Mathematics - 2018

सामान्य निर्देश : General Instructions :
इस प्रश्न पत्र में 20 प्रश्न हैं, जो तीन खण्डों अ, ब और स में बंटे हुए हैं। खण्ड अ में 10 प्रश्न हैं, जिनमें प्रत्येक 1 अंक का है, खण्ड ब में 12 प्रश्न हैं जिनमें प्रत्येक 4 अंक का है तथा खण्ड स में 7 प्रश्न हैं जिनमें प्रत्येक 6 अंक का है।
कैलकुलेटर के उपयोग की अनुमति नहीं है। आवश्यकता हो तो परीक्षार्थी के माँग पर लघुगणकीय अथवा सांख्यिकीय सारणी उपलब्ध कराई जा सकती है।

Section-A
(Objective Questions)

Q.1. Let * be the binary operation defined by a * b = 3a + b – 2. Find the value of 3 * 5.

Sol. $\because \quad a * b = 3a + b - 2$

$\therefore \quad 3 * 5 = 3\times 3 + 5 - 2$

$= 9 + 3 = 12$ Ans.

Q.2. Find the value of $\sin(\tan^{-1}x + \cot^{-1}x)$.

Sol. $\sin\left(\tan^{-1}x + \cot^{-1}x\right) = \sin\frac{\pi}{2} = 1$ Ans.

Q.3. Construct a (2×3) matrix whose elements are given by $a_{ij} = 2i + j$.

Sol. Let $A = [a_{ij}]_{2\times 3}$

$$A = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \end{bmatrix} \quad ...(i)$$

$\because \quad a_{ij} = 2\hat{i} + \hat{j}$

$\therefore \quad a_{11} = 2\times 1 + 1 = 3,\ a_{12} = 2\times 1 + 2 = 4,\ a_{13} = 2\times 1 + 3 = 5$

$a_{21} = 2\times 2 + 1 = 5,\ a_{22} = 2\times 2 + 6 = 6,\ a_{23} = 2\times 2 + 3 = 7$

$$A = \begin{bmatrix} 3 & 4 & 5 \\ 5 & 6 & 7 \end{bmatrix} \text{ Ans.}$$

Q.4. Find the value of x : $\begin{vmatrix} 8 & 3 \\ 4 & x \end{vmatrix} = \begin{vmatrix} x & 2 \\ 2 & 4 \end{vmatrix}$

Sol. $\begin{vmatrix} 8 & 3 \\ 4 & x \end{vmatrix} = \begin{vmatrix} x & 2 \\ 2 & 4 \end{vmatrix}$

$\Rightarrow \quad 8x - 12 = 4x - 4$

$\Rightarrow \quad 8x - 4x = 12 - 4$

$\Rightarrow \quad 4x = 8$

$\Rightarrow \quad x = \frac{8}{4} = 2$ Ans.

Q.5. Find the slope of the tangent to the curve $y = x^3 - x$ at x=2.

Sol. $y = x^3 - x$

$\frac{dy}{dx} = 3x^2 - 1 \quad ...(i)$

$\therefore$ Slope of tangent at $x = 2$ is $= \left[\frac{dy}{dx}\right]_{x=2}$

$= 3\times 2^2 - 1$ by (i)

$= 12 - 1 = 11$ Ans.

Q.6. Find $\frac{dy}{dx}$: $y = \cos\sqrt{\sin x}$

Sol. $\because \quad y = \cos\sqrt{\sin x}$

$\frac{dy}{dx} = \frac{d}{dx}\left\{\cos\sqrt{\sin x}\right\}$

$= -\sin\sqrt{\sin x} \times \frac{1}{2\sqrt{\sin x}} \times \cos x$

$= \frac{-\cos x.\sin\sqrt{\sin x}}{2\sqrt{\sin x}}$ Ans.

Q.7. Find the value of $\int \frac{e^{\tan^{-1}z}}{1+z^2}dz$.

Sol. Let $I = \int \frac{e^{\tan^{-1}z}}{1+z^2}dz \quad ...(i)$

Let $\tan^{-1}z = t \quad ...(ii)$

$\Rightarrow \quad \frac{1}{1+z^2}dz = dt \quad ...(iii)$

Using (ii) and (iii) in (i), we get

$I = \int e^t dt$

$= e^t + c$

$I = e^{\tan^{-1}z} + C$ Ans.

Q.8. Find a unit vector in the direction of vector $(6\hat{i} + 8\hat{j})$.

Sol. Let $\vec{a} = 6\hat{i} + 8\hat{j}$

$|\vec{a}| = \sqrt{6^2 + 8^2} = \sqrt{36 + 64} = \sqrt{100} = 10$

$\therefore$ Unit vector in the direction of given vector $\vec{a}$ is

$\hat{a} = \frac{\vec{a}}{|\vec{a}|}$

$= \frac{6\hat{i} + 8\hat{j}}{10} = \frac{6}{10}\hat{i} + \frac{8}{10}\hat{j}$

$= \frac{3}{5}\hat{i} + \frac{4}{5}\hat{j}$ Ans.

Q.9. Find the projection of $\vec{a} = 2\hat{i} + \hat{j} + \hat{k}$ on $\vec{b} = \hat{i} + 2\hat{j} + \hat{k}$.

Sol. $\vec{a} = 2\hat{i} + \hat{j} + \hat{k}$

$\vec{b} = \hat{i} + 2\hat{j} + \hat{k}$

$\therefore \quad \vec{a}.\vec{b} = 2\times 1 + 1\times 2 + 1\times 1 = 2 + 2 + 1 = 5$

$|\vec{b}| = \sqrt{1^2 + 2^2 + 1^2} = \sqrt{6}$

$\therefore$ Projection of $\vec{a}$ on $\vec{b} = \frac{\vec{a}.\vec{b}}{|\vec{b}|} = \frac{5}{\sqrt{6}}$ Ans.

Q.10. Find the direction ratios and direction cosines of the vector $\vec{r} = 2\hat{i} + 3\hat{j} + 4\hat{k}$.

Sol. $\vec{r} = 2\hat{i} + 3\hat{j} + 4\hat{k}$

Direction ratios of $\vec{r} = 2, 3, 4$

Direction cosines of

$\vec{r} = \frac{2}{\sqrt{2^2+3^2+4^2}}, \frac{3}{\sqrt{2^2+3^2+4^2}}, \frac{4}{\sqrt{2^2+3^2+4^2}}$

$= \frac{2}{\sqrt{29}}, \frac{3}{\sqrt{29}}, \frac{4}{\sqrt{29}}$ Ans.

Section-B (खण्ड-ब)

Q.11. Let $f: R \to R$ and $g: R \to R$ be defined by $f(x) = x^2 - 1$ and $g(x) = 2x + 3$. Find fog and gof.

Sol. $f: R \to R$ and $g: R \to R$ be defined by

$f(x) = x^2 - 1$ ...(i)

$g(x) = 2x + 3$ ...(ii)

$\therefore$ $fog(x) = f\{g(x)\}$

$= f(2x+3)$, by (ii)

$= (2x+3)^2 - 1$, by (i)

$= 4x^2 + 2\times 2x \times 3 + 3^2 - 1$

$= 4x^2 + 12x + 8$ Ans.

