Mathematics - 2019

सामान्य निर्देश : General Instructions :

इस प्रश्न-पत्र में 29 प्रश्न हैं, जो तीन खण्डों-अ, ब और स में बंटे हुए हैं। खण्ड-अ में 10 प्रश्न हैं, जिनमें प्रत्येक 1 अंक का है, खण्ड-ब में 12 प्रश्न हैं जिनमें प्रत्येक 4 अंक का है तथा खण्ड-स में 7 प्रश्न हैं जिनमें प्रत्येक 6 अंक का है।

कैलकुलेटर के उपयोग की अनुमति नहीं है। आवश्यकता हो तो परीक्षार्थी के माँग पर लघुगणकीय अथवा सांख्यिकीय सारणी उपलब्ध करायी जा सकती है।

Section-A
(Objective Questions)

1. Let R be set of real number. An operation * is defined on R by a * b = a + b + 2ab. Then find the value of 2 * 3.

Ans. $a * b = a + b + 2ab$

$2 * 3 = 2 + 3 + 2.2.3 = 5 + 12 = 17$ Ans.

2. Find the value of $\cot^{-1}(-1)$.

Ans. $\cot^{-1}(-1) = -\cot^{-1} = -\cot^{-1}\left(\cot\frac{\pi}{4}\right) = \frac{-\pi}{4}$ Ans.

3. If $\begin{bmatrix} 2x+y & 2 \\ 1 & x-2y \end{bmatrix} = \begin{bmatrix} 7 & 2 \\ 1 & 4 \end{bmatrix}$, then find the value of x and y.

Ans. $\begin{bmatrix} 2x+y & 2 \\ 1 & x-2y \end{bmatrix} = \begin{bmatrix} 7 & 2 \\ 1 & 4 \end{bmatrix}$

$\Rightarrow \quad 2x + y = 7 \quad ...(i) \times 2$

$x - 2y = 4 \quad ...(ii)$

$+$

$5x = 18 \Rightarrow x = \frac{18}{5}$

Put $x = \frac{18}{5}$ in (i)

$2 \times \frac{18}{5} + y = 7 \Rightarrow y = 7 - \frac{36}{5} = \frac{35-36}{5}$

$\Rightarrow y = \frac{-1}{5}$

$\therefore \quad x = \frac{18}{5},\ y = \frac{-1}{5}$ Ans.

4. Evaluate the determinant $\begin{vmatrix} \sin 70° & -\cos 70° \\ \sin 20° & \cos 20° \end{vmatrix}$.

Ans. $\begin{vmatrix} \sin 70° & -\cos 70° \\ \sin 20° & \cos 20° \end{vmatrix} = \sin 70° \cos 20° - (-\cos 70° \sin 20°)$

$= \sin 70° \cos 20° + \cos 70° \sin 20°$

$= \sin(70° + 20°)$

$= \sin 90°$

$= 1$

5. If y = log (sin x) then find $\frac{dy}{dx}$

Ans. $y = \log(\sin x)$

$\Rightarrow \frac{dy}{dx} = \frac{d}{2x}\{\log(\sin x)\}$

$= \frac{1}{\sin x} \times \cos x$

$\frac{dy}{dx} = \cot x$ Ans.

6. Find the value of $\int e^x(\tan x + \sec^2 x)dx$.

Ans. $\int e^x(\tan x + \sec^2 x)dx$

Let $f(x) = \tan x \Rightarrow f'(x) = \sec^2 x$

$\int e^x(\tan x + \sec^2 x)dx = \int e^x\{f(x) + f'(x)\}dx$

$= e^x f(x) + C$

$= e^x + \tan x + C$ Ans.

7. Find the slpe of the tangent to the curve $y = x^3 - 2x + 1$ at the point x = 2.

Ans. $y = x^3 - 2x + 1$

$\Rightarrow \frac{dy}{dx} = \frac{d}{dx}(x^3 - 2x + 1) = 3x^2 - 2.1 + 0 = 3x^2 - 2$

$\therefore$ Slope of tangent at n = 2 $= \left[\frac{dy}{dx}\right]_{x=2}$

$= 3 \times 2^{-2} - 2$

$= 10$ Ans.

8. **Find the value of λ for which the vector $\lambda\left(\hat{i}-\hat{j}+\hat{k}\right)$ is a unit vector.**

Ans. $\because$ $\lambda\left(\hat{i}-\hat{j}+\hat{k}\right)$ is a unit vector

$\Rightarrow \left|\lambda\left(\hat{i}-\hat{j}+\hat{k}\right)\right|=1$

$\Rightarrow \left|\lambda\hat{i}-\lambda\hat{j}+\lambda\hat{k}\right|=1$

$\Rightarrow \sqrt{\lambda^2+(-\lambda)^2+\lambda^2}=1$

$\Rightarrow \sqrt{3\lambda^2}=1 \Rightarrow \pm\sqrt{3}\lambda=1$

$\Rightarrow \lambda=\pm\frac{1}{\sqrt{3}}$ Ans.

9. **Given $|\vec{a}|=10$, $|\vec{b}|=2$ and $\vec{a}.\vec{b}=12$, then find the value of $|\vec{a}\times\vec{b}|$.**

Ans. $\vec{a}.\vec{b}=12$

$\Rightarrow |\vec{a}||\vec{b}|\cos\theta=12 \Rightarrow 10.2.\cos\theta=12$

$\Rightarrow \cos\theta=\frac{12}{20}=\frac{3}{5}$

$\therefore \quad \sin\theta=\sqrt{1-\cos^2\theta}=\sqrt{1-\left(\frac{3}{5}\right)^2}=\sqrt{\frac{25-9}{25}}$

$\Rightarrow \sin\theta=\sqrt{\frac{16}{25}}=\frac{4}{5}$

$\therefore |\vec{a}\times\vec{b}|=|\vec{a}||\vec{b}|\sin\theta=10.2.\frac{4}{5}=16$ Ans.

10. **Find the Cartesian equation of the plane whose vector equation is $\vec{r}.\left(2\hat{i}+5\hat{j}-6\hat{k}\right)=7$.**

Ans. Given equation of plane is $\vec{r}.\left(2\hat{i}+5\hat{j}-6\hat{k}\right)=7$

$\Rightarrow \left(x\hat{i}+y\hat{j}+z\hat{k}\right).\left(2\hat{i}+5\hat{j}-6\hat{k}\right)=7$

$\Rightarrow 2x+5y-6z=7$

Which is the requried cartesian equation of plane.

Section - B

11. **Let $f:R\to R$ be given by $f(x)=x^2-5x+4$, then find value of fof (1).**

Ans. $\because f(x)=x^2-5x+4$ ...(i)

$\therefore fof(1)=f\{f(1)\}$

$=f\{1^2-5.1+4\}$, by (i)

$=f(0)$

$=0^2-5.0+4$, by (i)

$=4$ Ans.

