Q1) Let $\boldsymbol{M}$ be any square matrix of arbitrary order n such that $\boldsymbol{M}^2 = \mathbf{0}$ and the nullity of $\boldsymbol{M}$ is 6. Then the maximum possible value of n (in integer) is ______

Q2) Let $X^1, X^2, \ldots, X^n$ be a random sample from a population $f(x; \theta)$, where θ is a parameter. Then which one of the following statements is NOT true?

(A) $\sum_{i=1}^{n} X_i$ is a complete and sufficient statistic for θ, if

$$f(x;\theta) = \frac{e^{-\theta}\theta^x}{x!}, \quad x = 0, 1, 2, \ldots, \quad \text{and} \quad \theta > 0$$

(B) $\left(\sum_{i=1}^{n} X_i, \sum_{i=1}^{n} X_i^2\right)$ is a complete and sufficient statistic for θ, if

$$f(x;\theta) = \frac{1}{\sqrt{2\pi}\,\theta} e^{-\frac{1}{2\theta^2}(x-\theta)^2}, -\infty < x < \infty, \quad \theta > 0$$

(C) $f(x;\theta) = \theta x^{\theta-1}, 0 < x < 1, \ \theta > 0$ has monotone likelihood ratio property in $\prod_{i=1}^{n} X_i$

(D) $X_{(n)} - X_{(1)}$ is ancillary statistic for θ if $f(x;\theta) = 1, \ 0 < \ \theta < x < \theta + 1$, where $X_{(1)} = \min\{X_1, X_2, \ldots, X_n\}$ and $X_{(n)} = \max\{X_1, X_2, \ldots, X_n\}$

Q3) At a telephone exchange, telephone calls arrive independently at an average rate of 1 call per minute, and the number of telephone calls follows a Poisson distribution. Five time intervals, each of duration 2 minutes, are chosen at random. Let p denote the probability that in each of the five time intervals at most 1 call arrives at the telephone exchange. Then e^{10} (in integer) is equal to _______

Q4) A random sample of size 4 is taken from the distribution with the probability density Function

$$f(x;\theta) = \begin{cases} \frac{2(\theta - x)}{\theta^2}, & 0 < x < \theta, \\ 0, & \text{elsewhere.} \end{cases}$$

If the observed sample values are 6, 5, 3, 6, then the method of moments estimate (in integer) of the parameter θ, based on these observations, is ______

Q5) A company sometimes stops payments of quarterly dividends. If the company pays the quarterly dividend, the probability that the next one will be paid is 0.7. If the company stops the quarterly dividend, the probability that the next quarterly dividend will not be paid is 0.5. Then the probability (rounded off to three decimal places) that the company will not pay quarterly dividend in the long run is ______.

Q6) Let $X_1, X_2, \dots, X_8$ be a random sample taken from a distribution with the probability density function

$$f_X(x) = \begin{cases} \frac{x}{8}, & 0 < x < 4, \\ 0, & \text{elsewhere.} \end{cases}$$

Let $F_8(x)$ be the empirical distribution function of the sample. If α is the variance of $F_8(2)$, then 128α (in integer) is equal to ______ .

Q7) Let $\boldsymbol{M}$ be a 3×3 real symmetric matrix with eigenvalues $-1, 1, 2$ and the corresponding unit eigenvectors $\boldsymbol{u}, \boldsymbol{v}, \boldsymbol{w}$, respectively. Let $\boldsymbol{x}$ and $\boldsymbol{y}$ be two vectors in $\mathbb{R}^3$ such that

$$\boldsymbol{Mx} = \boldsymbol{u} + 2(\boldsymbol{v} + \boldsymbol{w}) \quad \text{and} \quad \boldsymbol{M}^2\boldsymbol{y} = \boldsymbol{u} - (\boldsymbol{v} + 2\boldsymbol{w})$$

Considering the usual inner product in $\mathbb{R}^3$, the value of $|\boldsymbol{x} + \boldsymbol{y}|^2$, where $|\boldsymbol{x} + \boldsymbol{y}|$ is the length of the vector $\boldsymbol{x} + \boldsymbol{y}$, is

(A) 1.25

(B) 0.25

(C) 0.75

(D) 1

Q8) Consider the following infinite series:

$$S_1 := \sum_{n=0}^{\infty} (-1)^n \frac{n}{n^2 + 4} \qquad \text{and} \qquad S_2 := \sum_{n=0}^{\infty} (-1)^n \left(\sqrt{n^2 + 1} - n\right)$$

Which of the above series is/are conditionally convergent?

(A) S_1 only

(B) S_2 only

(C) Both S_1 and S_2

(D) Neither S_1 nor S_2

Q9) Let (X, Y, Z) be a random vector with the joint probability density function

$$f_{X,Y,Z}(x, y, z) = \begin{cases} \frac{1}{3}(2x + 3y + z), & 0 < x < 1, 0 < y < 1, 0 < z < 1 \\ 0, & \text{elsewhere.} \end{cases}$$

Then which one of the following points is on the regression surface of X on (Y, Z) ?

