark each (no negative marks).

Q.44	Let (X, Y) have a bivariate normal distribution with the joint probability density function $$f_{X,Y}(x,y) = \frac{1}{\pi} e^{\left(\frac{3}{2}xy - \frac{25}{32}x^2 - 2y^2\right)}, \quad -\infty < x, y < \infty.$$ Then $8\,E(XY)$ equals __________

Q.45	Let $f: \mathbb{R} \times \mathbb{R} \to \mathbb{R}$ be defined by $f(x, y) = 8x^2 - 2y$, where $\mathbb{R}$ denotes the set of all real numbers. If M and m denote the maximum and minimum values of f, respectively, on the set $\{(x, y) \in \mathbb{R} \times \mathbb{R} : x^2 + y^2 = 1\}$, then $M - m$ equals __________ (round off to 2 decimal places).

Q.46	Let $A = [a\ u_1\ u_2\ u_3]$, $B = [b\ u_1\ u_2\ u_3]$ and $C = [u_2\ u_3\ u_1\ a + b]$ be three 4×4 real matrices, where a, b, u_1, u_2 and u_3 are 4×1 real column vectors. Let $\det(A)$, $\det(B)$ and $\det(C)$ denote the determinants of the matrices A, B and C, respectively. If $\det(A) = 6$ and $\det(B) = 2$, then $\det(A + B) - \det(C)$ equals __________

Q.47	Let X be a random variable having the moment generating function $$M(t) = \frac{e^t - 1}{t(1 - t)}, \qquad t < 1.$$ Then $P(X > 1)$ equals __________ (round off to 2 decimal places).

Q.48	Let $\{X_n\}_{n\ge1}$ be a sequence of independent and identically distributed random variables each having uniform distribution on $[0, 3]$. Let Y be a random variable, independent of $\{X_n\}_{n\ge1}$, having probability mass function $$P(Y = k) = \begin{cases} \dfrac{1}{(e-1)k!}, & k = 1, 2, \dots, \\ 0, & \text{otherwise.} \end{cases}$$ Then $P(\max\{X_1, X_2, \dots, X_Y\} \le 1)$ equals __________ (round off to 2 decimal places).

Q.49	Let $\{X_n\}_{n\geq 1}$ be a sequence of independent and identically distributed random variables each having probability density function $$f(x) = \begin{cases} e^{-x}, & x > 0, \\ 0, & \text{otherwise}. \end{cases}$$ Let $X_{(n)} = \max\{X_1, X_2, \ldots, X_n\}$ for $n \geq 1$. If Z is the random variable to which $\{X_{(n)} - \log_e n\}_{n\geq 1}$ converges in distribution, as $n \to \infty$, then the median of Z equals __________ (round off to 2 decimal places).

Q.50	Consider an amusement park where visitors are arriving according to a Poisson process with rate 1. Upon arrival, a visitor spends a random amount of time in the park and then departs. The time spent by the visitors are independent of one another, as well as of the arrival process, and have common probability density function $$f(x) = \begin{cases} e^{-x}, & x > 0, \\ 0, & \text{otherwise}. \end{cases}$$ If at a given time point, there are 10 visitors in the park and p is the probability that there will be exactly two more arrivals before the next departure, then $\frac{1}{p}$ equals __________

Q.51	Let $\{0.90, 0.50, 0.01, 0.95\}$ be a realization of a random sample of size 4 from the probability density function $$f(x) = \begin{cases} \frac{\theta}{1-\theta} x^{(2\theta-1)/(1-\theta)}, & 0 < x < 1, \\ 0, & \text{otherwise}, \end{cases}$$ where $0.5 \leq \theta < 1$. Then the maximum likelihood estimate of θ based on the observed sample equals __________ (round off to 2 decimal places).

Q.52	Let a random sample of size 100 from a normal population with unknown mean μ and variance 9 give the sample mean 5.608. Let $\Phi(\cdot)$ denote the distribution function of the standard normal random variable. If $\Phi(1.96) = 0.975$, $\Phi(1.64) = 0.95$ and the uniformly most powerful unbiased test based on sample mean is used to test $H_0: \mu = 5.02$ against $H_1: \mu \neq 5.02$, then the p-value equals __________ (round off to 3 decimal places).

Q.53 Let X be a discrete random variable with probability mass function $p \in \{p_0, p_1\}$, where

x	7	8	9	10
$p_1(x)$	0.69	0.10	0.16	0.05
$p_0(x)$	0.90	0.05	0.04	0.01

To test $H_0: p = p_0$ against $H_1: p = p_1$, the power of the most powerful test of size 0.05, based on X, equals __________ (round off to 2 decimal places).

Q.54 Let $X_1, X_2, \dots, X_{10}$ be a random sample from a probability density function

$$f_\theta(x) = f(x - \theta), \quad -\infty < x < \infty,$$

where $-\infty < \theta < \infty$ and $f(-x) = f(x)$ for $-\infty < x < \infty$. For testing $H_0: \theta = 1.2$ against $H_1: \theta \neq 1.2$, let T^+ denote the Wilcoxon Signed-rank test statistic. If η denotes the probability of the event $\{T^+ < 50\}$ under H_0, then $32\,\eta$ equals __________ (round off to 2 decimal places).