12. **Prove that $\tan^{-1}\left(\frac{2}{5}\right)+\tan^{-1}\left(\frac{1}{3}\right)+\tan^{-1}\left(\frac{1}{12}\right)=\frac{\pi}{4}$**

Ans. L.H.S. $=\tan^{-1}\left(\frac{2}{5}\right)+\tan^{-1}\left(\frac{1}{3}\right)+\tan^{-1}\left(\frac{1}{12}\right)$

$=\tan^{-1}\left(\frac{\frac{2}{5}+\frac{1}{3}}{1-\frac{2}{5}\times\frac{1}{3}}\right)+\tan^{-1}\left(\frac{1}{12}\right)$

$=\tan^{-1}\left(\frac{\frac{6+5}{15}}{\frac{15-2}{15}}\right)+\tan^{-1}\left(\frac{1}{12}\right)$

$=\tan^{-1}\left(\frac{11}{13}\right)+\tan^{-1}\left(\frac{1}{12}\right)$

$=\tan^{-1}\left(\frac{\frac{11}{13}+\frac{1}{12}}{1-\frac{11}{13}\times\frac{1}{12}}\right)=\tan^{-1}\left(\frac{\frac{132+13}{13\times12}}{\frac{156-11}{13\times12}}\right)$

$=\tan^{-1}\left(\frac{145}{145}\right)$

$=\tan^{-1}$

$=\frac{\pi}{4}$ = R.H.S. Proved

13. **Prove that $\begin{vmatrix} a & a+b & a+b+c \\ 2a & 3a+2b & 4a+3b+2c \\ 3a & 6a+3b & 10a+6b+3c \end{vmatrix}=a^3$**

Ans. $\begin{vmatrix} a & a+b & a+b+c \\ 2a & 3a+2b & 4a+3b+2c \\ 3a & 6a+3b & 10a+6b+3c \end{vmatrix}$

Applying $R_2\to R_2-2R_1$, $R_3\to R_3-3R_1$

$=\begin{vmatrix} a & a+b & a+b+c \\ 0 & a & 2a+b \\ 0 & 3a & 7a+3b \end{vmatrix}$

Applying $R_3\to R_3-3R_2$

$$= \begin{vmatrix} a & a+b & a+b+c \\ 0 & a & 2a+b \\ 0 & 0 & a \end{vmatrix}$$

Expanding a long C_1

$$= a\begin{vmatrix} a & 2a+b \\ 0 & a \end{vmatrix} - 0 + 0$$

$= a(a^2 - 0) = a^3$ proved

OR

If $A = \begin{bmatrix} 3 & 1 \\ -1 & 2 \end{bmatrix}$, then show that $A^2 - 5A + 7I_2 = 0$.

Hence find A^{-1}

Ans. $A = \begin{bmatrix} 3 & 1 \\ -1 & 2 \end{bmatrix}$ ...(i)

$$A^2 = A.A = \begin{bmatrix} 3 & 1 \\ -1 & 2 \end{bmatrix}\begin{bmatrix} 3 & 1 \\ -1 & 2 \end{bmatrix} = \begin{bmatrix} 9-1 & 3+2 \\ -3-2 & -1+4 \end{bmatrix}$$

$A^2 = \begin{bmatrix} 8 & 5 \\ -5 & 3 \end{bmatrix}$ ...(ii)

$-5A = -5.\begin{bmatrix} 3 & 1 \\ -1 & 2 \end{bmatrix} = \begin{bmatrix} -15 & -5 \\ 5 & -10 \end{bmatrix}$ ...(iii)

$7I_2 = 7\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 7 & 0 \\ 0 & 7 \end{bmatrix}$ ...(iv)

$\therefore A^2 - 5A + 7I_2 = \begin{bmatrix} 8 & 5 \\ -5 & 3 \end{bmatrix} + \begin{bmatrix} -15 & -5 \\ 5 & -10 \end{bmatrix} + \begin{bmatrix} 7 & 0 \\ 0 & 7 \end{bmatrix}$

by (ii), (iii) and (iv)

$$= \begin{bmatrix} 8-15+7 & 5-5+0 \\ -5+5+0 & 3-10+7 \end{bmatrix}$$

$$= \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$$

$A^2 - 5A + 7I_2 = 0$ ...(v) Proved

$\Rightarrow A.A - 5A = 0 - 7I_2$

$\Rightarrow A.A - 5A = -7I_2$

Multiplying both isde by A^{-1}

$A.AA^{-1} - 5AA^{-1} = -7\,I_2\,A^{-1}$

$\Rightarrow A.I - 5I = -7\,A^{-1}$

$\Rightarrow A - 5I = -7A^{-1}$

$$\Rightarrow \begin{bmatrix} 3 & 1 \\ -1 & 2 \end{bmatrix} + (-5)\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = -7A^{-1}$$

$$\Rightarrow \begin{bmatrix} 3 & 1 \\ -1 & 2 \end{bmatrix} + (-5)\begin{bmatrix} -5 & 0 \\ 0 & -5 \end{bmatrix} = -7A^{-1}$$

$$\Rightarrow \begin{bmatrix} -2 & 1 \\ -1 & -3 \end{bmatrix} = -7A^{-1}$$

$$\Rightarrow A^{-1} = \frac{-1}{7}\begin{bmatrix} -2 & 1 \\ -1 & -3 \end{bmatrix}$$

$$A^{-1} = \begin{bmatrix} \frac{2}{7} & \frac{-1}{7} \\ \frac{1}{7} & \frac{3}{7} \end{bmatrix}$$ Ans.

14. Determine of f defined by $f(x) = \begin{cases} x^2 \sin\left(\frac{1}{x}\right) & if\ x \neq 0 \\ 0 & if\ x = 0 \end{cases}$ is a continuous function.

Ans. $f(x) = \begin{cases} x^2 \sin\left(\frac{1}{x}\right) & , if\ x \neq 0 \\ 0 & , if\ x = 0 \end{cases}$ ...(i)

R.H.L $= \lim_{x \to 0^+} f(x)$

$= \lim_{x \to 0^+} x^2 \sin\left(\frac{1}{x}\right)$, by (i)

$= \lim_{x \to 0} (0+h)^2 \sin\left(\frac{1}{0+h}\right)$

$= \lim_{x \to 0} h^2 \sin\left(\frac{1}{h}\right)$

$= 0^2 \times a$ finite quantity $(\because\ -1 \le \sin\theta \le 1)$

$= 0$ ...(ii)

$f(0) = 0$, by (i) ...(iii)

L.H.L. $= \lim_{x \to 0^-} f(x)$

$= \lim_{x \to 0^-} x^2 \sin\left(\frac{1}{x}\right)$, by (i)

$= \lim_{h \to 0} (0-h)^2 \sin\left(\frac{1}{0-h}\right)$

$= \lim_{h \to 0} h^2 \sin\left(-\frac{1}{h}\right)$

$= \lim_{h \to 0} h^2 \sin\left(\frac{1}{h}\right)$

$= 0^2 \times$ a finite quantity $\{\because -1 \le \sin\theta \le 1\}$

$= 0$

$\because$ R.H.L. $= f(0) =$ L.H.L.

i.e., $\lim_{x \to 0} f(x) = f(0) = \lim_{x \to 0} f(x)$

Hence, f(x) is continuous at $x = 0$ Ans.