(A) $\left(\frac{4}{7}, \frac{1}{3}, \frac{1}{3}\right)$

(B) $\left(\frac{6}{7}, \frac{2}{3}, \frac{2}{3}\right)$

(C) $\left(\frac{1}{2}, \frac{1}{3}, \frac{2}{3}\right)$

(D) $\left(\frac{1}{2}, \frac{2}{3}, \frac{1}{3}\right)$

Q10) A random sample X of size one is taken from a distribution with the probability density function

$$f(x;\theta) = \begin{cases} \frac{2x}{\theta^2}, & 0 < x < \theta, \\ 0, & \text{elsewhere.} \end{cases}$$

If x / θ is used as a pivot for obtaining the confidence interval for θ, then which one of the following is an 80% confidence interval (confidence limits rounded off to three decimal places) for θ based on the observed sample value $x = 10$?

(A) (10.541, 31.623)
(B) (10.987, 31.126)
(C) (11.345, 30.524)
(D) (11.267, 30.542)

Q11) Let $X1, X2, \ldots, X7$ be a random sample from a normal population with mean 0 and variance $\theta > 0$. Let

$$K = \frac{X_1^2 + X_2^2}{X_1^2 + X_2^2 + \cdots + X_7^2}$$

Consider the following statements:

(I) The statistics K and $X_1^2 + X_2^2 + \cdots + X_7^2$ are independent.

(II) $\frac{7K}{2}$ has an F-distribution with 2 and 7 degrees of freedom.

(III) $E(K^2) = \frac{8}{63}$.

Then which of the above statements is/are true?

(A) (I) and (II) only
(B) (I) and (III) only
(C) (II) and (III) only

(D) (I) only

Q12) Consider the following statements:
(I) Let a random variable X have the probability density function

$$f_X(x) = \frac{1}{2}e^{-|x|}, \qquad -\infty < x < \infty.$$

Then there exist $i.\ i.\ d.$ random variables X_1 and X_2 such that X and $X_1 - X_2$ have the same distribution
(II) Let a random variable Y have the probability density function

$$f_Y(y) = \begin{cases} \frac{1}{4}, & -2 < y < 2, \\ 0, & \text{elsewhere.} \end{cases}$$

Then there exist $i.\ i.\ d.$ random variables Y_1 and Y_2 such that Y and $Y_1 - Y_2$ have the same distribution.
Then which of the above statements is/are true?
(A) (I) only
(B) (II) only
(C) Both (I) and (II)
(D) Neither (I) nor (II)

Q13) Which of the following real valued functions is/are uniformly continuous on $[0, \infty)$?
(A) $\sin^2 x$
(B) $x \sin x$
(C) $\sin(\sin x)$
(D) $\sin(x \sin x)$

Q14) Two independent random samples, each of size 7, from two populations yield the following values:

Population 1	18	20	16	20	17	18	14
Population 2	17	18	14	20	14	13	16

If Mann-Whitney U test is performed at 5% level of significance to test the null hypothesis H_0: Distributions of the populations are same, against the alternative hypothesis H_1: Distributions of the populations are not same, then the value of the test statistic U (in integer) for the given data, is ______

Q15) Suppose a random sample of size 3 is taken from a distribution with the probability density function

$$f(x) = \begin{cases} 2x, & 0 < x < 1, \\ 0, & \text{elsewhere.} \end{cases}$$

If p is the probability that the largest sample observation is at least twice the smallest sample observation, then the value of p (rounded off to three decimal places) is ______

Q16) Let 0, 1, 1, 2, 0 be five observations of a random variable X which follows a Poisson distribution with the parameter $\theta > 0$. Let the minimum variance unbiased estimate of $P(X \leq 1)$, based on this data, be α. Then $5^4\alpha$ (in integer) is equal to ______

Q17) In a laboratory experiment, the behavior of cats are studied for a particular food preference between two foods A and B. For an experiment, 70% of the cats that had food A will prefer food A, and 50% of the cats that had food B will prefer food A. The experiment is repeated under identical conditions. If 40% of the cats had food A in the first experiment, then the percentage (rounded off to one decimal place) of cats those will prefer food A in the third experiment, is ___

Q18) A random sample of size 5 is taken from a distribution with the probability density function

$$f(x;\theta) = \begin{cases} \dfrac{3x^2}{\theta^3}, & 0 < x < \theta, \\ 0, & \text{elsewhere,} \end{cases}$$

where θ is an unknown parameter. If the observed values of the random sample are 3, 6, 4, 7, 5, then the maximum likelihood estimate of the 1/8th quantile of the distribution (rounded off to one decimal place) is ______

Q19) Consider a gamma distribution with the probability density function

$$f(x;\beta) = \begin{cases} \dfrac{1}{24\,\beta^5}\, x^4 e^{-x/\beta}, & x > 0, \\ 0, & \text{elsewhere,} \end{cases}$$

with $\beta > 0$. Then, for $\beta = 2$, the value of the Cramer-Rao lower bound (rounded off to one decimal place) for the variance of any unbiased estimator of β^2 based on a random sample of size 8 from this distribution, is ______

Q20) Let $Y_1 < Y_2 < \cdots < Y_n$ be the order statistics of a random sample of size n from a continuous distribution, which is symmetric about its mean μ. Then the smallest value of n (in integer) such that $P(Y_1 < \mu < Y) \geq 0.99$, is ______.

