Physics - 2016

General Instructions :

- Group-A has 15 objective type questions each of 1 mark. खण्ड-अ में 15 वस्तुनिष्ठ प्रश्न हैं, प्रत्येक 1 अंक का है।
- Group-B has 8 questions, each of 2 marks. खण्ड-ब में 8 प्रश्न हैं, प्रत्येक का मान 2 अंक है।
- Group-C has 8 questions, each of 3 marks. खण्ड-स में 8 प्रश्न हैं, प्रत्येक का मान 3 अंक है।
- Group-D has 3 questions, each of 5 marks. खण्ड-द में 3 प्रश्न हैं, प्रत्येक का मान 5 अंक है।

Group-A

Q.1. Answer the following questions :

(i) The unit for permittivity of free space (ϵ_0) is

(a) CN^2m^{-2} (b) CNm^{-1}
(c) $C^2N^{-1}m^{-2}$ (d) $CN^{-1}m^{-2}$.

Ans. (c) $C^2N^{-1}m^{-2}$

(ii) In an electric circuit the algebraic sum of currents meeting at a point is

(a) zero (b) infinity
(c) positive (d) negative

Ans. (a) zero

(iii) A wire carries a current of 1 μA. If charge on a electron is $1.5\times10^{-19}C$, then number of electrons flowing per second through the wire is

(a) 0.625×10^{13} (b) 0.625×10^{19}
(c) 1.6×10^{-19} (d) 1.6×10^{-13}

Ans. (a) 0.625×10^{13}

(iv) When an ammeter is shunted, its measuring range

(a) decreases (b) increases
(c) remains unchanged (d) none of these.

Ans. (b) increases

(v) A magnet of magnetic moment $\vec{M}$ is lying along a uniform magnetic field $\vec{B}$. The work done in rotating the magnet by 90° is

(a) zero (b) $\frac{MB}{2}$
(c) MB (d) $2MB$.

Ans. (d) $2MB$.

(vi) The ratio of root-mean-square value and peak value of alternating current is

(a) $\frac{1}{\sqrt{2}}$ (b) $\sqrt{2}$
(c) $\frac{1}{2}$ (d) $2\sqrt{2}$.

Ans. (a) $\frac{1}{\sqrt{2}}$

(vii) Speed of electromagnetic wave in free space is

(a) $\mu_0 \epsilon_0$ (b) $\sqrt{\mu_0 \epsilon_0}$
(c) $\frac{1}{\sqrt{\mu_0 \epsilon_0}}$ (d) $\frac{1}{\mu_0 \epsilon_0}$.

Ans. (c) $\frac{1}{\sqrt{\mu_0 \epsilon_0}}$

(viii) When a ray of light passes from one medium to another, which of the following does not change?

(a) Velocity (b) Frequency
(c) Refractive index (d) Wavelength.

Ans. (b) Frequency

(ix) Dimensions of PLanck constant (h) is

(a) MLT^{-1} (b) ML^2T^{-1}.
(c) ML^2T^{-2} (d) MLT^{-2}.

Ans. (c) ML^2T^{-2}

(x) Which of the following is not a fundamental particle?

(a) Neutron (b) Proton
(c) α-particle (d) Electron.

Ans. (c) α-particle

(xi) In an atom, electrons can revolve only in those orbits in which its angular momentum is integral multiple of

(a) $\frac{h}{2\pi}$ (b) $2\pi h$
(c) $\frac{2\pi}{h}$ (d) none of these.

Ans. (a) $\frac{h}{2\pi}$

(xii) As temperature increases, the resistance of a semiconductor

(a) increases (b) decreases
(c) remains constant
(d) depends upon the semiconductor.

Ans. (b) decreases

(xiii) The Boolean expression for OR gate is

(a) $A+B=Y$ (b) $A.B=Y$
(c) $\overline{A.B}=Y$ (d) $\overline{A}=A$

Ans. (c) $\overline{A.B}=Y$

(xiv) Radio waves are reflected from

(a) Ionosphere (b) Stratosphere
(c) Troposphere (d) Both (b) and (c).

Ans. (a) Ionosphere

(xv) Coaxial cable has a band width of

(a) 750 Hz (b) 750 kHz
(c) 750 MHz (d) 750 GHz.

Ans. (c) 750 MHz

Group-B

Answer the following question :

Q.2. What are electric field lines? Mention two of its properties.

Ans. **Electric field lines :** Electric field lines are curved or straight imaginary lines in the electric field such that the tangent at any point on the field lines gives the direction of the electric field at that point.