15. If $x = a(\theta - \sin\theta)$, $y = a(1 - \cos\theta)$, then find $\frac{dy}{dx}$.

Ans. $x = a(\theta - \sin\theta)$ ...(i)

$\Rightarrow \frac{dx}{d\theta} = \frac{d}{d\theta}\{a(\theta - \sin\theta)\} = a\left\{\frac{d}{d\theta}(\theta) - \frac{d}{d\theta}(\sin\theta)\right\}$

$\frac{dx}{d\theta} = a(1 - \cos\theta)$ ...(ii)

Again, $y = a(1 - \cos\theta)$ ...(iii)

$\Rightarrow \frac{dy}{d\theta} = \frac{d}{d\theta}\{a(1 - \cos\theta)\} = a\left\{\frac{d}{d\theta}(1) - \frac{d}{d\theta}(\cos\theta)\right\}$

$\Rightarrow \frac{dy}{d\theta} = a(0 - (-\sin\theta)) = a\sin\theta$ ...(iv)

We know that, $\frac{dy}{dx} = \frac{dy/d\theta}{\theta x/d\theta}$

$= \frac{a\sin\theta}{a(1-\cos\theta)}$ by (iv) and (ii)

$= \frac{2\sin\frac{\theta}{2}\cos\frac{\theta}{2}}{2\sin^2\frac{\theta}{2}}$

$\frac{dy}{dx} = \cot\frac{\theta}{2}$ Ans.

OR

If $x^y + y^x = 1$, find $\frac{dy}{dx}$.

Ans. We have, $x^y + y^x = 1$

$\Rightarrow u + v = 1$ (let) ...(i)

$\Rightarrow \frac{du}{dx} + \frac{dv}{dx} = \frac{d}{dx}(1) = 0$ ...(ii)

Now, $u = x^y$

$\Rightarrow \log u = \log x^y$

$\Rightarrow \log u = y\log x$

$\Rightarrow \frac{d}{dx}(\log u) = \frac{d}{dx}(y\log x)$

$\Rightarrow \frac{1}{u}\frac{du}{dx} = y \cdot \frac{1}{x} + \log x \frac{dy}{dx}$

$\Rightarrow \frac{du}{dx} = u\left(\frac{y}{x} + \log x \frac{dy}{dx}\right)$

$\Rightarrow \frac{du}{dx} = x^y\left(\frac{y}{x} + \log x . \frac{dy}{dx}\right)$ ...(iii)

Again, $v = y^x$

$\Rightarrow \log v = \log y^x$

$\Rightarrow \log v = x.\log y$

$\Rightarrow \frac{d}{dx}(\log v) = \frac{d}{dx}(x.\log y)$

$\Rightarrow \frac{1}{v}.\frac{dv}{dx} = x.\frac{d}{dx}(\log y) + \log y.\frac{d}{dx}(x)$

$\Rightarrow \frac{dv}{dx} = v\left(x.\frac{1}{y}\frac{dy}{dx} + \log y.1\right)$

$\Rightarrow \frac{dv}{dx} = y^x\left(\frac{x}{y}\frac{dy}{dx} + \log y\right)$ ...(iv)

Using (iii) and (iv) in (ii), we get

$x^y\left(\frac{y}{x} + \log x.\frac{dy}{dx}\right) + y^x\left(\frac{x}{y}\frac{dy}{dx} + \log y\right) = 0$

$\Rightarrow x^y.\frac{y}{x} + x^y.\log x.\frac{dy}{dx} + y^x.\frac{x}{y}\frac{dy}{dx} + y^x.\log y = 0$

$\Rightarrow \frac{dy}{dx}\left(x^y.\log x + y^{x-1}x\right) = -\left(y^x.\log y + x^{y-1}.y\right)$

$\therefore \frac{dy}{dx} = -\frac{\left(y^x \log y + x^{y-1}.y\right)}{\left(x^y \log xl + y^{x-1}.x\right)}$ Ans.

16. Find the interval on which the function $f(x) = 2x^3 - 21x^2 + 36x - 40$, is increasing or decreaisng.

Ans. $f(x) = 2x^3 - 21x^2 + 36x - 40$ ...(i)

$\Rightarrow f'(x) = 6x^2 - 42x + 36$

$\Rightarrow f'(x) = 6(x^2 - 7x + 6)$

$= 6\{x^2 - 6x - x + 6\} = 6\{x(x-6) - (x-6)\}$

$f'(x) = 6(x-6)(x-1)$ ...(ii)

For $f(x)$ to be increasing, we must have $f'(x) > 0$

$\Rightarrow 6(x-6)(x-1) > 0$

$\Rightarrow (x-6)(x-1) > 0$

+ve   -ve   +ve

$-\infty$ 1   6 ∞

$\Rightarrow x \in (-\infty, 1) \cup (6, \infty)$

$\therefore f(x)$ is increasing in $(-\infty, 1) \cup (6, \infty)$

Again for $f(x)$ to be decreasing, we must have $f'(x) < 0$

$\Rightarrow 6(x-6)(x-1) < 0$

$\Rightarrow (x-6)(x-1) < 0$

+ve   -ve   +ve

$-\infty$ 1   6 ∞

$\Rightarrow n \in (1, 6)$

$\therefore f(x)$ is decreasing in $(1, 6)$ Ans.

OR/ अथवा

A particle moves along the curve $y = \frac{2}{3}x^3 + 1$. Find the points on the curve at which y-co-ordinate is changing as twice as x-co-ordinate.

Ans. Given curve is, $y = \frac{2}{3}x^3 + 1$ ...(i)

A/g, $\frac{dy}{dt} = 2.\frac{dx}{dt}$

$\Rightarrow \frac{d}{dt}\left(\frac{2}{3}x^3 + 1\right) = 2\frac{dx}{dt}$, by ...(i)

$\Rightarrow \frac{2}{3} \times 3x^2 \frac{dx}{dt} + 0 = 2\frac{dx}{dt}$

$\Rightarrow 2x^2 \frac{dx}{dt} = 2\frac{dx}{dt}$

$\Rightarrow x^2 = 1$

$\Rightarrow x = \sqrt{1} = \pm 1$

Put $x = \pm 1$ in (i), we get $y = \frac{2}{3} \times (\pm 1)^3 + 1$

$= \pm\frac{2}{3} + 1$

$= \frac{2}{3} + 1, \ \frac{-2}{3} + 1$

$y = \frac{5}{3}, \frac{1}{3}$

$\therefore$ Required pionts on the curve are $\left(1, \frac{5}{3}\right)$ and $\left(-1, \frac{1}{3}\right)$

17. Find the value of $\int \frac{x+3}{x^2 - 2x - 5} dx$

Ans. Let $I = \int \frac{x+3}{x^2 - 2x - 5} dx$ ...(i)

Let $x + 3 = A \cdot \frac{d}{dx}\left(x^2 - 2x - 5\right) + B$

$\Rightarrow x + 3 = A(2x - 2) + B$ ...(ii)

Equating the coefficient to like terms,. we get $1 = 2A$

$\Rightarrow A = \frac{1}{2}$

$3 = -2A + B \Rightarrow B = 3 + 2A = 3 + 2 \times \frac{1}{2} = 4$

Putting the value of A and B in (ii), we get

$x + 3 = \frac{1}{2}(2x - 2) + 4$ ...(iii)

Using (iii) in (i), we get $I = \int \frac{\frac{1}{2}(2x-2) + 4}{x^2 - 2x - 5} dx$

$= \frac{1}{2}\int \frac{2x - 2}{x^2 - 2x - 5} dx + 4\int \frac{1}{x^2 - 2x - 5} dx$

Let $x^2 - 2x - 5 = t$

$\Rightarrow (2x - 2)dx = dt$

$I = \frac{1}{2}\int \frac{dt}{t} + 4\int \frac{dx}{x^2 - 2.x.1 + 1^2 - 1^2 - 5}$

$= \frac{1}{2}\log|t| + 4\int \frac{dx}{(x-1)^2 - \left(\sqrt{6}\right)^2} + C_1$

$= \frac{1}{2}\log\left|x^2 - 2x - 5\right| + 4 \times \frac{1}{2\sqrt{6}} \log\left|\frac{x - 1 - \sqrt{6}}{x - 1 + \sqrt{6}}\right| + C$ Ans.