Again $gof(x) = g\{f(x)\}$

$= g\{x^2 - 1\}$, by (i)

$= 2\times(x^2-1) + 3$, by (ii)

$= 2x^2 - 2 + 3$

$= 2x^2 + 1$ Ans.

Q.12. Show that $\tan^{-1}\frac{1}{3} + \tan^{-1}\frac{1}{5} + \tan^{-1}\frac{1}{7} + \tan^{-1}\frac{1}{8} = \frac{\pi}{4}$.

Sol. L.H.S. $= \tan^{-1}\frac{1}{3} + \tan^{-1}\frac{1}{5} + \tan^{-1}\frac{1}{7} + \tan^{-1}\frac{1}{8}$

$= \tan^{-1}\left(\frac{\frac{1}{3}+\frac{1}{5}}{1-\frac{1}{3}\times\frac{1}{5}}\right) + \tan^{-1}\left(\frac{\frac{1}{7}+\frac{1}{8}}{1-\frac{1}{7}\times\frac{1}{8}}\right)$ $\left[\because \tan^{-1}x + \tan^{-1}y = \tan^{-1}\frac{x+y}{1-xy}\right]$

$= \tan^{-1}\left(\frac{\frac{5+3}{15}}{\frac{15-1}{15}}\right) + \tan^{-1}\left(\frac{\frac{8+7}{56}}{\frac{56-1}{56}}\right)$

$= \tan^{-1}\left(\frac{8}{14}\right) + \tan^{-1}\left(\frac{15}{55}\right)$

$= \tan^{-1}\left(\frac{4}{7}\right) + \tan^{-1}\left(\frac{3}{11}\right)$

$= \tan^{-1}\left(\frac{\frac{4}{7}+\frac{3}{11}}{1-\frac{4}{7}\times\frac{3}{11}}\right) = \tan^{-1}\left(\frac{\frac{44+21}{77}}{\frac{77-12}{77}}\right)$

$= \tan^{-1}\left(\frac{65}{65}\right) = \tan^{-1} = \frac{\pi}{4}$ =R.H.S. Proved

Q.13. Prove that $\begin{vmatrix} a-b-c & 2a & 2a \\ 2b & b-c-a & 2b \\ 2c & 2c & c-a-b \end{vmatrix} = (a+b+c)^3$

Sol. Let $\Delta = \begin{vmatrix} a-b-c & 2a & 2a \\ 2b & b-c-a & 2b \\ 2c & 2c & c-a-b \end{vmatrix}$

Applying $R_1 \to R_2 + R_3$

$= \begin{vmatrix} a+b+c & a+b+c & a+b+c \\ 2b & b-c-a & 2b \\ 2c & 2c & c-a-b \end{vmatrix}$

Taking $(a+b+c)$ common from R_1

$= (a+b+c)\begin{vmatrix} 1 & 1 & 1 \\ 2b & b-c-a & 2b \\ 2c & 2c & c-a-b \end{vmatrix}$

Applying $C_1 \to C_1 - C_3$, $C_2 \to C_2 - C_3$

$= (a+b+c)\begin{vmatrix} 0 & 0 & 1 \\ 0 & -b-c-a & 2b \\ c+a+b & c+a+b & c-a-b \end{vmatrix}$

Taking $(a+b+c)$ common from C_1 and C_2 respectively

$= (a+b+c)^3\begin{vmatrix} 0 & 0 & 1 \\ 0 & -1 & 2b \\ 1 & 1 & c-a-b \end{vmatrix}$

Expanding along R_1

$= (a+b+c)^3\left\{0 - 0 + 1\begin{vmatrix} 0 & -1 \\ 1 & 1 \end{vmatrix}\right\}$

$= (a+b+c)^3(1(0+1))$

$= (a+b+c)^3 \times 1$

$= (a+b+c)^3$ proved.

OR

Prove that $\begin{vmatrix} 1 & a & a^2 \\ 1 & b & b^2 \\ 1 & c & c^2 \end{vmatrix} = (a-b)(b-c)(c-a)$

Sol. Let $\Delta = \begin{vmatrix} 1 & a & a^2 \\ 1 & b & b^2 \\ 1 & c & c^2 \end{vmatrix}$

Applying $R_1 \to R_1 - R_3$, $R_2 \to R_2 - R_3$

$= \begin{vmatrix} 0 & a-c & a^2-c^2 \\ 0 & b-c & b^2-c^2 \\ 1 & c & c^2 \end{vmatrix}$

Taking $(a-c)$ and $(b-c)$ common from R_1 & R_2 respectively

$= (a-c)(b-c)\begin{vmatrix} 0 & 1 & a+c \\ 0 & 1 & b+c \\ 1 & c & c^2 \end{vmatrix}$

Applying $R_1 \to R_1 - R_2$

$$= (a-c)(b-c)\begin{vmatrix} 0 & 0 & a-b \\ 0 & 1 & b+c \\ 1 & c & c^2 \end{vmatrix}$$

Expanding along R_1, we get

$$= (a-c)(b-c)\left\{0 - 0 + (a-b)\begin{vmatrix} 0 & 1 \\ 1 & c \end{vmatrix}\right\}$$

$$= (a-c)(b-c)\{(a-b)(0-1)\}$$

$$= (a-c)(b-c)(a-b)(-1)$$

$$= (a-b)(b-c)(c-a) \qquad \text{Proved}$$

Q.14. Find the value of k so that the function $f(x)$ is continuous at $x = 5$.

$$f(x) = \begin{cases} kx+1, & \text{if } x \le 5 \\ 3x-5, & \text{if } x > 5 \end{cases}$$

Sol. We have, $f(x) = \begin{cases} kx+1, & \text{if } x \le 5 \\ 3x-5, & \text{if } x > 5 \end{cases}$ ...(i)

$\because$ $f(x)$ is continuous at $x = 5$

$\therefore$ $\lim_{x\to 5^+} f(x) = \lim_{x\to 5^-} f(x) = f(5)$

$\Rightarrow$ $\lim_{x\to 5^+}(3x-5) = \lim_{x\to 5^-}(kx+1) = k\times 5+1$ by. (i)

$\Rightarrow$ $\lim_{x\to 5^+}(3x-5) = 5k+1$

$\Rightarrow$ $\lim_{h\to 0}[3(5+h)-5] = 5k+1$

$\Rightarrow$ $3(5+0)-5 = 5k+1$

$\Rightarrow$ $15-5 = 5k+1$

$\Rightarrow$ $10 = 5k+1$

$\Rightarrow$ $5k = 10-1$

$k = \frac{9}{5}$ Ans.

Q.15. Differentiate $\sqrt{\frac{(x-1)(x-2)}{(x-3)(x-4)(x-5)}}$ with respect to x.