Properties of electric field lines :

1. The electric field lines begin from positive charge and terminate or end on negative charge.
2. Electric field lines are imaginary lines.
3. The tangent at any point on an electric field at that point.
4. Two electric field lines cannot cross each other.
5. The number of lines per unit cross-sectional area perpendicular to the field lines is proportional to the magnitude of the intensity of electric field in that region.
6. Electrostatic field lines do not form closed loop.
7. In a charge free region, electric field lines can be taken to be continuous curves without any break.
8. Electric field lines contract lengthwise to represent attraction between two unlike charges.
9. Electric field line exert lateral pressure on each other to represent repulsion between like charges.
10. Electric field lines are perpendicular to the surface of a positively or negatively charged sphere.
11. Electric field lines are not allowed to pass through a conductor because the interior of a conductor is free from the influence of the electric field.
12. Electric field lines are allowed to pass through a non-conductor or dielectric.

Q.3. Define *emf* of a cell. Mention two differences between emf and terminal potential difference.

Ans. Emf of a cell : It is define as the potential difference between it terminals when no current is drawn frow the cell

	E.M.F.		Terminal Potential Diff.
1.	The potential diff. between if terminal when no current is drawn from the cell	1.	The potential difference between it terminal in a closed circuit (i.e., when current is drawn from the cell)
2.	The word e.m.f. is reserved for the potential difference of an electric source.		The word potential diff. is used for the measurement made between any two points of the electric circuit.

Q.4. Define critical angle. Mention the conditions for total internal reflection of light.

Ans. Critical Angle : When the light ray passes from denser medium to rarer medium then the angle of incidence at which angle of refraction becomes 90° is caled critical angle.

Condition for total internal reflection :

(i) The ray of light must travel from denser to rare medium.

(ii) Angle of incidence in densor medium must be greater than critical angle.

Q.5. What are microwaves? Mention two of its applications.

Ans. Microwaves : To wavelength range of microwaves is form 0.1 m to 1mm. The frequency range of microwaves is 10^{9} Hz to 10^{12} Hz. They are produced by oscillating electrons in a cavity. The commonly used of oscillators to produce microwaves are Klystron, Magnetron and Gum diodes.

Application :

1. Microwaves are used in a radar system for aircraft navigation.
2. These are used for atomic and molecular research.
3. These are used in microwaves ovens for cooking and warming food.
4. Microwaves are used for communication by cellular phones.
5. Microwaves are used in weather radar.

Q.6. Define coefficient of self induction and coefficient of mutual induction.

Ans. Coefficient of self induction : Co-efficient fo self induction is the magnetic flux linked with a coil, when a unit current flow through it.

Co-efficient of mutual induction: The co-efficient of mutual induction of a pair of coils is the magnetic flux linked with one of the coils when a unit current passes through the other coil.

Q.7. What is Rutherford's nuclear model of atom?

Ans. Rutherford Atomic Model :

(i) Entire positive charge and almost the ohle mass of the atom is concentrated in a small central core called nucleus.

(ii) The nucleus is surrounded by a suitable number of electrons so that the atom is electrically neutral.

(iii) The electron revolve around the nucleus in various orbits. The necessary centripetal force is provided to them by the electrostatic force of attraction between the electrons and the nucleus.

Q.8. Define mass defect and nuclear binding energy.

Ans. Mass defect : Difference between the sum of the mass of proton and neutron and that of the necleus is called mass defect.

$$\Delta M = (mp + mn) - M_N$$

where, mp = *mass of proton.*

mn = *mass of mutron.*

M_N = *mass of nucleus.*

Nuclear binding energy : The total energy required to disintegrate the nucleus into its constituent particles is called binding energy of the nucleus or nuclear binding energy.

$$E_b = \Delta mC^2$$

Q.9. Why is the base region of a transistor thin and lightly doped? Explain.

Ans. The majority carrier forms the omitter region move toward the collector region through the base. If the base region of a transistor is made thick and heavily doped, then most of the majority carriers will combine with the charge carrier in the base while passing through it. Hence, only smal no. of majority carrier will reach the collector region. As a result, the output or collector current will be decreased considerabley. Therefore, have region of a transistor is made t hin and lightly doped, so that minimum no. of majority carrier passing through the base get neutrolized and large output is available.

Group-C

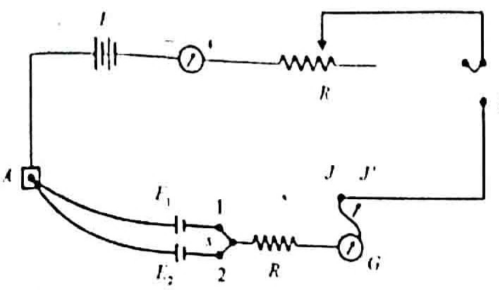
Answer the following questions :

Q.10. Explain the principle of a potentiometer. Draw the circuit diagram for the comparison of emf of two primary cells using a potentiometer.