18. Evaluate $\int \frac{2x - 1}{(x-1)(x+2)(x-3)} dx$

Ans. Let $I = \int \frac{2x-1}{(x-1)(x+2)(x-3)} dx$ ...(i)

Let $\frac{2x-1}{(x-1)(x+2)(x-3)} \equiv \frac{A}{x-1} + \frac{B}{x+2} + \frac{C}{x-3}$ ...(ii)

$\Rightarrow \frac{2x-1}{(x-1)(x+2)(x-3)} \equiv \frac{A(x+2)(x-3) + B(x-1)(x-3) + C(x-1)(x+2)}{(x-1)(x+2)(x-3)}$

$\Rightarrow 2x - 1 \equiv A(x+2)(x-3) + B(x-1)(x-3) + C(x-1)(x+2)$

Put $x = 1$ in (iii)

$2 \times 1 - 1 = A(1+2)(1-3) + B.O + C.O$

$\Rightarrow 1 = A \times -6 \Rightarrow A = \frac{-1}{6}$

Put x = −2 in (iii)

$$2\times(-2)-1 = A.0 + B(-2-1)(-2-3) + C.0$$

$$\Rightarrow -5 = B\times(-3)\times(-5) \Rightarrow B = \frac{-5}{15} = \frac{-1}{3}$$

Put x = 3 in (iii)

$$2\times3-1 = A.0 + B.0 + C.(3-1)(3+2)$$

$$\Rightarrow 5 = C\times2\times5 \Rightarrow C = \frac{5}{10} = \frac{1}{2}$$

Putting the vlaue of A, B and C in (ii), we get

$$\frac{2x-1}{(x-1)(x+2)(x-3)} = \frac{-1/6}{x-1} + \frac{(-1/3)}{x+2} + \frac{1/2}{x-3} \quad ...(iv)$$

Using (iv) in (i), we get

$$I = \int\left[\frac{-1/6}{x-1} + \frac{(-1/3)}{x+2} + \frac{1/2}{x-3}\right]dx$$

$$= \frac{-1}{6}\int\frac{dx}{x-1} - \frac{1}{3}\int\frac{dx}{x+2} + \frac{1}{2}\int\frac{dx}{x-3}$$

$$I = -\frac{1}{6}\log|x-1| - \frac{1}{3}\log|x+2| + \frac{1}{2}\log|x-3| + C$$

Ans.

OR

Evaluate $\int_2^3 x^2 dx$ as limit of a sum.

Ans. Let $I = \int_2^3 x^2 dx$

Here $f(x) = x^2,\ a = 2,\ b = 3$

$\therefore\ xh = b - a = 3 - 2 = 1$

Integrating (i) as the limit of sums

$$I = \lim_{h\to0} h\left[f(a) + f(a+h) + f(a+2h) + ... + f\{a + (n-1)\}h\right]$$

$$= \lim_{h\to0} h\left[f(2) + f(2+h) + f(2+2h) + ... + f(2+(n-1))h\right]$$

$$= \lim_{h\to0} h\left[2^2 + (2+h)^2 + (2+2h)^2 + ... + \{2+(n-1)h\}^2\right]$$

$$= \lim_{h\to0} h\left[2^2 + \left(2^2 + 2.2.h + h^2\right) + \left(2^2 + 2.2.2h + 2^2h^2\right) + ...\right.$$

$$\left. + h^2\left\{1^2 + 2^2 + ... + (n-1)^2\right\}\right]$$

$$\lim_{h\to0} h\left[\left\{2^2 + 2^2 + 2^2 + ... + 2^2\right\} + 2.2.h\{1 + 2 + ... + (n-1)\}\right.$$

$$\left. + h^2\left\{1^2 + 2^2 + ... + (n-1)^2\right\}\right]$$

$$= \lim_{h\to0} h\left[n\times2^2 + 4h\times\frac{n(n-1)}{2} + h^2\times\frac{n(n-1)(2n-1)}{6}\right]$$

$$= \lim_{h\to0} h\left[4nh + 2nh(nh-h) + \frac{nh(nh-h)(2nh-h)}{6}\right]$$

$$= \lim_{h\to0} h\left[4\times1 + 2\times1(1-h) + \frac{1(1-h)(2.1-h)}{6}\right]$$

$$= 4 + 2(1-0) + \frac{1(1-0)(2-0)}{6}$$

$$= 6 + \frac{2}{6} = \frac{18+1}{3}$$

$$= \frac{19}{3}\ \text{Ans.}$$

19. Evaluate $\int_0^4 \frac{\sqrt{x}}{\sqrt{4-x}+\sqrt{x}}dx$.

Ans. Let $I = \int_0^4 \frac{\sqrt{x}}{\sqrt{4-x}+\sqrt{x}}dx \quad ...(i)$

$$= \int_0^4 \frac{\sqrt{4-x}}{\sqrt{4-(4-x)}+\sqrt{4-x}}dx$$

$$\left\{\because \int_0^a f(x)dx = \int_0^a f(a-x)dx\right\}$$

$$I = \int_0^4 \frac{\sqrt{4-x}}{\sqrt{x}+\sqrt{4-x}}dx \quad ...(ii)$$

Adding (i) and (ii), we get

$$I + I = \int_0^4\left[\frac{\sqrt{x}}{\sqrt{4-x}+\sqrt{x}} + \frac{\sqrt{4-x}}{\sqrt{x}+\sqrt{4-x}}\right]dx$$

$$\Rightarrow 2I = \int_0^4\left[\frac{\sqrt{x}+\sqrt{4-x}}{\sqrt{4-x}+\sqrt{x}}\right]dx$$

$$= \int_0^4 dx$$

$$= [x]_0^4$$

$$2I = 4 - 0$$

$\Rightarrow I = \frac{4}{2} = 2$

$I = 2$ Ans.

20. If $\vec{a} = 3\hat{i} + 2\hat{j} - 4\hat{k}$ and $\vec{b} = \hat{i} + 2\hat{j} + 3\hat{k}$ then find (i) $2\vec{a} + \vec{b}$ (ii) $\vec{a} \cdot \vec{b}$ (iii) $\vec{a} \times \vec{b}$ (iv) $|\vec{a} - \vec{b}|$.

Ans. We have, $\vec{a} = 3\hat{i} + 2\hat{j} - 4\hat{k}$

$\vec{b} = \hat{i} + 2\hat{j} + 3\hat{k}$

(i) $2\vec{a} + \vec{b} = 2(3\hat{i} + 2\hat{j} - 4\hat{k}) + \hat{i} + 2\hat{j} + 3\hat{k}$

$= 6\hat{i} + 4\hat{j} - 8\hat{k} + \hat{i} + 2\hat{j} + 3\hat{k}$

$= 7\hat{i} + 6\hat{j} - 5\hat{k}$

(ii) $\vec{a}.\vec{b} = (3\hat{i} + 2\hat{j} - 4\hat{k}).(\hat{i} + 2\hat{j} + 3\hat{k})$

$= 3 \times 1 + 2 \times 2 + (-4) \times 3$

$= 3 + 4 - 12$

$= -5$ Ans.