Q21) If $P(x, y, z)$ is a point which is nearest to the origin and lies on the intersection of the surfaces $z = xy + 5$ and $x + y + z = 1$. Then the distance (in integer) between the origin and the point P is ______

Q22) Let $\Phi(\cdot)$ denote the cumulative distribution function of a standard normal random variable. If the random variable X has the cumulative distribution function

$$F(x) = \begin{cases} \Phi(x) & if\ x < -1 \\ \Phi(x+1) & if\ x \geq -1, \end{cases}$$

then which one of the following statements is true?

(A) $P(X \leq -1) = 1/2$
(B) $P(X = -1) = 1/2$
(C) $P(X < -1) = 1/2$
(D) $P(X \leq 0) = ½$

Q23) Suppose that X has the probability density function

$$f(x) = \begin{cases} \dfrac{\lambda^\alpha}{\Gamma(\alpha)} x^{\alpha-1} e^{-\lambda x} & if\ \ x > 0 \\ 0 & otherwise, \end{cases}$$

where $\alpha > 0$ and $\lambda > 0$. Which one of the following statements is NOT true?

(A) $E(X)$ exists for all $\alpha > 0$ and $\lambda > 0$
(B) Variance of X exists for all $\alpha > 0$ and $\lambda > 0$
(C) $E\ (1/X)$ exists for all $\alpha > 0$ and $\lambda > 0$
(D) $E(\log e(1 + X))$ exists for all $\alpha > 0$ and $\lambda > 0$

Q24) Suppose that there are 5 boxes, each containing 3 blue pens, 1 red pen and 2 black pens. One pen is drawn at random from each of these 5 boxes. If the random variable X_1 denotes the total number of blue pens drawn and the random variable X_2 denotes the total number of red pens drawn, then $P(X_1 = 2, X_2 = 1)$ equals

(A) 5/36
(B) 5/18
(C) 5/12
(D) 5/9

Q25) Suppose that x is an observed sample of size 1 from a population with probability density function $f(\cdot)$. Based on x, consider testing

$$H_0: f(y) = \frac{1}{\sqrt{2\pi}} e^{-\frac{y^2}{2}} \; ; \; y \in \mathbb{R} \quad \text{against} \quad H_1: f(y) = \frac{1}{2} e^{-|y|} \; ; \; y \in \mathbb{R}.$$

Then which one of the following statements is true?
(A) The most powerful test rejects H_0 if $|x| > c$ for some $c > 0$
(B) The most powerful test rejects H_0 if $|x| < c$ for some $c > 0$
(C) The most powerful test rejects H_0 if $||x| - 1| > c$ for some $c > 0$
(D) The most powerful test rejects H_0 if $||x| - 1| < c$ for some $c > 0$

Q26) Let A be a 2×2 real matrix such that $AB = BA$ for all 2×2 real matrices B. If trace of A equals 5, then determinant of A (rounded off to two decimal places) equals _______________

Q27) Two defective bulbs are present in a set of five bulbs. To remove the two defective bulbs, the bulbs are chosen randomly one by one and tested. If X denotes the minimum number of bulbs that must be tested to find out the two defective bulbs, then $P(X = 3)$ (rounded off to two decimal places) equals ______________

Q28) Let A be an $n \times n$ real matrix. Consider the following statements.
(I) If A is symmetric, then there exists $c \geq 0$ such that $A + cI_n$ is symmetric and positive definite, where I_n is the $n \times n$ identity matrix
(II) If A is symmetric and positive definite, then there exists a symmetric and positive definite matrix B such that $A = B^2$.
Which of the above statements is/are true?
(A) Only (I)
(B) Only (II)
(C) Both (I) and (II)
(D) Neither (I) nor (II)

Q29) Let (X, Y) have joint probability mass function

$$p(x, y) = \begin{cases} \dfrac{c}{2^{x+y+2}} & if\ x = 0, 1, 2, \ldots; \quad y = 0, 1, 2, \ldots; \quad x \neq y \\ 0 & otherwise. \end{cases}$$

Then which one of the following statements is true?
(A) $c = 1/2$
(B) $c = 1/4$
(C) $c > 1$
(D) X and Y are independent

Q30) Which of the following sets is/are countable?

(A) The set of all functions from $\{1, 2, 3, \ldots, 10\}$ to the set of all rational numbers

(B) The set of all functions from the set of all natural numbers to $\{0, 1\}$

(C) The set of all integer valued sequences with only finitely many non-zero terms

(D) The set of all integer valued sequences converging to 1