Ans. Principle of Potentiometer : It work on the principle that potential difference across any part of a uniform wire is directly proportional to the length of that portion, when a constant current flows through the wire

$$V = Kl$$

Diagram for the comparison of e.m.f of two primary cells using a potentiometer:



Q.11. Derive an expression for the magnetic dipole moment of orbital electron of an atom.

Ans. Magnetic Orbital Dipole Moment of a revolving electron (Atom as a magnetic dipole): In an atom, electrons revolve around the nucleus in circular orbits. The movement of the electron is circular orbit around the nucleus in anticlockwise is equivalent to the flow of current in the orbit in clockwise direction. Thus, the orbit of electrons is considered as tiney current loop.

If an electron revolves in anti-clockwise direction as shown in fig. 42, the angular momentum vector $\vec{L}(=\vec{r}m_e\vec{\upsilon})$ acts along the normal to the plane of orbit in upward direction and it magnitude is given by

$$L = m_e \upsilon r \quad or \quad \upsilon r \frac{L}{m_e}$$

where, m is the mass of electron, υ is the velocity and r is the radius of orbit.

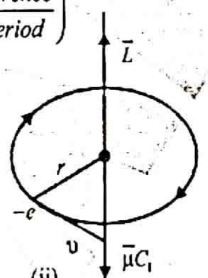
Orbital motion of electron in anticlockwise direction, is equivalent to the flow of conventional current in clockwise direction.

$$I = \frac{e}{T}$$

where, e is the change on an electron and T is the period of orbit motion.

But, $T = \dfrac{2\pi r}{\upsilon}$ $\quad \left(As\,\upsilon = \dfrac{circumference}{Time\ period}\right)$

$$I = \frac{e}{\frac{2\pi r}{\upsilon}} = \frac{e\upsilon}{2\pi r}$$



orbit magnetic moment of a current loop.

$$\mu_l = I \times A = \frac{e\upsilon}{2\pi r} \times \pi r^2 = \frac{e\upsilon r}{2}$$

using (i) $\mu_l \dfrac{e}{2m_e} L$ (orbital motion) ...(ii)

In vector notation, $\vec{\mu}_l = -\left(\dfrac{e}{2m_e}\right)\vec{L}$

From eq^n. (ii)

$$\frac{\mu_l}{L} = \frac{e}{2m_e} = \frac{1.6\times10^{-19}}{2\times9.11\times10^{-31}} = 8.8\times10^{10} C\ Kg^{-1}$$

Q.12. A circular coil of wire consisting of 100 turns, each of radius 8.0 cm, carries a current of 0.40A. What is the magnitude of magnetic field at the centre of the coil?

Ans. Given,

$N = 100 \qquad T = 0.40A$

$R = 8.0\ cm = 0.8m$

we know that,

$$B = \frac{\mu_0}{\omega t}\frac{2\pi NT}{R}$$

$$= 10^{-7} \times 2 \times \frac{22}{7} \times \frac{100\times4}{0.8} \times \frac{100}{10}$$

$$= \frac{44}{7} \times \frac{4}{8} \times 10^{-7}$$

$$= 3.14 \times 10^{-4} T$$

Q.13. What is polarisation of light? State and explain Brewster's law.

Ans. Polarisation of light : During the propagation of light waves, the particles of medium, vibrate about their mean position in all direction in the plane prependicular to the direction of propagation of light waves. When the vibration of the particles take place, only in one direction, it known as polarisation.

Brewster's law : According to Brewster's law the refractive index of the refractive medium (μ) is numerically equal to the tangent of the angle of polarisation Op.

i.e., $\mu = \tan OP$

where $Op = Brewester\ angle$

Q.14. In Young's experiment, the slits are 0.03 cm apart and screen is 1.5 m away from the surface of the slits. The distance between the central fringe and the fourth bright fringe is 1 cm. Find the wavelength of light used.

Ans. Here,

$d = 0.03cm = 3\times10^{-4}m$

$D = 1.5m$

$m = 4,\ y = 1cm = 1\times10^{-2}m.$

Now, $y = \dfrac{m\lambda\theta}{d}$

$$\therefore \quad \lambda = \frac{yd}{mD} = \frac{1\times10^{-2}\times3\times10^{-4}}{4\times1.5}$$

$$= 0.5\times10^{-6}$$

$$= 5\times10^{-7}m$$

Or, $\quad wave\ length, \lambda = 5000A.$

Q.15. What is photoelectric effect? Establish Einstein's equation for photoelectric effect.