(iii) $\vec{a} \times \vec{b} = (3\hat{i} + 2\hat{j} - 4\hat{k}) \times (\hat{i} + 2\hat{j} + 3\hat{k})$

$$= \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 3 & 2 & -4 \\ 1 & 2 & 3 \end{vmatrix}$$

$= (6+8)\hat{i} - (9+4)\hat{j} + (6-2)\hat{k}$

$\vec{a} \times \vec{b} = 14\hat{i} - 13\hat{j} + 4\hat{k}$

(iv) $\vec{a} - \vec{b} = (3\hat{i} + 2\hat{j} - 4\hat{k}) - (\hat{i} + 2\hat{j} + 3\hat{k})$

$\vec{a} - \vec{b} = 2\hat{i} - 7\hat{k}$

$|\vec{a} - \vec{b}| = \sqrt{2^2 + (-7)^2} = \sqrt{4 + 49} = \sqrt{53}$ unit Ans.

21. Find the angle between the planes $\vec{r}.(3\hat{i} - 6\hat{j} + 2\hat{k}) = 7$ and $\vec{r}.(2\hat{i} + 2\hat{j} - 2\hat{k}) = 5$.

Ans. Given Planes are

$\vec{r}.(3\hat{i} - 6\hat{j} + 2\hat{k}) = 7$

$\vec{r}.(2\hat{i} + 2\hat{j} - 2\hat{k}) = 5$

$\Rightarrow 3x - 6y + 2z = 7$ ...(i)

$2x + 2y - 2z = 5$ ...(ii)

$a_1 = 3,\ b_1 = -6,\ c_1 = 2$

$a_2 = 2,\ b_2 = 2,\ c_2 = -2$

Let θ is the angle between two given planes.

$$\therefore \cos\theta = \left| \frac{a_1a_2 + b_1b_2 + c_1c_2}{\sqrt{a_1^2 + b_1^2 + c_1^2}\ \sqrt{a_2^2 + b_2^2 + c_2^2}} \right|$$

$$= \left| \frac{3.2 + (-6).2 + 2 \times (-2)}{\sqrt{3^2 + (-6)^2 + 2^2}\ \sqrt{2^2 + 2^2 + (-2)^2}} \right|$$

$$= \left| \frac{6 - 12 - 4}{\sqrt{49}\ \sqrt{12}} \right|$$

$$= \frac{10}{7 \times 2\sqrt{3}}$$

$\theta = \cos^{-1}\left(\frac{5}{7\sqrt{3}}\right)$ Ans.

OR

Find the value of p so that the lines $\frac{1-x}{3} = \frac{7y-14}{2p} = \frac{z-3}{2}$ and $\frac{7-7x}{3p} = \frac{y-5}{1} = \frac{6-z}{5}$ are perpendicular.

Ans. Given lines are $\frac{1-x}{3} = \frac{7y-14}{2p} = \frac{z-3}{2}$

$\frac{7-7x}{3p} = \frac{y-5}{1} = \frac{6-z}{5}$

$\Rightarrow \frac{x-1}{-3} = \frac{y-2}{\frac{2p}{7}} = \frac{z-3}{2}$ ...(i)

$\frac{x-1}{\frac{-3p}{7}} = \frac{y-5}{1} = \frac{z-6}{-5}$ ...(ii)

$a_1 = -3,\ b_1 = \frac{2p}{7},\ c_1 = 2$

$a_2 = \frac{-3p}{7},\ b_2 = 1,\ c_2 = -5$

$\because$ The two lines (i) and (ii) are perpendicular

$\therefore a_1a_2 + b_1b_2 + c_1c_2 = 0$

$\Rightarrow -3 \times \frac{-3p}{7} + \frac{2p}{7} \times 1 + 2 \times -5 = 0$

$\Rightarrow \frac{7p + 2p - 70}{7} = 0$

$\Rightarrow 11p - 70 = 0 \Rightarrow p = \frac{70}{11}$ Ans.

22. **Two cards are drawn at random and without replacement from a pack of 52 cards. Find the probability that both are black.**

Ans. Here $n(s) = 52C_2$

Let E = event to getting 2 black cards

$\therefore\ n(E) = 26C_2$

$$P(E) = \frac{n(E)}{n(s)} = \frac{26C_2}{52C_2}$$

$$= \frac{\dfrac{\lfloor 26}{\lfloor 2 \lfloor 2}}{\dfrac{\lfloor 52 \times 52}{\lfloor 2 \lfloor 50}} = \frac{25 \times 26}{51 \times 52}$$

$$\therefore P(E) = \frac{25}{102} \quad \text{Ans.}$$

$$C_{12} = -M_{12} = -\begin{vmatrix} 1 & 3 \\ 2 & 1 \end{vmatrix} = -(1-6) = 5$$

$$C_{13} = M_{13} = \begin{vmatrix} 1 & -2 \\ 2 & -1 \end{vmatrix} = -1+4 = 3$$

$$C_{21} = M_{21} = -\begin{vmatrix} 1 & 1 \\ -1 & 1 \end{vmatrix} = -(1+1) = -2$$

$$C_{22} = M_{22} = -\begin{vmatrix} 1 & 1 \\ 2 & 1 \end{vmatrix} = 1-2 = -1$$

$$C_{23} = M_{23} = -\begin{vmatrix} 1 & 1 \\ 2 & -1 \end{vmatrix} = -(-1-2) = 3$$

$$C_{31} = M_{31} = \begin{vmatrix} 1 & 1 \\ -2 & 3 \end{vmatrix} = 3+2 = 5$$

$$C_{32} = M_{32} = -\begin{vmatrix} 1 & 1 \\ 1 & 3 \end{vmatrix} = -(3-1) = -2$$

$$C_{33} = M_{33} = \begin{vmatrix} 1 & 1 \\ 1 & -2 \end{vmatrix} = -2-1 = -3$$

$$\therefore\ A^{-1} = \frac{1}{|A|} Adj\,A = \frac{1}{|A|}\begin{bmatrix} C_{11} & C_{21} & C_{31} \\ C_{12} & C_{22} & C_{32} \\ C_{13} & C_{23} & C_{33} \end{bmatrix}$$

$$\Rightarrow\ A^{-1} = \frac{1}{9}\begin{bmatrix} 1 & -2 & 5 \\ 5 & -1 & -2 \\ 3 & 3 & -3 \end{bmatrix} \qquad \text{...(iii)}$$

Using (iii) and (i) in equation (ii), we get

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \frac{1}{9}\begin{bmatrix} 1 & -2 & 5 \\ 5 & -1 & -2 \\ 3 & 3 & -3 \end{bmatrix}\begin{bmatrix} 3 \\ 2 \\ 2 \end{bmatrix}$$

$$= \frac{1}{9}\begin{bmatrix} 1\times 3+(-2)\times 2+5\times 2 \\ 5\times 3+(-1)\times 2+(-2)\times 2 \\ 3\times 3+3\times 2+(-3)\times 2 \end{bmatrix}$$

$$= \frac{1}{9}\begin{bmatrix} 9 \\ 9 \\ 9 \end{bmatrix} = \begin{bmatrix} \frac{1}{9}\times 9 \\ \frac{1}{9}\times 9 \\ \frac{1}{9}\times 9 \end{bmatrix}$$

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$$

Hence, x = 1, y = 1, z = 1 which are the required solution.