Sol. Let $y = \sqrt{\frac{(x-1)(x-2)}{(x-3)(x-4)(x-5)}}$

Taking log on both sides

$$\log y = \log\sqrt{\frac{(x-1)(x-2)}{(x-3)(x-4)(x-5)}} = \log\left(\frac{(x-1)(x-2)}{(x-3)(x-4)(x-5)}\right)^{\frac{1}{2}}$$

$$= \frac{1}{2}\log\left[\frac{(x-1)(x-2)}{(x-3)(x-4)(x-5)}\right] \quad \{\because \log m^n = n\log m\}$$

$$\log y = \frac{1}{2}[\log(x-1)(x-2) - \log(x-3)(x-4)(x-5)]$$

$\Rightarrow$ $\log y = \frac{1}{2}[\{\log(x-1)+\log(x-2)\} - \{\log(x-3)+\log(x-4)+\log(x-5)\}]$

Differentiating both sides w.r.t. 'x'

$$\frac{d}{dx}(\log y) = \frac{d}{dx}\left[\frac{1}{2}\{\log(x-1)+\log(x-2)-\log(x-3)-\log(x-4)-\log(x-5)\}\right]$$

$$\Rightarrow \frac{1}{y}\times\frac{dy}{dx} = \frac{1}{2}\left[\frac{1}{x-1}\times(1-0) + \frac{1}{(x-2)}\times(1-0) - \frac{1}{(x-3)}\times(1-0) - \frac{1}{(x-4)}\times(1-0) - \frac{1}{x-5}\times(1-0)\right]$$

$$\frac{dy}{dx} = \frac{y}{2}\left[\frac{1}{x-1} + \frac{1}{x-2} - \frac{1}{x-3} - \frac{1}{x-4} - \frac{1}{x-5}\right]$$

$$\frac{dy}{dx} = \frac{1}{2}\sqrt{\frac{(x-1)(x-2)}{(x-3)(x-4)(x-5)}}\left[\frac{1}{x-1} + \frac{1}{x-2} - \frac{1}{x-3} - \frac{1}{x-4} - \frac{1}{x-5}\right] \text{Ans.}$$

OR

If $x^y + y^x = 1$, find $\frac{dy}{dx}$.

Sol. We have, $x^y + y^x = 1$

$\Rightarrow$ $u + v = 1$ (let) ...(i)

$\therefore$ $\frac{du}{dx} + \frac{dv}{dx} = \frac{d}{dx}(1)$

$\Rightarrow$ $\frac{du}{dx} + \frac{dv}{dx} = 0$ ...(ii)

Now $u = x^y$

$\Rightarrow$ $\log u = \log x^y$

$\Rightarrow$ $\log u = y\log x$

Diff. both sides w.r.t. 'x'

$$\frac{d}{dx}(\log u) = \frac{d}{dx}(y\log x)$$

$\Rightarrow$ $\frac{1}{u}\cdot\frac{du}{dx} = y\times\frac{1}{x} + \log x\times\frac{dy}{dx}$

$\Rightarrow$ $\frac{du}{dx} = u\left(\frac{y}{x} + \log x.\frac{dy}{dx}\right)$

$\Rightarrow$ $\frac{du}{dx} = x^y\left(\frac{y}{x} + \log x.\frac{dy}{dx}\right)$ ...(iii)

Again $v = y^x$

$\Rightarrow$ $\log v = \log y^x$

$\Rightarrow$ $\log v = x\log y$

Diff. both sides w.r.t. 'x'

$$\frac{d}{dx}(\log v) = \frac{d}{dx}(x.\log y)$$

$\Rightarrow$ $\frac{1}{v}\cdot\frac{dv}{dx} = x\times\frac{1}{y}\frac{dy}{dx} + \log y\times 1$

$\Rightarrow$ $\frac{dv}{dx} = v\left(\frac{x}{y}\frac{dy}{dx} + \log y\right)$

$\frac{dv}{dx} = y^x\left(\frac{x}{y}\frac{dy}{dx} + \log y\right)$ ...(iv)

Using (iii) and (iv) in (ii), we get

$$x^y\left(\frac{y}{x} + \log x.\frac{dy}{dx}\right) + y^x\left(\frac{x}{y}\frac{dy}{dx} + \log y\right) = 0$$

$\Rightarrow$ $x^y.\frac{y}{x} + x^y.\log x.\frac{dy}{dx} + y^x.\frac{x}{y}\frac{dy}{dx} + y^x.\log y = 0$

$\Rightarrow$ $\frac{dy}{dx}[x^y\log x + y^{x-1}.x] = -x^{y-1}.y - y^x\log y$

$\therefore$ $\frac{dy}{dx} = -\left(\frac{x^{y-1}.y + y^x\log y}{x^y\log x + y^{x-1}.x}\right)$ Ans.

Q.16. Find the intervals on which the function $f(x) = (x+1)^3(x-1)^3$ is (a) increasing, (b) decreasing.

Sol. $f(x) = (x+1)^3(x-1)^3$ ...(i)

$\Rightarrow f'(x) = (x+1)^3 \cdot 3(x-1)^2 + (x-1)^3 \cdot 3(x+1)^2$

$= 3(x-1)^2(x+1)^2[(x+1)+(x-1)]$

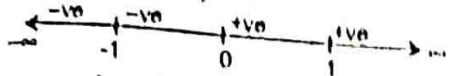
$= 3(x-1)^2(x+1)^2 \times 2x$

$f'(x) = 6x(x-1)^2(x+1)^2$ ...(ii)

For $f(x)$ to be increasing, $f'(x) > 0$

$\Rightarrow 6x(x-1)^2(x+1)^2 > 0$, by (ii)

$\Rightarrow x(x-1)^2(x+1)^2 > 0$



$\Rightarrow x \in]0, \infty[$

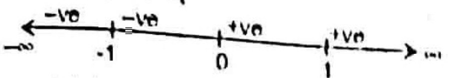
$\therefore$ $f(x)$ is increasing in $]0, \infty[$

Again for $f(x)$ to be decreasing $f'(x) < 0$

$\Rightarrow 6x(x-1)^2(x+1)^2 < 0$, by (ii)

$\Rightarrow x(x-1)^2(x+1)^2 < 0$

$\Rightarrow x \in]-\infty, 0[$



$\therefore$ $f(x)$ is decreasing in $]-\infty, 0[$

OR

A stone is dropped into a quiet lake and waves move in circles at a speed of 4 cm per second. At the instant, when the radius of the circular wave is 10 cm, how fast is the enclosed area increasing?

Sol. Let at any time 't', r is the radius to circular wave and A is the enclosed area.

$\frac{A}{q}\frac{dr}{dt} = 4\ cm/sec$

$\left[\frac{dA}{dt}\right]_{r=10cm} = ?$

$\because A = \pi r^2$

$\Rightarrow \frac{dA}{dt} = \frac{d}{dt}(\pi r^2)$

$\Rightarrow \frac{dA}{dt} = \pi \times 2r\frac{dr}{dt}$

$\therefore \left[\frac{dA}{dt}\right]_{r=10} = \pi \times 2 \times 10 \times 4$

$= 80\pi\ cm^2/sec$ Ans.