Ans. Photoelectric Effect : The phenomenos of photoelectric effect was first revealed by Hemrich Hertz in 1887. He observed that when light of suitable frequency falls on a metal surface electrons are produced. This is known as photoelectric emission. It is spontencous process.

Einstein photoelectric eqn. : Einstein described the phenomonon of photoelectric effect on the basis of the law of conservation of energy. According to Einstean, the energy of photon falling on a surface is used in the following ways.

(i) in ejecting out the electrons.

(ii) giving kinetic energy to the emitted electrons.

$$E = h\upsilon = WtK.E$$

In V_0 be the threshold frequency for the surface of a metal, then minimum energy required for emitting electrons for emitting electrons from the surface of the metal or the work function.

$$W = h\upsilon_0$$

$$\therefore \quad h\upsilon = \upsilon_0 + K.E.$$

but, $K.E. = \dfrac{1}{2}mv^2$

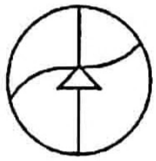
$$\therefore \quad h\upsilon = h\upsilon_0 + \frac{1}{2}mv^2$$

or, $\frac{1}{2}mv^2 = h(\upsilon - \upsilon_0)$

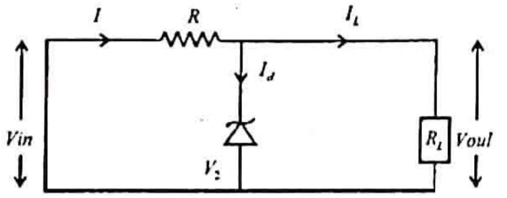
eq. (i) represent the Einstein's photoelectric eq^n. If υ (frequency of incident light) is less than υ_0 (threshold frequency) $K.E. = -Ve$. wh ich is not possible, therefore in such situation thermoelectric emission does not take place.

Q.16. What is a Zener diode? Explain its action as a voltage regulator.

Ans. Zener Diode : Zener diode is specially designed junction diode which can operate in the region of reverse breakdown voltage without being damaged. It is used in providing constant voltage supply.



· A zener diode when working in the breakdown region can serve as a voltage regulator.



In the figure,

Vin = Input d.c. voltage whose variations are to be regulated.

The zenor diode is reverse connected across Vin.

When P.d. across the diode is greater than V_z, it conducts and draws relatively large current through the series rasistance R. The load resistance R_L across which a constant voltage vout is required. is connected in parallel with the diode. The total current in parallel with the diode. The total current I passing through R equals the sum of diode current and load current.

Q.17. What is modulation? Why is modulation needed?

Ans. Modulation : The process of superimposition audio frequency signal over high frequency carrier waves is called modulation.

Need of modulation :

(i) Decrease the height (or size) of Transmitting Antenna.

(ii) Effective power radiated by an antenna

(iii) Mixing up of signals from different transmitters.

(iv) To superimpose the low frequency into high frequency.

Group-D

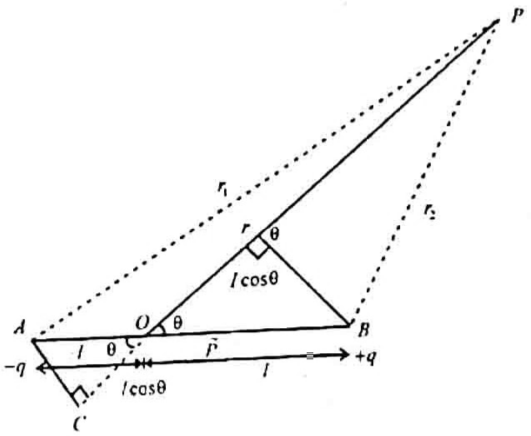
Answer the following questions:

Q.18. Define electric potential. Derive an expression for electric potential at any point due an electric dipole.

Ans. Electric Potential : Electric potential at a point is defined as the amount of work down in bringing a unit positive charge from infinity to that point.

Expression for electric potential at any point due to an electric dipole :

Consider any point P at a distance r from the centre (O) of the electrid dipole AB. Let OP makes an angle θ with the vector dipole vector moment $\vec{p}$ and r_1, r_2 be the distance of point P from $-q$ charge and $+q$ charge respectively. (Fig.)



Potential at P due to $-q$ charge,

$$V_1 = \frac{1}{4\pi\epsilon_0}\frac{(-q)}{r_1} = -\frac{1}{4\pi\epsilon_0}\frac{q}{r_1}$$

Potential at P due to $+q$ charge

$$V_2 = \frac{1}{4\pi\epsilon_0}\frac{q}{r_2}$$

$\therefore$ Potential at P due to the dipole,

$V = V_1 + V_2$ (principle of superposition)

or, $$V = -\frac{1}{4\pi\epsilon_0}\frac{q}{r_1} + \frac{1}{4\pi\epsilon_0}\frac{q}{r_2}$$

or, $$V = -\frac{q}{4\pi\epsilon_0}\left[\frac{1}{r_2} - \frac{1}{r_1}\right] \quad ...(i)$$

Draw a perpendicular from A which meets the line OP at C when produced backward. Also draw $BD \perp$ on OP.