OR

If $2P(A) = P(B) = \frac{5}{13}$ and $P(A/B) = \frac{2}{5}$, then find $P(A \cup B)$.

Ans. We have, $2P(A) = P(B) = \frac{5}{13}$

$$\Rightarrow\ P(A) = \frac{5}{26},\ P(B) = \frac{5}{13}$$

Also $P\left(\frac{A}{B}\right) = \frac{2}{5}$

We know that $P\left(\frac{A}{B}\right) = \frac{P(A \cap B)}{P(B)}$

$$\Rightarrow\ \frac{2}{5} = \frac{P(A \cap B)}{\frac{5}{13}}$$

$$\because\ P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

$$= \frac{5}{26} + \frac{5}{13} - \frac{2}{13}$$

$$= \frac{5+10-4}{26}$$

$$P(A \cup B) = \frac{11}{26} \qquad \text{Ans.}$$

Section - C

23. Using matrices, solve the following system of equations:
$x+y+z=3$, $x-2y+3z=2$, $2x-y+z=2$.

Ans. We have, $x+y+z=3$

$x-2y+3z=2$

$2x-y+z=2$

$$\Rightarrow \begin{bmatrix}1 & 1 & 1\\1 & -2 & 3\\2 & -1 & 1\end{bmatrix}\begin{bmatrix}x\\y\\z\end{bmatrix}=\begin{bmatrix}3\\2\\2\end{bmatrix}$$

$\Rightarrow A\times=B$ (let) ...(i)

$$\text{Now } |A|=\begin{vmatrix}1 & 1 & 1\\1 & -2 & 3\\2 & -1 & 1\end{vmatrix}=1(-2+3)-1(1-6)+1(-1+4)$$

$=1+5+3$

$|A|=9\neq 0$

This $\Rightarrow$ A^{-1} exists and solution is given by

$X=A^{-1}.B$ ...(ii)

OR

Find the inverse of the matrix using elementary operations:

$$A=\begin{bmatrix}0 & 1 & 2\\1 & 2 & 3\\3 & 1 & 1\end{bmatrix}.$$

Ans. Let $A=\begin{bmatrix}0 & 1 & 2\\1 & 2 & 3\\3 & 1 & 1\end{bmatrix}$

We know that, A = I . A

$$\Rightarrow \begin{bmatrix}0 & 1 & 2\\1 & 2 & 3\\3 & 1 & 1\end{bmatrix}=\begin{bmatrix}1 & 0 & 0\\0 & 1 & 0\\0 & 0 & 1\end{bmatrix}.A$$

Applying $R_1\to R_1+R_2$, $R_3\to R_3-3R_2$

$$\begin{bmatrix}1 & 3 & 5\\1 & 2 & 3\\0 & -5 & -8\end{bmatrix}=\begin{bmatrix}1 & 1 & 0\\0 & 1 & 0\\0 & -3 & 1\end{bmatrix}.A$$

Applying $R_2\to R_2-R_1$

$$\begin{bmatrix}1 & 3 & 5\\0 & -1 & -2\\0 & -5 & -8\end{bmatrix}=\begin{bmatrix}1 & 1 & 0\\-1 & 0 & 0\\0 & -3 & 1\end{bmatrix}.A$$

Applying $R_1\to R_1+3R_2$, $R_3\to R_3-5R_2$

$$\begin{bmatrix}1 & 0 & -1\\0 & -1 & -2\\0 & 0 & 2\end{bmatrix}=\begin{bmatrix}-2 & 1 & 0\\-1 & 0 & 0\\5 & -3 & 1\end{bmatrix}.A$$

Applying $R_3\to\frac{1}{2}R_3$, $R_2\to(-1)R_2$

$$\begin{bmatrix}1 & 0 & -1\\0 & 1 & 2\\0 & 0 & 1\end{bmatrix}=\begin{bmatrix}-2 & 1 & 0\\1 & 0 & 0\\5/2 & -3/2 & 1/2\end{bmatrix}.A$$

Applying $R_1\to R_1+R_3$, $R_2\to R_2-2R_3$

$$\begin{bmatrix}1 & 0 & 0\\0 & 1 & 0\\0 & 0 & 1\end{bmatrix}=\begin{bmatrix}\frac{1}{2} & -\frac{1}{2} & \frac{1}{2}\\-4 & 3 & -1\\\frac{5}{2} & \frac{-3}{2} & \frac{1}{2}\end{bmatrix}.A$$

$\Rightarrow I=A^{-1}.A$

$$\text{Hence } A^{-1}=\begin{bmatrix}\frac{1}{2} & -\frac{1}{2} & \frac{1}{2}\\-4 & 3 & -1\\\frac{5}{2} & \frac{-3}{2} & \frac{1}{2}\end{bmatrix}\text{ Ans.}$$

24. Find the absolute maximum and absolute minimum of a function f given by $f(x)=2x^3-15x^2+36x+1$ in the interval [1, 5].

Ans. $f(x)=2x^3-15x^2+36x+1$ in $[1,5]$ ...(i)

$\Rightarrow f'(x)=6x^2-30x+36$ ...(ii)

$\Rightarrow f''(x)=12x-30$ ...(iii)

For maxima or minima, we must have $f'(x)=0$

$\Rightarrow 6x^2-30x+36=0$

$\Rightarrow 6(x^2-5x+6)=0$

$\Rightarrow x^2-5x+6=0$

$\Rightarrow x^2-2x-3x+6=0$

$\Rightarrow x(x-2)-3(x-2)=0$

$\Rightarrow (x-2)(x-3)=0$

$\Rightarrow x=2,3$

Now $f''(2)=12\times 2-30$, by (iii) $=-6<0$

This x = 2 is the point of local maxima

$\therefore$ Local maximum value = f(2)

$=2\times 2^3-15\times 2^2+36\times 2+1$

$=16-60+72+1$

[illegible]

Absolute maximum value of [illegible]

= Greatest Among local maximum value [illegible]

= [illegible]

= [illegible] Ans

Absolute minimum value of [illegible]

= Least Among local minimum value [illegible]

= $f(1)$

= 24 Ans

25. Find the area of the region bounded by the parabola $y^2 = 4x$ and the line $x = 3$.

Ans. Given equation of parabola and line are

$y^2 = 4x$ ...(i)

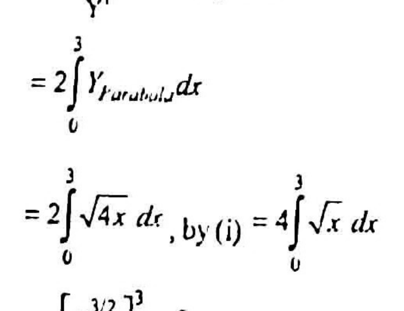
and $x = 3$ ...(ii)

Parabola (i) is a right handed parabola having vertex at (0, 0)

line (ii) is a st. line parallel to Y-axis at a distance 3 unit from it.