Q.17. Find the value of $\int \frac{2x+1}{\sqrt{x^2+2x-1}}dx$.

Sol. Let $I = \int \frac{2x+1}{\sqrt{x^2+2x-1}}dx$ ...(i)

Let $2x+1 = A\frac{d}{dx}(x^2+2x+1)+B$

$\Rightarrow 2x+1 = A(2x+2)+B$ ...(ii)

Equating the coefficient of like terms

$2 = 2A \Rightarrow A = 1$

and $1 = 2A+B \Rightarrow B = 1-2A$

$= 1-2\times1$

$\Rightarrow B = -1$

Putting the value of A and B in (ii), we get

$2x+1 = 1(2x+2) - 1$ ...(iii)

Using (iii) in (i), we get $I = \int \frac{1(2x+2)-1}{\sqrt{x^2+2x-1}}dx$

$\Rightarrow I = \int \frac{2x+2}{\sqrt{x^2+2x-1}}dx - \int \frac{1}{\sqrt{x^2+2x-1}}dx$

Let $x^2+2x-1 = t$

$\Rightarrow (2x+2)dx = dt$

$I = \int \frac{dt}{\sqrt{t}} - \int \frac{1}{\sqrt{x^2+2x+1+1^2-1^2-1}}dx$

$= \int t^{-\frac{1}{2}}dt - \int \frac{1}{\sqrt{(x+1)^2-(\sqrt{2})^2}}dx$

$= \frac{t^{\frac{1}{2}}}{\left(\frac{1}{2}\right)} - \log\left|(x+1)+\sqrt{(x+1)^2-(\sqrt{2})^2}\right| + C$

$= 2\sqrt{t} - \log\left|(x+1)+\sqrt{x^2+2x-1}\right| + C$

$\Rightarrow I = 2\sqrt{x^2+2x-1} - \log\left|x+1+\sqrt{x^2+2x-1}\right| + C$

Q.18. Find the value of $\int \frac{\cos x}{(1-\sin x)(2-\sin x)}dx$.

Sol. Let $I = \int \frac{\cos x}{(1-\sin x)(2-\sin x)}dx$ ...(i)

Let $\sin x = t$

$\Rightarrow \cos x\, dx = dt$

$\therefore I = \int \frac{dt}{(1-t)(2-t)}$ ...(ii)

Let $\frac{1}{(1-t)(2-t)} = \frac{A}{1-t} + \frac{B}{2-t}$ ...(iii)

$\Rightarrow \frac{1}{(1-t)(2-t)} = \frac{A(2-t)+B(1-t)}{(1-t)(2-t)}$

$\therefore 1 = A(2-t)+B(1-t)$ ...(iv)

Put $t = 2$ in (iv)

$1 = A\times0 + B(1-2) \Rightarrow 1 = -B$

$\therefore B = -1$

Again put $t = 1$ in (iv)

$1 = A(2-1) + B\times D \Rightarrow 1 = A$

Putting the value of A and B in (iii), we get

$\frac{1}{(1-t)(2-t)} = \frac{1}{1-t} - \frac{1}{2-t}$ ...(v)

Using (v) in (ii), we get, $I = \int\left(\frac{1}{1-t} - \frac{1}{2t}\right)dt$

$= \int \frac{1}{1-t}dt - \int \frac{1}{2-t}dt$

$= \frac{\log|1-t|}{-1} - \frac{\log|2-t|}{-1} + C$

$= \log|2-t| - \log|1-t| + C$

$= \log|2-\sin x| - \log|1-\sin x| + C$

Ans.

OR

Evaluate $\int_0^5 (x+1)dx$ as a limit of a sum.

Sol. Let $I = \int_0^5 (x+1)dx$ ...(i)

here $a = 0,\ b = 5,\ f(x) = x+1$ ...(ii)

$\therefore \quad xh = b - a = 5 - 0 = 5$

Integrating as the limit of sums

$$I = \lim_{h\to 0} h\left[f(a) + f(a+h) + f(a+2h) + \ldots\ldots\ldots + f\{a+(n-1)h\}\right]$$

$$= \lim_{h\to 0} h\left[f(0) + f(0+h) + f(0+2h) + \ldots\ldots + f\{0+(n-1)h\}\right]$$

$$= \lim_{h\to 0} h\left[f(0) + f(h) + f(2h) + \ldots\ldots + f((n-1)h)\right]$$

$$= \lim_{h\to 0} h\left[(0+1) + (h+1) + (2h+1) + \ldots\ldots + \{(n-1)h+1\}\right]$$

$$= \lim_{h\to 0} h\left[\{h + 2h + \ldots\ldots + (n-1)\} + \{1+1+1+\ldots\ldots+1\}\right]$$

$$= \lim_{h\to 0} h\left[(1+2+\ldots\ldots+(n-1))h + n\times 1\right]$$

$$= \lim_{h\to 0} h\left[\frac{n(n-1)}{2}h + n\right] = \lim_{h\to 0}\left[\frac{nhn(xh-h)}{2} + nh\right]$$

$$= \lim_{h\to 0}\left[\frac{5(5-h)}{2} + 5\right] = \frac{5(5-0)}{2} + 5$$

$$= \frac{25+10}{2} = \frac{35}{2} \quad \text{Ans.}$$

Q.19. Find the value of $\int_0^{\pi/2} \frac{\cos^3 x}{\cos^3 x + \sin^3 x}dx$.

Sol. Let $I = \int_0^{\pi/2} \frac{\cos^3 x}{\cos^3 x + \sin^3 x}dx$ ...(i)

$$\Rightarrow \quad I = \int_0^{\pi/2} \frac{\cos^3\left(\frac{\pi}{2} - x\right)}{\cos^3\left(\frac{\pi}{2} - x\right) + \sin^3\left(\frac{\pi}{2} - x\right)}dx$$

$$\Rightarrow \quad I = \int_0^{\pi/2} \frac{\sin^3 x}{\sin^3 x + \cos^3 x}dx \quad \text{...(ii)}$$

Adding (i) and (ii), we get

$$I + I = \int_0^{\pi/2} \frac{\cos^3 x}{\cos^3 x + \sin^3 x}dx + \int_0^{\pi/2} \frac{\sin^3 x}{\sin^3 x + \cos^3 x}dx$$

$$\Rightarrow \quad 2I = \int_0^{\pi/2} \frac{\cos^3 x + \sin^3 x}{\cos^3 x + \sin^3 x}dx$$

$$= [x]_0^{\frac{\pi}{2}}$$

$$2I = \frac{\pi}{2} - 0$$

$$\Rightarrow \quad I = \frac{\pi}{4} \quad \text{Ans.}$$

Q.20. If $\vec{a} = 2\hat{i} + 2\hat{j} + 3\hat{k}$, $\vec{b} = -\hat{i} + 2\hat{j} + \hat{k}$ and $\vec{c} = 3\hat{i} + \hat{j}$ are such that $\left(\vec{a} + \lambda\vec{b}\right)$ is perpendicular to $\vec{c}$, then find the value λ.