Then,

$r_1 = AP \cong CP = OP + OC = r + l\cos\theta$

($\because$ from ΔAOC, $OC = l\cos\theta$)

$r_2 = BP \cong DP = OP - OD = r - l\cos\theta$

($\because$ from ΔBOD, $OD = l\cos\theta$)

Substituting the values of r_1 and r_2 in eq^n. (i), we get,

$$V = \frac{q}{4\pi\epsilon_0}\left(\frac{1}{(r - l\cos\theta)} - \frac{1}{(r + l\cos\theta)}\right)$$

$$= \frac{q}{4\pi\epsilon_0}\left(\frac{r + l\cos\theta - r + l\cos\theta}{(r^2 - l^2\cos^2\theta)}\right)$$

$$V = \frac{q}{4\pi\epsilon_0}\left(\frac{2l\cos\theta}{r^2 - l^2\cos^2\theta}\right) = \frac{q2l\cos\theta}{4\pi\epsilon_0(r^2 - l^2\cos^2\theta)}$$

$$V = \frac{P\cos\theta}{4\pi\epsilon_0(r^2 - l^2\cos^2\theta)} \quad (\because \text{dipole moment, } P = q.2l)$$

...(10)

If $r >> l$, then eq^n. (10) becomes

$$V = \frac{P\cos\theta}{4\pi\epsilon_0 r^2} \quad ...(11)$$

Since,

$P\cos\theta = \vec{P}.\hat{r}$, where $\hat{r}$ is unit vector directed along OP.

$$\therefore \quad V = \frac{\vec{P}\hat{r}}{4\pi\epsilon_0 r^2} \quad \text{for } r >> l \quad ...(12)$$

Special case :

1. If point P lies on the axial line of the dipole i.e, $\theta = 0°$ the eq^n. (11) becomes $V = \frac{P}{4\pi\epsilon_0 r^2}$ or $V \propto \frac{1}{r^2}$ [$\because \cos 0° = 1$]

2. If point P lies on the equatorial line of the dipole i.e. $\theta = 90°$

$V = 0 \quad [\because \cos 90° = 0]$

Potential due to a dipole is zero at all points on the equitorial line of the dipole.

Or,

Q. State Gauss's theorem in electrostatics. Use this theorem to derive expressions for electric field due to uniformly charged thin spherical shell at a point (i) outside and, (ii) inside the shell.

Ans. Gauss's theorem : The total electric flux passing through a closed surface is equal to $1/\epsilon_0$ times the charge enclosed within it.

Expression for the electric field intensity at a point due to a uniformly charged spherical shall (hollow sphere):

Let, $+q$ = the amount of charge which is uniformly distributed over a hollow spherical conductor.

R = the radius of the hollow spherical conductor.

O = Centre

(i) Outside the shell

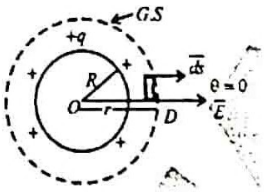
Let P = a point at a distance r from O which lies outside i.e., r > R, where electric field intensity is to be determined.

Let us draw a Gaussian surface through P with O as centre such that OP = r (radius).

$\overline{E}$ = the electric field intensity at P

$\overline{ds}$ = the outward normal.

It is clear that angle between $\overline{E}$ & $\overline{ds}$ is 0° at every point on the Gaussian surface.



Total electric on the Gaussian surface.

Total electric flux across G.S.-

$$\phi E = \oint E ds \cos\theta = \oint E ds \cos 0° = E \oint ds$$

$$= Es = E \times 4\pi r^2 \quad ..(1)$$

By Gauss's theorem,

$$\phi E = \frac{1}{\epsilon^0} \times q \quad ..(2)$$

Equating (1) & (2) we get

$$E \times 4\pi r^2 = \frac{1}{\epsilon^0} q$$

$$E = \frac{1}{4\pi \epsilon^0} \frac{q}{r^2} \quad ...(3)$$

In vector form

$$\overline{E} = \frac{1}{4\pi \epsilon^0} \frac{q}{r^2} \hat{r} \quad \text{(where } \hat{r} = \text{ unit vector of } r\text{)}$$

This is the required expression for electric field intensity when r > R.