Required Area = Area of shaded region OABCO

= 2 × Ar. of shaded region OBCO by symmetricity of curve.



$$= 2\int_0^3 Y_{Parabola}\,dx$$

$$= 2\int_0^3 \sqrt{4x}\,dx \text{, by (i)} = 4\int_0^3 \sqrt{x}\,dx$$

$$= 4\left[\frac{x^{3/2}}{3/2}\right]_0^3 = \frac{8}{3}\left[3^{3/2} - 0^{3/2}\right]$$

[illegible]

[illegible] AB

[illegible]

$$y - 0 = \frac{2-0}{2-1}(x-1) \Rightarrow Y_{AB} = 2x - 2 \quad ...(ii)$$

Equation of side BC

$$y - 2 = \frac{1-2}{3-2}(x-2) \Rightarrow y = -1(x-2) + 2$$

$$\Rightarrow Y_{BC} = -x + 4 \quad ...(iii)$$

Equation of side AC

$$Y - 0 = \frac{1-0}{3-1}(x-1) \Rightarrow Y_{AC} = \frac{1}{2}(x-1) \quad ...(iv)$$

∴ Required Area = Area of ΔABC

= Area of region ALBA + Area of region BLMC - Area of region AMCA

$$= \int_1^2 Y_{AB}\,dx + \int_2^3 Y_{BC}\,dx - \int_1^3 Y_{AC}\,dx$$

$$= \int_1^2 (2x-2)\,dx + \int_2^3 (-x+4)\,dx - \int_1^3 \frac{1}{2}(x-1)\,dx,$$

by (i),(ii)&(iii)

$$= \left[\frac{2x^2}{2} - 2x\right]_1^2 + \left[\frac{-x^2}{2} + 4x\right]_2^3 - \frac{1}{2}\left[\frac{x^2}{2} - x\right]_1^3$$

$$\Rightarrow y \times \log x = \int \frac{2}{x^2} \times \log x\,dx$$

$$\Rightarrow y\log x = 2\left[\log x \int \frac{1}{x^2}dx\right]\left[\left\{\frac{d}{dx}(\log x)\right\}\left\{\int \frac{1}{x^2}dx\right\}dx\right]$$

$$\Rightarrow y\log x = 2\left[\log x \times \frac{-1}{x} \int \frac{1}{x} \times \frac{-1}{x} dx\right]$$

$$= \frac{-2\log x}{x} + 2\int x^{-2} dx$$

$$y\log x = \frac{-2\log x}{x} + 2\times\frac{x^{-2}+1}{-2+1} + C$$

$$\Rightarrow y = \frac{-2}{x} - \frac{2}{x\log x} + \frac{C}{\log x}$$

Which is the required solution.

26. Solve the differential equation:

$$x.\log x.\frac{dy}{dx} + y = \frac{2}{x}.\log x$$

Ans. We have, $x\log x.\frac{dy}{dx} + y = \frac{2}{x}.\log x$

$$\Rightarrow \frac{dy}{dx} + \frac{y}{x\log x} = \frac{2\log x}{x.x\log x}$$

$$\Rightarrow \frac{dy}{dx} + \frac{y}{x\log x} = \frac{2}{x^2}$$

Which is a line are differential equation in y.

$$\therefore P = \frac{1}{x\log x}, Q = \frac{2}{x^2}$$

$$\text{I.F.} = e^{\int P dx} = e^{\int \frac{1}{x\log x} dx} = e^{\int \frac{1}{t} dt} = e^{\log t}$$

$$= t = \log x$$

I.F. = log x

Now, solution of (i) is given by $y \times I.F. = \int \theta \times I.F. dx?$

OR

Solve the differential equation:

$$x.\frac{dy}{dx} - y = x.\tan\left(\frac{y}{x}\right), \text{given that } y = \frac{\pi}{2} \text{ when } x = 1.$$

Ans. $x\frac{dy}{dx} - y = x\tan\left(\frac{y}{x}\right)$, given that $y = \frac{\pi}{2}$ when x = 1

$$\Rightarrow \frac{dy}{dx} - \frac{y}{x} = \tan\left(\frac{y}{x}\right) \quad ...(i)$$

$$\text{Let } \frac{y}{x} = v \Rightarrow y = vx \quad ...(ii)$$

$$\frac{dy}{dx} = v.1 + x\frac{dv}{dx} \quad ...(iii)$$

Using (ii) and (iii) in (i), we get

$$v + x\frac{dv}{dx} - v = \tan v$$

$$\Rightarrow x\,dv = \tan v\,dx$$

$$\Rightarrow \int \frac{dv}{\tan v} = \int \frac{dx}{x} \Rightarrow \int \cot v\,dv = \int \frac{dx}{x}$$

$$\Rightarrow \log\sin v = \log x + \log C$$

$$\Rightarrow \log\sin\frac{y}{x} = \log cx \Rightarrow \sin\frac{y}{x} = Cv$$

$$\Rightarrow \frac{y}{x} = \sin^{-1}(cx) \Rightarrow y = x\sin^{-1}(cx)$$

$$= \left[\left(2^2 - 2\times2\right) - \left(1^2 - 2\times1\right)\right] + \left[\left(-\frac{1}{2}(3^2) + 4\times3\right) - \left(\frac{-1}{2}\times(2)^2 + 4\times2\right)\right]$$

$$-\frac{1}{2}\left[\left(\frac{1}{2}\times3^2 - 3\right) - \left(\frac{1^2}{2} - 1\right)\right]$$

$$= [0+1] + \left[\frac{-9}{2} + 12 + 2 - 8\right] - \frac{1}{2}\left[\frac{9}{2} - 3 - \frac{1}{2} + 1\right]$$

$$= 1 + 6 - \frac{9}{2} - \frac{1}{2}\left(\frac{9}{2} - \frac{1}{2} - 2\right)$$

$$= 7 - \frac{9}{2} - \frac{1}{2}\left(\frac{8}{2} - 2\right) = 7 - \frac{9}{2} - \frac{1}{2}\times2$$

$$= 6 - \frac{9}{2} = \frac{12-9}{2}$$

$$= \frac{3}{2} \text{ Square unit.}$$

27. Find the shortest distance between the lines $\vec{r} = \hat{i} + 2\hat{j} + 3\hat{k} + \lambda\left(2\hat{i} + 3\hat{j} + 4\hat{k}\right)$ **and** $\vec{r} = 2\hat{i} + 4\hat{j} + 5\hat{k} + t\left(3\hat{i} + 4\hat{j} + 5\hat{k}\right)$

Ans. Given lines are

$$\vec{r} = \hat{i} + 2\hat{j} + 3\hat{k} + \lambda\left(2\hat{i} + 3\hat{j} + 4\hat{k}\right) \quad ...(i)$$

$$\vec{r} = 2\hat{i} + 4\hat{j} + 5\hat{k} + t\left(3\hat{i} + 4\hat{j} + 5\hat{k}\right) \quad ...(ii)$$

Here $\vec{a}_1 = \hat{i} + 2\hat{j} + 3\hat{k}, \vec{b}_1 = 2\hat{i} + 3\hat{j} + 4\hat{k}$

$\vec{a}_2 = 2\hat{i} + 4\hat{j} + 5\hat{k}, \vec{b}_2 = 3\hat{i} + 4\hat{j} + 5\hat{k}$

$$\vec{a}_2 - \vec{a}_1 = \hat{i} + 2\hat{j} + 2\hat{k} \quad ...(iii)$$

$$\vec{b}_1 \times \vec{b}_2 = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & 3 & 4 \\ 3 & 4 & 5 \end{vmatrix} = \hat{i}(15-16) - \hat{j}(10-12) + \hat{k}(8-9)$$

[illegible]

$\vec b_1 \times \vec b_2 =$ [illegible]

$\therefore |\vec b_1 \times \vec b_2| = \sqrt{\ldots}$ [illegible] ...(v)

Now $(\vec a_2 - \vec a_1)\cdot(\vec b_1 \times \vec b_2) =$ [illegible]

$= 1 + 4 - 2$ [illegible]

$\therefore (\vec a_2 - \vec a_1)\cdot(\vec b_1 \times \vec b_2) = 1$ ...(vi)

We know that shortest distance

$$= \left|\frac{(\vec a_2 - \vec a_1)\cdot(\vec b_1 \times \vec b_2)}{|\vec b_1 \times \vec b_2|}\right|$$

$$= \left|\frac{1}{\sqrt{6}}\right|, \text{ by (v) and (vi)}$$

$$= \frac{1}{\sqrt{6}} \times \frac{\sqrt{6}}{\sqrt{6}}$$

$$= \frac{\sqrt{6}}{6} \text{ unit} \qquad \text{Ans.}$$

28. A card from a pack of 52 cards is lost. From the remaining cards of the pack two cards are drawn and are found to be both diamonds. Find the probability of the lost card being a diamond.