Sol. We have, $\vec{a} = 2\hat{i} + 2\hat{j} + 3\hat{k}$

$\vec{b} = -\hat{i} + 2\hat{j} + \hat{k}$

$\vec{c} = 3\hat{i} + \hat{j}$

A/q. $\therefore \left(\vec{a} + \lambda\vec{b}\right) \perp \vec{c}$

$$\Rightarrow \quad \left(\vec{a} + \lambda\vec{b}\right)\cdot\vec{c} = 0$$

$$\Rightarrow \quad \left[\left(2\hat{i} + 2\hat{j} + 3\hat{k}\right) + \lambda\left(-\hat{i} + 2\hat{j} + \hat{k}\right)\right]\left(3\hat{i} + \hat{j}\right) = 0$$

$$\Rightarrow \quad \left[(2-\lambda)\hat{i} + (2+2\lambda)\hat{j} + (3+\lambda)\hat{k}\right]\left(3\hat{i} + \hat{j}\right) = 0$$

$$\Rightarrow \quad (2-\lambda).3 + (2+2\lambda).1 = 0$$

$$\Rightarrow \quad 6 - 3\lambda + 2 + 2\lambda = 0$$

$$\Rightarrow \quad 8 - \lambda = 0$$

$$\therefore \quad \lambda = 8 \quad \text{Ans.}$$

Q.21. Find the acute angle between the following planes:

$$2x - y + z + 8 = 0$$

$$x + y + 2z - 14 = 0$$

Sol. Given planes are

$2x - y + z + 8 = 0$

$x + y + 2z - 14 = 0$

here, $a_1 = 2, b_1 = -1, c_1 = 1$

$a_2 = 1, b_2 = 1, c_2 = 2$

$$\therefore \quad \cos\theta = \left|\frac{a_1a_2 + b_1b_2 + c_1c_2}{\sqrt{a_1^2 + b_1^2 + c_1^2}\sqrt{a_2^2 + b_2^2 + c_2^2}}\right|$$

$$= \left|\frac{2\times1 + (-1)\times1 + 1\times2}{\sqrt{2^2 + (-1)^2 + 1^2}\sqrt{1^2 + 1^2 + 2^2}}\right| = \left|\frac{3}{\sqrt{6}\sqrt{6}}\right| = \frac{3}{6}$$

$$\Rightarrow \quad \cos\theta = \frac{1}{2}$$

$$\Rightarrow \quad \cos\theta = \cos\frac{\pi}{3}$$

$$\therefore \quad \theta = \frac{\pi}{3} \quad \text{Ans.}$$

OR

Find the angle between the following pair of lines:

$\vec{r} = 3\hat{i} + 2\hat{j} - 4\hat{k} + \lambda\left(\hat{i} + 2\hat{j} + 2\hat{k}\right)$ and

$\vec{r} = 5\hat{i} - 2\hat{j} + \mu\left(3\hat{i} + 2\hat{j} + 6\hat{k}\right)$

Sol. Given lines are

$\vec{r} = 3\hat{i} + 2\hat{j} - 4\hat{k} + \lambda\left(\hat{i} + 2\hat{j} + 2\hat{k}\right)$

$\vec{r} = 5\hat{i} - 2\hat{j} + \lambda\left(3\hat{i} + 2\hat{j} + 6\hat{k}\right)$

Here, $a_1 = 1, b_1 = 2, c_1 = 2$

$a_2 = 3, b_2 = 2, c_2 = 6$

Let θ is the angle between two lines

$$\therefore \quad \cos\theta = \left|\frac{a_1a_2 + b_1b_2 + c_1c_2}{\sqrt{a_1^2 + b_1^2 + c_1^2}\sqrt{a_2^2 + b_2^2 + c_2^2}}\right|$$

$$= \left|\frac{1\times3 + 2\times2 + 2\times6}{\sqrt{1^2 + 2^2 + 2^2}\sqrt{3^2 + 2^2 + 6^2}}\right|$$

$$\Rightarrow \quad \cos\theta = \frac{19}{\sqrt{9}\sqrt{49}}$$

$$\cos\theta = \frac{19}{3\times7}$$

$$\theta = \cos\left(\frac{19}{21}\right) \quad \text{Ans.}$$

Q.22. Let A and B be events such that $P(A)=\frac{7}{18}, P(B)=\frac{9}{13}$ and $P(A\cap B)=\frac{4}{13}$. Find:

(i) $P(A/B)$ (ii) $P(B/A)$

(iii) $P(A\cup B)$ (iv) $P\left(\frac{\bar{B}}{\bar{A}}\right)$

Sol. $P(A)=\frac{7}{18},\ P(B)=\frac{9}{13},\ P(A\cap B)=\frac{4}{13}$

(i) $P\left(\frac{A}{B}\right)=\frac{P(A\cap B)}{P(B)}$

$=\frac{\frac{4}{13}}{\frac{9}{13}}=\frac{4}{9}$ Ans.

(ii) $P\left(\frac{B}{A}\right)=\frac{P(B\cap A)}{P(A)}=\frac{P(A\cap B)}{P(A)}$

$=\frac{\frac{4}{13}}{\frac{7}{18}}=\frac{4}{13}\times\frac{18}{7}$

$=\frac{72}{91}$ Ans.

(iii) $P(A\cup B)=P(A)+P(B)-P(A\cap B)$

$=\frac{7}{18}+\frac{9}{13}-\frac{4}{13}$

$=\frac{91+162-72}{18\times 13}$

$=\frac{91+90}{234}$

$=\frac{181}{234}$ Ans.

(iv) $P\left(\frac{\bar{B}}{\bar{A}}\right)=\frac{P(\bar{B}\cap\bar{A})}{P(\bar{A})}=\frac{P(\overline{A\cup B})}{1-P(A)}$

$=\frac{1-P(A\cup B)}{1-P(A)}$

$=\frac{1-\frac{181}{234}}{1-\frac{7}{18}}=\frac{\frac{234-181}{234}}{\frac{18-7}{18}}=\frac{53}{234}\times\frac{8}{11}$

$=\frac{53}{143}$ Ans.

OR

Two dice are thrown. Find the probability that the numbers appeared has a sum 8, if it is known that the second die always exhibits 4.

Sol. Here $S=\begin{Bmatrix}(1,1)(1,2)\ldots\ldots(1,6)\\(2,1)(2,2)\ldots\ldots(2,6)\\\ldots\ldots\\\ldots\ldots\\\ldots\ldots\\(3,1)(3,2)\ldots\ldots(3,6)\end{Bmatrix}$

$x(5)=36$

Let A = event of getting the sum 8.

$=\{(2,6),(3,5)(4,4)(5,3)(6,2)\}$

$x(A)=5$

B = event that the second die always exhibits 4.

$=\{(1,4),(2,4)(3,4)(4,4)(5,4)(6,4)\}$

$x(B)=6$

$A\cap B=\{(4,4)\}$

$x(A\cap B)=1$

Required Probability $=P\left(\frac{A}{B}\right)$

$=\frac{P(A\cap B)}{P(B)}=\frac{x(A\cap B)/x(s)}{x(B)/x(s)}$

$=\frac{x(A\cap B)}{x(B)}$

$=\frac{1}{6}$ Ans.