Q.55 Consider the multiple linear regression model

$$Y_i = \beta_0 + \beta_1 x_{1,i} + \beta_2 x_{2,i} + \cdots + \beta_{22} x_{22,i} + \epsilon_i, \qquad i = 1, 2, \dots, 123,$$

where, for $j = 0, 1, 2, \dots, 22$, β_j's are unknown parameters and ϵ_i's are independent and identically distributed $N(0, \sigma^2)$, $\sigma > 0$, random variables.
If the sum of squares due to regression is 338.92, the total sum of squares is 522.30 and R^2_{adj} denotes the value of adjusted R^2, then $100\,R^2_{adj}$ equals __________ (round off to 2 decimal places).

END OF THE QUESTION PAPER

Graduate Aptitude Test in Engineering (GATE 2021)

Answer Keys and Marks for Subject/Paper: Statistics (ST)

Q. No.	Session	Question Type MCQ/MSQ/NAT	Section Name	Answer Key/Range	Marks	Negative Marks
1	4	MCQ	GA	C	1	1/3
2	4	MCQ	GA	B	1	1/3
3	4	MCQ	GA	C	1	1/3
4	4	MCQ	GA	C	1	1/3
5	4	MCQ	GA	B	1	1/3
6	4	MCQ	GA	C	2	2/3
7	4	MCQ	GA	D	2	2/3
8	4	MCQ	GA	A	2	2/3
9	4	MCQ	GA	C	2	2/3
10	4	MCQ	GA	A	2	2/3
1	4	MCQ	ST	A	1	1/3
2	4	MCQ	ST	C	1	1/3
3	4	MCQ	ST	A	1	1/3
4	4	MCQ	ST	B	1	1/3
5	4	MCQ	ST	A	1	1/3
6	4	MCQ	ST	A	1	1/3
7	4	MCQ	ST	A	1	1/3
8	4	MCQ	ST	B	1	1/3
9	4	MCQ	ST	A	1	1/3
10	4	NAT	ST	2 to 2	1	0

GATE 2019 Answer Key for Statistics (ST)

Q. No.	Session	Question Type MCQ/MSQ/NAT	Section Name	Answer Key/Range	Marks	Negative Marks
11	4	NAT	ST	33.50 to 34.50	1	0
12	4	NAT	ST	2 to 2	1	0
13	4	NAT	ST	1.70 to 1.75	1	0
14	4	NAT	ST	0.32 to 0.35	1	0
15	4	NAT	ST	7 to 7	1	0
16	4	NAT	ST	0.32 to 0.35	1	0
17	4	NAT	ST	7 to 7	1	0
18	4	NAT	ST	0.50 to 0.50	1	0
19	4	NAT	ST	0.27 to 0.35	1	0
20	4	NAT	ST	50 to 50	1	0
21	4	NAT	ST	0.98 to 1.00	1	0
22	4	NAT	ST	8.60 to 8.75	1	0
23	4	NAT	ST	0.14 to 0.18	1	0
24	4	NAT	ST	1.00 to 1.00	1	0
25	4	NAT	ST	6.31 to 6.35	1	0
26	4	MCQ	ST	C	2	2/3
27	4	MCQ	ST	D	2	2/3
28	4	MCQ	ST	A	2	2/3
29	4	MCQ	ST	B	2	2/3
30	4	MCQ	ST	C	2	2/3
31	4	MCQ	ST	B	2	2/3
32	4	MCQ	ST	D	2	2/3
33	4	MCQ	ST	B	2	2/3

GATE 2019 Answer Key for Statistics (ST)

Q. No.	Session	Question Type MCQ/MSQ/NAT	Section Name	Answer Key/Range	Marks	Negative Marks
34	4	MCQ	ST	C	2	2/3
35	4	MCQ	ST	C	2	2/3
36	4	MCQ	ST	B	2	2/3
37	4	MCQ	ST	B	2	2/3
38	4	MCQ	ST	C	2	2/3
39	4	MCQ	ST	A	2	2/3
40	4	MCQ	ST	D	2	2/3
41	4	MCQ	ST	C	2	2/3
42	4	MCQ	ST	D	2	2/3
43	4	MCQ	ST	C	2	2/3
44	4	NAT	ST	3 to 3	2	0
45	4	NAT	ST	10.10 to 10.15	2	0
46	4	NAT	ST	72 to 72	2	0
47	4	NAT	ST	0.60 to 0.66	2	0
48	4	NAT	ST	0.20 to 0.26	2	0
49	4	NAT	ST	0.32 to 0.42	2	0
50	4	NAT	ST	143 to 143	2	0
51	4	NAT	ST	0.50 to 0.50	2	0
52	4	NAT	ST	0.045 to 0.055	2	0
53	4	NAT	ST	0.20 to 0.22	2	0
54	4	NAT	ST	31.60 to 31.80	2	0
55	4	NAT	ST	57.00 to 57.40	2	0