(01)

Mathematics - 2020

Full Marks : 100 **(Time: 3 Hours)** **Pass Marks : 33**

General Instructions:

The question paper consists of 29 questions divided into three Sections - A, B and C.
Section - A comprises 10 questions of 1 mark each.
Section - B comprises 12 questions of 4 marks each and
Section - C comprises 7 questions of 6 marks each.
Use of calculator is not permitted. However, you may ask for logarithmic and statistical tables, if required.

Section - A

1. Let * be the binary operation on Z+, defined by $a * b = |a-b|$. Find the value of $3 * 7$. 1

Ans. $a * b = |a-b|$

$\therefore \quad 3*7 = |3-7| = |-4| = 4$ Ans.

2. Evaluate $\sin\left\{\frac{\pi}{3} - \sin^{-1}\left(-\frac{1}{2}\right)\right\}$. 1

Ans. $\sin\left\{\frac{\pi}{3} - \sin^{-1}\left(-\frac{1}{2}\right)\right\}$

$$= \sin\left\{\frac{\pi}{3} - \sin^{-1}\left(-\sin\frac{\pi}{6}\right)\right\}$$

$$= \sin\left\{\frac{\pi}{3} - \sin^{-1}\left(\sin\left(\frac{-\pi}{6}\right)\right)\right\}$$

$$= \sin\left\{\frac{\pi}{3} - \left(\frac{-\pi}{6}\right)\right\} = \sin\left\{\frac{\pi}{3} + \frac{\pi}{6}\right\}$$

$$= \sin\frac{\pi}{2} = 1 \quad \text{Ans.}$$

3. Find the values of x and y, where $\begin{bmatrix} 2x+y & 3y \\ 6 & 4 \end{bmatrix} = \begin{bmatrix} 6 & 0 \\ 6 & 4 \end{bmatrix}$ 1

Ans. $\begin{bmatrix} 2x+y & 3y \\ 6 & 4 \end{bmatrix} = \begin{bmatrix} 6 & 0 \\ 6 & 4 \end{bmatrix}$

$\therefore \quad 3y = 0 \Rightarrow y = \frac{0}{3} = 0$

$2x + y = 6$

$\Rightarrow \quad 2x + 0 = 6 \Rightarrow 2x = 6 \Rightarrow x = \frac{6}{2} = 3$

$\therefore \quad x = 3, y = 0$ Ans.

4. Find minors of all the elements of $\begin{vmatrix} 2 & -4 \\ 0 & 3 \end{vmatrix}$ 1

Ans. $\begin{vmatrix} 2 & -4 \\ 0 & 3 \end{vmatrix}$

$M_{11} = |3| = 3$, $M_{12} = |0| = 0$

$M_{21} = |-4| = -4$, $M_{22} = |2| = 2$ Ans.

5. Find the slope of the tangent to the curve $x^2 + y^2 = 25$ at point $(-3, 4)$. 1

Ans. $x^2 + y^2 = 25$

Dift. both sides w.r.t 'x'

$$2x + 2y\frac{dy}{dx} = 0$$

$$\frac{dy}{dx} = \frac{-2x}{2y}$$

$\therefore$ Slope of tangent at $(-3, 4) = \left[\frac{dy}{dx}\right]_{(-3,4)} = \frac{-(-3)}{4} = \frac{3}{4}$ Ans.

6. Find $\frac{dy}{dx} : y = \tan^{-1}x + \cot^{-1}x$. 1

Ans. $y = \tan^{-1}x + \cot^{-1}x$

$\Rightarrow \quad y = \frac{\pi}{2}$

$\therefore \quad \frac{dy}{dx} = \frac{d}{dx}\left(\frac{\pi}{2}\right) = 0$ Ans.

7. Solve: $\int a^{3\log_a x} dx$. 1

Ans. $\int a^{3\log_a x} dx = \int a^{\log_a x^3} dx$

$= \int x^3 dx = \frac{x^4}{4} + C$ Ans.

8. For what value of p the vectors $3\hat{i} + 2\hat{j} + 9\hat{k}$ and $\hat{i} + p\hat{j} + 3\hat{k}$ are parallel. 1

Ans. Let $\vec{a} = 3\hat{i} + 2\hat{j} + 9\hat{k}$

$\vec{b} = \hat{i} + p\hat{j} + 3\hat{k}$

A/q, $\because$ $\vec{a}$ & $\vec{b}$ are parablel

$\therefore \quad \vec{a} \times \vec{b} = 0$

$$\Rightarrow \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 3 & 2 & 9 \\ 1 & p & 3 \end{vmatrix} = 0$$

$\Rightarrow \quad (6-9p)\hat{i} - (9-9)\hat{j} + (3p-2)\hat{k} = 0.\hat{i} + 0.\hat{j} + 0.\hat{k}$

$\Rightarrow \quad 6 - 9p = 0 \quad \Rightarrow 9p = 6$

$\Rightarrow \quad p = \frac{6}{9} \quad \therefore p = \frac{2}{3}$ Ans.