Section-C (खण्ड-स)

23. Solve the system of linear equations using matrix method.

$x+y+z=6$

$y+3z=11$

$x-2y+z=0$

Sol. $x+y+z=6$

$y+3z=11$

$x-2y+z=0$

$$\begin{bmatrix}1&1&1\\0&1&3\\1&-2&1\end{bmatrix}\begin{bmatrix}x\\y\\z\end{bmatrix}=\begin{bmatrix}6\\11\\0\end{bmatrix}$$

$\Rightarrow$ $AX=B$ (let) ...(i)

$\therefore$ $|A|=\begin{vmatrix}1&1&1\\0&1&3\\1&-2&1\end{vmatrix}=1(1+6)-1(0-3)+1(0-1)$

$=7+3-1$

$\Rightarrow$ $|A|=9\neq 0$

This $\Rightarrow$ A^{-1} exists and solution is given by

$X=A^{-1}B$ ...(ii)

Now, $C_{11}=M_{11}=\begin{vmatrix}1&3\\-2&1\end{vmatrix}=1+6=7$

$C_{12}=M_{12}=-\begin{vmatrix}0&3\\1&1\end{vmatrix}=-(0-3)=3$

$C_{13}=M_{13}=\begin{vmatrix}0&1\\1&-2\end{vmatrix}=0-1=-1$

$C_{21}=M_{21}=-\begin{vmatrix}1&1\\-2&1\end{vmatrix}=-(1+2)=-3$

$C_{22}=M_{22}=\begin{vmatrix}1&1\\1&1\end{vmatrix}=1-1=0$

$C_{23}=M_{23}=-\begin{vmatrix}1&1\\1&-2\end{vmatrix}=-(-2-1)=3$

$C_{31}=M_{31}=\begin{vmatrix}1&1\\1&3\end{vmatrix}=3-1=2$

$$C_{32} = M_{32} = -\begin{vmatrix} 1 & 1 \\ 0 & 3 \end{vmatrix} = -(3-0) = -3$$

$$C_{33} = M_{33} = \begin{vmatrix} 1 & 1 \\ 0 & 1 \end{vmatrix} = 1-0 = 1$$

$$A' = \frac{1}{|A|} adj\ A$$

$$= \frac{1}{9}\begin{bmatrix} c_{11} & c_{21} & c_{31} \\ c_{12} & c_{22} & c_{32} \\ c_{13} & c_{23} & c_{33} \end{bmatrix}$$

$$\Rightarrow \quad A' = \frac{1}{9}\begin{bmatrix} 7 & -3 & 2 \\ 3 & 0 & -3 \\ -1 & 3 & 1 \end{bmatrix}$$

Using (i) and (iii) in (ii), we get

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \frac{1}{9}\begin{bmatrix} 7 & -3 & 2 \\ 3 & 0 & -3 \\ -1 & 3 & 1 \end{bmatrix}\begin{bmatrix} 6 \\ 11 \\ 0 \end{bmatrix}$$

$$= \frac{1}{9}\begin{bmatrix} 7\times6+(-3)\times11+2\times0 \\ 3\times6+0\times11+(-3)\times0 \\ -1\times6+3\times11+1\times0 \end{bmatrix} = \begin{bmatrix} \frac{1}{9}\times9 \\ \frac{1}{9}\times18 \\ \frac{1}{9}\times27 \end{bmatrix}$$

$$\Rightarrow \quad \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$$

$\therefore \quad x=1, y=2, z=3$ Ans.

OR

Obtain the inverse of the matrix using elementary operations:

$$A = \begin{bmatrix} 2 & 0 & -1 \\ 5 & 1 & 0 \\ 0 & 1 & 3 \end{bmatrix}$$

Sol. $$A = \begin{bmatrix} 2 & 0 & -1 \\ 5 & 1 & 0 \\ 0 & 1 & 3 \end{bmatrix}$$

We know that $A = I.A$

$$\begin{bmatrix} 2 & 0 & -1 \\ 5 & 1 & 0 \\ 0 & 1 & 3 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}.A$$

Applying $R_1 \to \frac{1}{2}R_1$

$$\begin{bmatrix} 1 & 0 & -\frac{1}{2} \\ 5 & 1 & 0 \\ 0 & 1 & 3 \end{bmatrix} = \begin{bmatrix} \frac{1}{2} & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}.A$$

Applying $R_2 \to R_2 - 5R_1$

$$\begin{bmatrix} 1 & 0 & -\frac{1}{2} \\ 0 & 1 & \frac{5}{2} \\ 0 & 1 & \frac{2}{3} \end{bmatrix} = \begin{bmatrix} \frac{1}{2} & 0 & 0 \\ \frac{-5}{2} & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}.A$$

Applying $R_3 \to R_3 - R_2$

$$\begin{bmatrix} 1 & 0 & -\frac{1}{2} \\ 0 & 1 & \frac{5}{2} \\ 0 & 0 & \frac{1}{2} \end{bmatrix} = \begin{bmatrix} \frac{1}{2} & 0 & 0 \\ \frac{-5}{2} & 1 & 0 \\ \frac{5}{2} & -1 & 1 \end{bmatrix}$$

Applying $R_3 \to 2.R_3$

$$\begin{bmatrix} 1 & 0 & -\frac{1}{2} \\ 0 & 1 & \frac{5}{2} \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} \frac{1}{2} & 0 & 0 \\ \frac{-5}{2} & 1 & 0 \\ 5 & -2 & 2 \end{bmatrix}.A$$

Applying $R_1 \to R_1 + \frac{1}{2}R_3$

$R_2 \to R_2 - \frac{5}{2}R_3$

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 3 & -1 & 1 \\ -15 & 6 & -5 \\ 5 & -2 & 2 \end{bmatrix}.A$$

$I = A^{-1}.A$

$$\therefore \quad A^{-1} = \begin{bmatrix} 3 & -1 & 1 \\ -15 & 6 & -5 \\ 5 & -2 & 2 \end{bmatrix} \quad \text{Ans.}$$

24. Find the maxium and minimum values of given function $f(x) = 2x^3 - 21x^2 + 36x - 20$

Sol. $f(x) = 2x^3 - 21x^2 + 36x - 20$ ...(i)

$\Rightarrow \quad f'(x) = 6x^2 - 42x + 36$ ...(ii)

$\Rightarrow \quad f''(x) = 12x - 42$ ...(iii)