Ans. Let E_1 = event that the lost card being a diamand.

E_2 = event that the lost card being a heart.

E_3 = event that the lost card being a spade.

E_4 = event that the lost card being a club.

$$P(E_1) = \frac{13C_1}{52C_1} = \frac{13}{52} = \frac{1}{4}$$

Similarly, $P(E_2) = P(E_3) = P(E_4) = \frac{13C_1}{52C_1} = \frac{13}{52} = \frac{1}{4}$

Let E = event that the two cards drawn are both diamonds

$P\left(\frac{E}{E_1}\right) = P$ (both diamonds given that lost card being a diamond)

$$= \frac{12C_2}{51C_2} = \frac{\frac{\lfloor 12}{\lfloor 2\,\lfloor 10}}{\frac{\lfloor 51}{\lfloor 2\,\lfloor 49}} = \frac{11\times 12}{50\times 51}$$

$P\left(\frac{E}{E_2}\right) = P$ (both diamonds given that the lost card being a heart)

$$= \frac{13C_2}{51C_2} = \frac{12\times 13}{50\times 51}$$

[illegible]

$P\left(\frac{E}{E_4}\right) = P$ (both diamonds given that the lost card being a club)

$$= \frac{13C_2}{51C_2} = \frac{12\times 13}{50\times 51}$$

Required probability $= P\left(\frac{E_1}{E}\right)$

$$= \frac{P(E_1).P\left(\frac{E}{E_1}\right)}{P(E_1).P\left(\frac{E}{E_1}\right) + P(E_2).P\left(\frac{E}{E_2}\right) + P(E_3).P\left(\frac{E}{E_3}\right) + P(E_4).P\left(\frac{E}{E_4}\right)}$$

(From baye's theorem)

$$= \frac{\frac{1}{4}\times\frac{11\times 12}{50\times 51}}{\frac{1}{4}\times\frac{11\times 12}{50\times 51} + \frac{1}{4}\times\frac{12\times 13}{50\times 51} + \frac{1}{4}\times\frac{12\times 13}{50\times 51} + \frac{1}{4}\times\frac{12\times 13}{50\times 51}}$$

$$= \frac{\frac{1}{4}\times\frac{11\times 12}{50\times 51}}{\frac{1}{4}\left[\frac{11\times 12 + 12\times 13 + 12\times 13 + 12\times 13}{50\times 51}\right]}$$

$$= \frac{132}{132 + 156 + 156 + 156}$$

$$= \frac{132}{600} = \frac{11}{50} \qquad \text{Ans.}$$

OR

A person buys 50 tickets of a lottery, in each of which his chance of winning a prize is $\frac{1}{100}$. What is the probability that he will win a prize (a) at least once, (b) exactly once, (c) at least twice?

Ans. Here $n = 50$, $p = \frac{1}{100}$

$$\therefore q = 1 - p = 1 - \frac{1}{100} = \frac{99}{100}$$

(a) P (at least once) $= P(x \geq 1)$

$= P(x = 1) + P(x = 2) + \dots + P(x = 50)$

$= 1 - P(x = 0)$

$= 1 - 50C_0\left(\frac{1}{100}\right)^0\left(\frac{99}{100}\right)^{50-0}$ $\left[\because P(x = r) = {}^nC_r p^r q^{n-r}\right]$

$= 1 - 1\times 1\times\frac{99^{50}}{100^{50}} = \frac{(100)^{50} - (99)^{50}}{(100)^{50}}$

(b) P (exactly once) = P (x = 1)

$= 50C_1\times\left(\frac{1}{100}\right)^1\times\left(\frac{99}{100}\right)^{49}$

$= 50\times\frac{1}{100}\times\left(\frac{99}{100}\right)^{49}$

(c) P (at least twice) = P (x ≥ 2)

$= P(x = 2) + P(x = 3) + \dots + P(x = 50)$

$= 1 - P(x = 0) - P(x = 1)$

$= 1 - 50C_0\left(\frac{1}{100}\right)^0\left(\frac{99}{100}\right)^{50} - 50C_1\times\left(\frac{1}{100}\right)\times\left(\frac{99}{100}\right)^{49}$

$= 1 - 1.1.\left(\frac{99}{100}\right)^{50} - 50\times\frac{1}{100}\times\left(\frac{99}{100}\right)^{49} = \frac{(100)^{50} - (99)^{50} - 50\times(99)^{49}}{(100)^{50}}$

29. Solve the following LPP graphically:
Minimize Z = 20x + 50y

Subject to constraints $x + 2y \geq 10$

$3x + 4y \leq 24$

$x \geq 0,\ y \geq 0$

Ans. Minimize Z = 20x + 50y

Subject to constraints $x + 2y \geq 10$

$3x + 4y \leq 24$

$x \geq 0,\ y \geq 0$

Table for x + 2y = 10

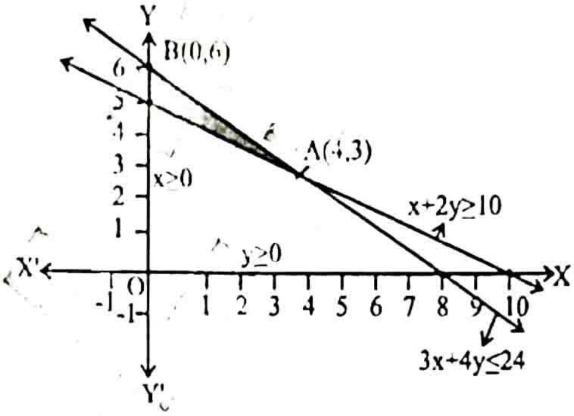
x	0	10
y	5	0

Table for 3x + 4y = 24

x	0	8
y	6	0

x = 0 is Y-axis

y = 0 is X-axis



from the graph, we canclude that the shaded region CABC is the solution set,

Co-ordinate of point C is (0, 5)

Co-ordinate of point A is (4, 3)

Co-ordinate of point A is A (0, 6)

At the point C (0, 5), $Z = 20\times 0 + 50\times 5 = 250$

At the point C (4, 3), $Z = 20\times 4 + 50\times 3 = 230$

At the point C (0, 6), $Z = 20\times 0 + 50\times 6 = 300$

Clearly, Z is minimum at the point A(4, 3)

∴ x = 4 and y = 3 is the required solution.

Ans.