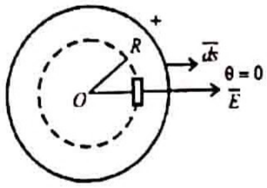
(ii) Inside the shell :

Let P = a point at O at a distance of r from O such that r < R, where electric intensity is to be determined.

Let $\widehat{E}$ = the electric intensity at P.

let us draw the G.S. through P with O as centre such athat OP = r (radius).

It is clear that angle between $\overline{E}$ & $\overline{ds}$ is 0°.



Total flux across G.S.

$$\phi E = \oint_s E ds \cos 0° = E \oint ds = ES$$

$$\Rightarrow \oint E = E \times 4\pi r^2 \quad ...(1)$$

By Gauss's theorem

$$\oint E = \frac{1}{\epsilon_0} q \quad ...(2)$$

Equating (1) & (2) we get

$$E \times 4\pi r^2 = \frac{1}{\epsilon_0} \times q = 0 \; [ere \; q = 0 \; inside \; G.S.]$$

$\therefore \quad E = 0$

Hence, electric intensity at a point inside the shell is zero.

Q.19. State Faraday's law of electromagnetic induction. Derive e x p r *emf* induced in a coil rotating uniformly in a uniformmagnetic field. Mention the nature of this *emf*.

Ans. Faraday's law of electromagnetic induction :

First Law: Whenever magnetic flux linked with a conductor changes, an e.m.f. is induced in it.

Second Law: The magnitude of the e.m.f. is directly proportional to the rate of change of magnetic flux linked with the conductor.

Formula for e.m.f. induced in coil rotating in a uniform magnetic field.

Let a coil having number of turns N, area A be placed in a uniform Magnetic field (B) such that at any instant normal to the plane of coil makes an angle θ with MF.

If the coil is rotated at a uniform angular speed (ω), then $\theta = \omega t$.

The magnetic flux linked with the coil at any instant,

$$\phi = NBA \cos\theta$$

$$\phi = NBA \cos \omega t$$

The e.m.f. induced,

$$e = \frac{-d\phi}{dt}$$

$$= \frac{d}{dt}(NBA \cos \omega t)$$

$$= -NBA \frac{d}{dt}(\cos \omega t)$$

$$= -NBA(-\sin \omega t) \times \omega$$

or, $e = NBA\omega \sin \omega t$.

The emf induced varies periodically with time both in magnitude and direction. The emf induced is maximum,

When $\theta = \omega t = 90°$

$$e\max = \varepsilon_0 = NBA\omega$$

$$\therefore \quad \varepsilon = \varepsilon_0 \sin \omega t.$$

Or,

Q. Derive expression for instantaneous current in a series L-C-R circuit if connected to an alternating emf. Obtain the condition for resonance.

Ans. Let a series LCR circuit put across an alternating source of voltage V given by

$$V = V_0 \sin \omega t \quad ...(i)$$

But V = Potential difference across Lt Potential difference across C + Potential difference accross R.

i.e. $V = \frac{Ldl}{dt} + \frac{q}{c} + IR$

Where, I is instantaneous current at instant t and q is charge store in the capacitor.

Using enq. (i), we get

$$V_c \sin \omega t = \frac{L\,dI}{dt} + \frac{q}{C} + IR \quad \text{...(ii)}$$

Since, $I = \frac{dq}{dt}$ we have $\frac{dI}{dt} = \frac{d^2q}{dt^2}$ ...(iii)

using (iii) in (ii) we have

$$V_c \sin \omega t = \frac{L\,d^2q}{dt^2} + R\frac{dq}{dt} + \frac{q}{C}$$

Equation (iv) is similar to that of equation of damped simple harmonic motion suppose the solution of this eqn. is,

$$q = q_0 \sin(\omega t + \theta)$$

Differentiating w.r.t. to t, we have,

$$\frac{dq}{dt} = q_0\omega\cos(\omega t + \theta)$$

Then, $\frac{d^2q}{dt^2} = -q_0\omega^2 \sin(\omega t + \theta)$

Substituting this value in eqn. (iv), we have

$$V_c \sin \omega t = q_0\omega\left[R\cos(\omega t + \theta) - \omega t\sin(\omega t + q) + \frac{1}{\omega C}\sin(\omega t + \theta)\right] \quad \text{...(v)}$$

As. $\omega L = X_L$ and $\frac{1}{\omega C} = X_C$, we have

$$V_c \sin \omega t = q_0 W\left[R\cos(\omega t + \theta) + (X_C - X_L)\sin(\omega t + \theta)\right] \quad \text{...(vi)}$$