For maximum and minimum value of

$f'(x) = 0$

$\Rightarrow \quad 6x^2 - 42x + 36 = 0$

$\Rightarrow \quad 6(x^2 - 7x + 6) = 0$

$\Rightarrow \quad x^2 - 6x - x + 6 = \frac{0}{6} = 0$

$\Rightarrow \quad x(x-6) - (x-6) = 0$

$\Rightarrow \quad (x-6)(x-1) = 0$

$\Rightarrow \quad x = 1, 6$

From (ii), $f''(1) = 12\times1 - 42 = -30 < 0$

This $\Rightarrow$ x = 1 is the point of maxima

$\therefore$ Maximum value of $f(x) = f(1)$

$= 2\times1^3 - 21\times1^2 + 36\times1 - 20$

$= 38 - 41$

$= -3$ Ans

Again, from (ii), $f''(6) = 12\times 6 - 42 = 72 - 42$

$\Rightarrow f''(6) = 30 > 0$

This $\Rightarrow x = 6$ is the point of minima

$\therefore$ Minimum value of $f(x) = f(6)$

$= 2\times 6^3 - 21\times 6^2 + 36\times 6 - 20$

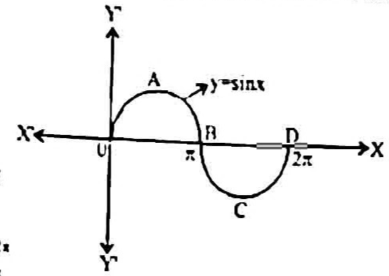
$= 432 - 756 + 216 - 20 = -776 + 648$

$= -128$ Ans.

25. Find the area bounded by the curve $y = \sin x$ between $x = 0$ and $x = 2\pi$.

Sol. Given curve is $y = \sin x$

Required Area = Area of the shaded region OAB + Area of the shaded region BCDB



$= \int_0^{\pi} y\,dx + \int_{\pi}^{2\pi} -y\,dx$

$= \int_0^{\pi} \sin x\,dx - \int_{\pi}^{2\pi} \sin x\,dx$

$= [-\cos x]_0^{\pi} - [-\cos x]_{\pi}^{2\pi}$

$= -(\cos\pi - \cos 0) + [\cos 2\pi - \cos\pi]$

$= -(-1-1) + [1-(-1)]$

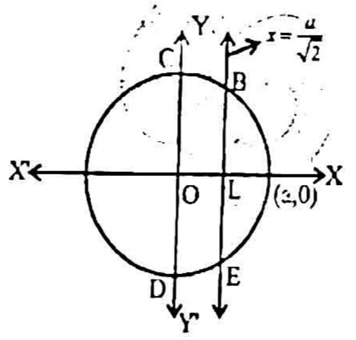
$= 2 + 2 = 4$ sq. unit Ans.

OR / अथवा

Find the area of the smaller part of the circle $x^2 + y^2 = a^2$ cut off by the straight line $x = \frac{a}{\sqrt{2}}$.

Sol. Given circle is $x^2 + y^2 = a^2$ ...(i)

It is a circle having centre at (0, 0) and radius a.



Again, $x = \frac{a}{\sqrt{2}}$ is a straight line parallel to Y-axis at a distance $\frac{a}{\sqrt{2}}$ from Y-axis.

Required Area = Area of shaded region DEBCD

$= 2\times$ Area of region OLBCO (by symmetricity of curve)

$= 2\times \int_0^{a/\sqrt{2}} y_{(circle)}\,dx$

$= 2\int_0^{a/\sqrt{2}} \sqrt{a^2 - x^2}\,dx$

$= 2\left[\frac{x}{2}\sqrt{a^2 - x^2} + \frac{a^2}{2}\sin^{-1}\frac{x}{a}\right]_0^{a/\sqrt{2}}$

$= 2\left[\left\{\frac{a/\sqrt{2}}{2}\sqrt{a^2 - \left(\frac{a}{\sqrt{2}}\right)^2} + \frac{a^2}{2}\sin^{-1}\frac{a/\sqrt{2}}{a}\right\} - \left\{\frac{0}{2}\sqrt{a^2 - 0^2} + \frac{a^2}{2}\sin^{-1}\frac{0}{a}\right\}\right]$

$= 2\left[\frac{a}{2\sqrt{2}}\sqrt{a^2 - \frac{a^2}{2}} + \frac{a^2}{2}\sin^{-1}\left(\frac{1}{\sqrt{2}}\right) - 0\right]$

$= 2\times\frac{a}{2\sqrt{2}}\times\frac{a}{\sqrt{2}} + \frac{2\times a^2}{2}\times\frac{\pi}{4}$

$= \frac{a^2}{2} + \frac{\pi a^2}{4}$ Ans.

26. Solve the differential equation:

$(x-y)dy - (x+y)dx = 0$

Sol. $(x-y)dy - (x+y)dx = 0$

$\Rightarrow (x-y)dy = (x+y)dx$

$\Rightarrow \frac{dy}{dx} = \frac{x+y}{x-y}$ ...(i)

Which is a homogeneous differential equation

$\therefore$ Let $y = vx$ ...(ii)

$\frac{dy}{dx} = v + x\frac{dv}{dx}$ ...(iii)

Using (ii) and (iii) in (i), we get $v + x\frac{dv}{dx} = \frac{x+vx}{x-vx}$

$\Rightarrow x.\frac{dv}{dx} = \frac{x(1+v)}{x(1-v)} - v$

$= \frac{1+v-v+v^2}{1-v}$

$\Rightarrow \int\frac{1-v}{1+v2}dv = \int\frac{dx}{x}$

Let $1+v^2 = t \Rightarrow 2vdv = dt$

$vdv = \frac{dt}{2}$

$\Rightarrow \tan^{-1}v - \frac{1}{2}\int\frac{dt}{t} = \log x + \log C$

$\tan^{-1} t - \frac{1}{2}\log t = -\log x + \log c$ [illegible]

$\Rightarrow \tan^{-1}\frac{y}{x} - \frac{1}{2}\log\left(1+t^2\right) = -\log x + \log c$

$\Rightarrow \tan^{-1}\frac{y}{x} - \frac{1}{2}\log\left(1+\frac{y^2}{x^2}\right) = -\log x + \log c$

$\Rightarrow \tan^{-1}\frac{y}{x} - \frac{1}{2}\log\left(\frac{x^2+y^2}{x^2}\right) = -\log x + \log c$

$\Rightarrow \tan^{-1}\frac{y}{x} - \frac{1}{2}\log\left(x^2+y^2\right) + \frac{1}{2}\log x^2 = -\log x + \log c$

$\Rightarrow \tan^{-1}\frac{y}{x} - \log\sqrt{x^2+y^2} + \frac{1}{2}\times 2\log x = -\log x + \log c$

$\Rightarrow \tan^{-1}\frac{y}{x} = \log cx + \log\sqrt{x^2+y^2} - \log x$

$\Rightarrow \tan^{-1}\frac{y}{x} = \log\frac{cx\sqrt{x^2+y^2}}{x}$

$\therefore \tan^{-1}\frac{y}{x} = \log c\sqrt{x^2+y^2}$

$\Rightarrow c\sqrt{x^2+y^2} = e^{\tan^{-1}\frac{y}{x}}$

Which is the required solution.

OR

Find The general solution of the differential equation:

$$\left(1+x^2\right)dy + 2xy\,dx = \cot x\,dx\,(x \neq 0)$$

Sol. $\left(1+x^2\right)dy + 2xy\,dx = \cot x\,dx\,(x \neq 0)$

$\Rightarrow \frac{dy}{dx} + \frac{2x}{1+x^2}\cdot y = \frac{\cot x}{1+x^2}$ ...(i)

Which is a linear differential equation in y.

Here $P = \frac{2x}{1+x^2}$, $Q = \frac{\cot x}{1+x^2}$

$\therefore I.F. = e^{\int P dx} = e^{\int \frac{2x}{1+x^2}dx} = e^{\log(1+x^2)}$

$\Rightarrow I.F = \left(1+x^2\right)$

Now solution of equation (i) is given by $Y \times I.F. = \int Q \times I.F.dx$

$\Rightarrow y \times \left(1+x^2\right) = \log|\sin x| + C$

Which is the required solution. Ans.

27. Find shortest distance between the lines $\vec{r} = \hat{i}+2\hat{j}+\hat{k}+\lambda\left(\hat{i}-\hat{j}+\hat{k}\right)$ and $\vec{r} = 2\hat{i}-\hat{j}-\hat{k}+\mu\left(2\hat{i}+\hat{j}+2\hat{k}\right)$

Sol. Given lines are $\vec{r} = \left(\hat{i}+2\hat{j}+\hat{k}\right)+\lambda\left(\hat{i}-\hat{j}+\hat{k}\right)$

$\vec{r} = \left(2\hat{i}-\hat{j}-\hat{k}\right)+\mu\left(2\hat{i}+\hat{j}+2\hat{k}\right)$

Here $\vec{a}_1 = \hat{i}+2\hat{j}+\hat{k}$, $\vec{b}_1 = \hat{i}-\hat{j}+\hat{k}$

$\vec{a}_2 = 2\hat{i}-\hat{j}-\hat{k}$, $\vec{b}_2 = 2\hat{i}+\hat{j}+2\hat{k}$

$\vec{a}_2 - \vec{a}_1 = \hat{i} - 3\hat{j} - 2\hat{k}$

$\vec{b}_1 \times \vec{b}_2 = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & -1 & 1 \\ 2 & 1 & 2 \end{vmatrix}$

$= (-2-1)\hat{i} - (2-2)\hat{j} + (1+2)\hat{k}$

$\vec{b}_1 \times \vec{b}_2 = -3\hat{i} + 3\hat{k}$

$\therefore \left|\vec{b}_1 \times \vec{b}_2\right| = \sqrt{(-3)^2 + 3^2} = \sqrt{18} = 3\sqrt{2}$

$\therefore$ Shortest Distance $= \left|\frac{\left(\vec{a}_2 - \vec{a}_1\right)\cdot\left(\vec{b}_1 \times \vec{b}_2\right)}{\left|\vec{b}_1 \times \vec{b}_2\right|}\right|$

$= \left|\frac{\left(\hat{i} - 3\hat{j} - 2\hat{k}\right)\cdot\left(-3\hat{i} + 3\hat{k}\right)}{3\sqrt{2}}\right|$

$= \left|\frac{1\times -3 + (-3)\times 0 + (-2)\times 3}{3\sqrt{2}}\right|$

$= \left|\frac{-9}{3\sqrt{2}}\right| = \frac{3}{\sqrt{2}}$

$= \frac{3\sqrt{2}}{2}$ unit Ans.

28. An insurance company insured 2000 scooter drivers, 4000 car drivers and 6000 truck drivers. The probabilities of an accident involving a scooter, a car and a truck are 0.01, 0.03 and 0.15 respectively. One of the insured drivers meets with an accident. What is the probability that he is a scooter driver?

Sol. Let E_1 =event that the insured person is a scooter drivers

E_2 = event that the insured person is a car driver.

E_3 = event that the insured person is a truck driver.

$P(E_1) = \frac{2000}{2000+4000+6000} = \frac{2000}{12000} = \frac{1}{6}$

$P(E_2) = \frac{4000}{2000+4000+6000} = \frac{4000}{12000} = \frac{1}{3}$

$P(E_3) = \frac{6000}{2000+4000+6000} = \frac{6000}{12000} = \frac{1}{2}$

Let E = event that the insured driver meets with an accident

From question, $P\left(\frac{E}{E_1}\right) = 0.01$, $P\left(\frac{E}{E_2}\right) = 0.03$, $P\left(\frac{E}{E_3}\right) = 0.15$

Required Probability = $P\left(\frac{E_1}{E}\right)$

$$= \frac{P(E_1).P\left(\frac{E}{E_1}\right)}{P(E_1).P\left(\frac{E}{E_1}\right)+P(E_2).P\left(\frac{E}{E_2}\right)+P(E_3)\left(\frac{E}{E_3}\right)}$$

$$= \frac{\frac{1}{6}\times 0.01}{\frac{1}{6}\times 0.01+\frac{1}{3}\times 0.03+\frac{1}{2}\times 0.15}$$

$$= \frac{\frac{0.01}{6}}{\frac{0.01+0.06+0.45}{6}} = \frac{0.01}{0.52}$$

$$P\left(\frac{E_1}{E}\right)=\frac{1}{52}$$ Ans.

OR

Six coins are tossed simultaneously. Find the probability of getting-

(i) 3 heads

(ii) at least one head

(iii) not more than 3 heads.

Sol. 1

29. **Maximize** $Z = 20x + 10y$

Subject to $1.5x + 3y \le 42$

$3x + y \le 24$

and $x, y \ge 0$

Sol. We have, maximize $Z = 20x + 10y$ ...(i)

Subject to $1.5x + 3y \le 42$

$3x + y \le 24$

and $x, y \ge 0$

Table for $1.5x + 3y = 42$

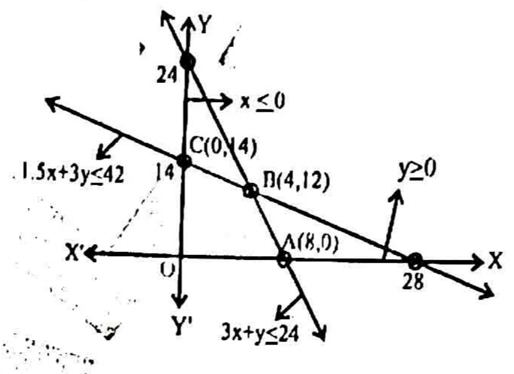
x	0	28
y	14	0

$x = 0$ is Y-axis

Table for $3x + y = 24$

x	0	8
y	24	0

$y = 0$ is X-axis



From graph, we conclude that the shaded region OABCO is the solution set.

Clearly, Co-ordinates of point O is (0, 0), point A is (8, 0), point B is (4, 12) and point C is (0, 14)

At point O (0, 0), $Z = 20\times 0 + 10\times 0 = 0$

At point A (8, 0), $Z = 20\times 8 + 10\times 0 = 160$

At point B (4, 12), $Z = 20\times 4 + 10\times 12 = 200$

At point C (0, 14) $Z = 20\times 0 + 10\times 14 = 140$

Thus, Z is Maximum at the point B (4, 12)

∴ Required solution is $x = 4, y = 12$

Also maximize $Z = 200$ Ans.