Multiplying and dividing eqn. (vi) by $Z = \sqrt{R^2 + (X_C - X_L)^2}$, we get,

$$V_c \sin \omega t = q_0\omega Z\left[\frac{R}{Z}\cos(\omega t + \theta) + \frac{X_C - X_L}{Z}\sin(\omega t + \theta)\right] \quad \text{...(vii)}$$

If $\frac{R}{Z} = \cos\phi$ and $\frac{X_C - X_L}{Z} = \sin\phi$ ...(viii)

Then, $\frac{\sin\phi}{\cos\phi} = \frac{X_C - X_L}{R} = \tan\phi$

or, $\phi = \tan^{-1}\frac{X_C - X_L}{R}$

using (viii) in (vii), we get

$$V_c \sin \omega t = q_0\omega z\left[\cos\phi\cos(\omega t + \theta) + \sin\phi\sin(\omega t + \theta)\right]$$

or, $V_c \cos\left(\omega t - \frac{\pi}{2}\right) = q_0\omega Z(\omega t + \theta - \phi)$ ...(ix)

comparing both side of (viii), we find that

$V_0 = q_0\omega Z$ i.e.,

But, $q_0 W = I_o$ So $V_o = I_o Z$ or $I_o = \frac{V_o}{Z}$ ..(x)

and $(\theta - \phi) = -\frac{\pi}{2}$ i.e., $\theta = \phi - \frac{\pi}{2}$

current in the circuit, $I = \frac{dq}{dt} = \frac{d}{dt}[q_o \sin(\omega t + \theta)]$

i.e. $I = q_o\omega\cos(\omega t + \theta)$

substituting $\theta = \left[\phi - \frac{\pi}{2}\right]$, we get

$$I = I_o \cos\left(\omega t + \phi - \frac{\pi}{2}\right)$$

i.e., $I = I_o \sin(\omega t + \phi)$

from eqn. (viii)

$$\cos^2\phi + \sin^2\phi = \frac{R^2 + (X_C - X_L)^2}{Z^2}$$

Q.20. For refraction at any spherical surface, establish the relation,

$\frac{\mu_2}{v} - \frac{\mu_1}{u} = \frac{\mu_2 - \mu_1}{R}$; **where terms have usual meanings.**

Ans. Consider a convex spherical refracting surface of refractive index n_2 (fig.) Let it be placed in a rare medium of refractive index n_1.

$(n_2 > n_1)$

A point object O lies on the principle axis at a distance U from the pole of the convex refracting surface in rarer medium. A ray of light from O indicence the convex surface at A. Ac is the normal to the convex surface. After refraction at A, the ray enters the denser medium and bends towards the normal. The refracted ray meets the principal axis at I which is the real image of the object O. The distance of the image I from the pole of the convex surface is V.

Step I : Determination of i and r. Let α, β and γ be the angles made by the incident ray, refracted ray and the normal respectively with the principal axis. Draw An perpendicular on the pricipal axis.

Form ΔAOC, $i = \alpha + \gamma$ ...(i)

and from ΔAIC, $\gamma = r + \beta$ or $r = \gamma - \beta$ ...(ii)

α, γ and β are small angles (assumption),

using the relation $\theta = \frac{1}{r}$, we get

$$\alpha = \frac{AN}{NO}, \ \gamma = \frac{AN}{NC} \text{ and } \beta = \frac{AN}{Ni} \quad \text{...(iii)}$$

Then, $i = \frac{AN}{NO} + \frac{AN}{NC} \simeq \frac{AN}{PO} + \frac{AN}{PC}$ ...(iv)

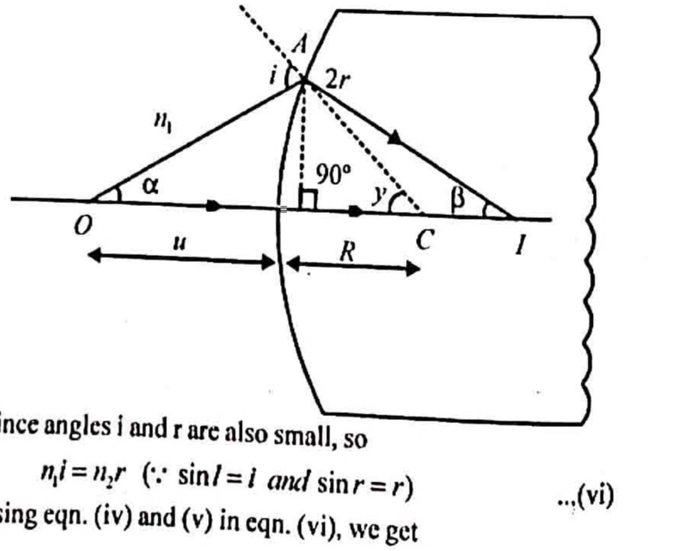
and $r = \frac{AN}{NC} - \frac{AN}{NI} \simeq \frac{AN}{PC} - \frac{AN}{PI}$ ...(v)

Since a perture of the spherical surface is assumed to be small, so point N lies very close to point P.

$\therefore$ $No \simeq PO$, $NC \simeq PC$ and $NI \simeq PI$

Step 2 : Now according to snell's law,

$$\frac{\sin i}{\sin r} = \frac{n_2}{n_1} \text{ or } n_1 \sin i = n_2 \sin r.$$



since angles i and r are also small, so

$n_1 i = n_2 r$ ($\because$ $\sin I = i$ and $\sin r = r$) ...(vi)

using eqn. (iv) and (v) in eqn. (vi), we get

$$n_1\left[\frac{AN}{PO}+\frac{AN}{PC}\right]=n_2\left[\frac{AN}{PC}-\frac{AN}{PI}\right]$$

or, $$\frac{n_1}{PO}+\frac{n_2}{PI}=\frac{n_2-n_1}{PC} \qquad \text{(vii)}$$

Step 3 : Applying new cartesian sign conventions,

$$PO=-u,\ PC=R,\ PI=v$$

Hence eqn. (vii) can be written as

$$\frac{-n_1}{u}+\frac{n_2}{v}=\frac{n_2-n_1}{R}$$

Or,

Q. State Huygens' principle. Establish the laws of refraction of light using Huygens' principle.

Ans. Huygen's Principal :

(i) Each source of light is a centre of disturbance from which waves spread in all direction. All particles equidistant from the source known as wavefront.

(ii) Every point on a wavefront is a source of new disturbance which produces secondary wavelets. These wavelets are spherical and travel with the speed of light in all directions in that medium.

(iii) Only forward envelop enclosing the tangents at the secondary wavelets at any instant gives the new position of wavefront.

Law of Refraction on the basis of Huygen's principal:

Let AB be the plane wavefront incidence on the boundry XY separating two media of refractive indices n_1 and n_2 (n_2/n_1) at A (Fig.). Let v_1 and v_2 be the speeds of light in the twomedia (I and II) respectively (v_1/v_2).

According to Huygen's principle, each point on the plane wavefront AB acts as a source of new disturbance. The position of the wavefront AB after time t is given by the wavefrot CD in the denser medium.

Time taken by wavelets to reach from B to C and from A to D is given by

$$t=\frac{BC}{v_1}=\frac{AD}{v_2}$$

CD will be the true refracted wavefront if the secondary wavelets arising from any point (say P) or the incident wavefront take the same time to reach the wavefront CD at point R via point Q.

Time takes by the secondary wavelets to go from point P to point R is given by

$$t=\frac{PQ}{v_1}+\frac{QR}{v_2}$$

from ΔPAQ, $\quad \sin i=\frac{PQ}{AQ}$ or $PQ=AQ\sin i \qquad$ (ii)

from ΔQCR, $\quad \sin r=\frac{QR}{QC}$ or $QR=QC\sin r$

$$QR=(AC-AQ)\sin r \qquad \text{(iii)}$$

Substituting the value of eqn. (ii) and (iii) in eqn. (i), we get

$$t=\frac{AQ\sin i}{v_1}+\frac{(AC-AQ)\sin r}{v_2}=AQ\left(\frac{\sin i}{v_1}-\frac{\sin r}{v_2}\right)+\frac{AC\sin r}{v_2} \qquad \text{...(iv)}$$

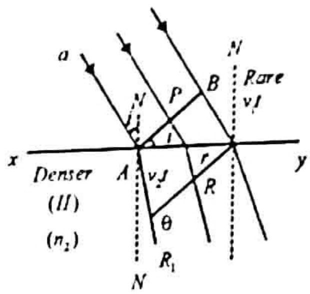
Since this time is independent of the position of point Q, so that

term $AQ=\left(\frac{\sin i}{v_1}-\frac{\sin r}{v_2}\right)$

i.e.,

$$AQ=\left(\frac{\sin i}{v_1}-\frac{\sin r}{v_2}\right)=0$$

or, $$\frac{\sin i}{\sin r}=\frac{v_1}{v_2}$$



multiplying and dividing R.H.S. by C, velocity of light in vacum, we get

$$\frac{\sin i}{\sin r}=\frac{\left(\frac{C}{v_2}\right)}{\left(\frac{C}{v_1}\right)}=\frac{n_2}{n_1} \qquad \text{...(v)}$$

where n_1 and n_2 are the refractive indices of medium 1 and medium 2 respectively. Eqn. (v) can be written as

$$\boxed{n_1\sin i=n_2\sin r}$$

which is snell's law of refraction.

□□□