9. Find the order and degree of the differential equation

$$\left(\frac{dy}{dx}\right)^4 + 3\frac{d^2y}{dx^2} = 0. \qquad 1$$

Ans. Order = 2, Degree = 1

10. Find the direction ratio and direction cosines of the line

$$\frac{x-5}{3} = \frac{y-1}{4} = \frac{z-4}{5}. \qquad 1$$

Ans. Direction ratios of the line = 3, 4, 5

Direction cosines of the line

$$= \frac{3}{\sqrt{3^2+4^2+5^2}}, \frac{4}{\sqrt{3^2+4^2+5^2}}, \frac{5}{\sqrt{3^2+4^2+5^2}}$$

$$= \frac{3}{\sqrt{50}}, \frac{4}{\sqrt{50}}, \frac{5}{\sqrt{50}}$$

$$= \frac{3}{5\sqrt{2}}, \frac{4}{5\sqrt{2}}, \frac{5}{5\sqrt{2}} \qquad \text{Ans.}$$

Section - B

11. If the function $f : R \rightarrow R$ be given by $f(x) = x^2 + 2$ and $g : R \rightarrow R$ be given by $g(x) = \frac{x}{x-1}, x \neq 1$, find fog and gof. 4

Ans. $f : R \rightarrow R$ given by $f(x) = x^2 + 2$ ...(i)

$g : R \rightarrow R$ given by $g(x) = \frac{x}{x-1}, x \neq 1$ ..(ii)

$\therefore \quad fog(x) = f\{g(x)\} = f\left(\frac{x}{x-1}\right)$, by (ii)

$$= \left(\frac{x}{x-1}\right)^2 + 2 = \frac{x^2}{(x-1)^2} + 2$$

$$= \frac{x^2+2(x-1)^2}{(x-2)^2} = \frac{x^2+2(x-1)^2}{(x-1)^2} = \frac{x^2+2(x^2-2x+1)}{(x-1)^2}$$

$$= \frac{x^2+2(x^2-2x+1)}{(x-1)^2} = \frac{2x^2-2x+1}{(x-1)^2}$$

$$= \frac{x^2+2x^2-4x+2}{(x-1)^2}$$

$\therefore \; fog(x) = \frac{3x^2-4x+2}{(x-1)^2}$ Ans.

Again, $gof(x) = g\{t(x)\}$

$= g\{x^2+2\}$, by (i)

$= \frac{x^2+2}{x^2+2-1}$, by (ii)

$gof(x) \; \frac{x^2+2}{x^2+1}$ Ans.

12. Prove that $\tan^{-1}\frac{2}{3} + \sin^{-1}\frac{3}{5} = \tan^{-1}\frac{17}{6}$ 4

Ans. L.H.S. $= \tan^{-1}\frac{2}{3} + \sin^{-1}\frac{3}{5}$

$$= \tan^{-1}\frac{2}{3} + \tan^{-1}\frac{3}{4} \qquad \left\{\because \sin^{-1}x = \tan^{-1}\frac{x}{\sqrt{1-x^2}}\right\}$$

$$= \tan^{-1}\left(\frac{\frac{2}{3}+\frac{3}{4}}{1-\frac{2}{3}\cdot\frac{3}{4}}\right)$$

$$= \tan^{-1}\left(\frac{\frac{8+9}{12}}{\frac{12-6}{12}}\right)$$

$$= \tan^{-1}\left(\frac{17}{6}\right)$$

= R.H.S. Proved.

13. Prove that

$$\begin{vmatrix} x+y+2z & x & y \\ z & y+z+2x & y \\ z & x & z+x+2y \end{vmatrix} = 2(x+y+z)^3 \qquad 4$$

Ans. Let $\Delta = \begin{vmatrix} x+y+2z & x & y \\ z & y+z+2x & y \\ z & x & z+x+2y \end{vmatrix}$

Applying $C_1 \rightarrow C_1 + C_2 + C_3$

$$= \begin{vmatrix} 2(x+y+z) & x & y \\ 2(x+y+z) & y+z+2x & y \\ 2(x+y+z) & x & z+x+2y \end{vmatrix}$$

Taking $2(x+y+z)$ from C_1

$$= 2(x+y+z)\begin{vmatrix} 1 & x & y \\ 1 & y+z+2x & y \\ 1 & x & z+x+2y \end{vmatrix}$$

Applying $R_1 \rightarrow R_1 - R_3$, $R_2 \rightarrow R_2 - R_3$

$$= 2(x+y+z)\begin{vmatrix} 0 & 0 & -z-x-y \\ 0 & y+z+x & -z-x-y \\ 1 & x & z+x+2y \end{vmatrix}$$

Expanding along R_1

$$= 2(x+y+z)\{0-0+(-z-x-y)(0-(y+z+x))\}$$

$$= 2(x+y+z)\{-(x+y+z)\times-(x+y+z)\}$$

$$= 2(x+y+z)(x+y+z)^2$$

$$= 2(x+y+z)^3 \qquad \text{proved}$$

OR

Prove that

$$\begin{vmatrix} \alpha & \alpha^2 & \beta+\gamma \\ \beta & \beta^2 & \gamma+\alpha \\ \gamma & \gamma^2 & \alpha+\beta \end{vmatrix} = (\alpha+\beta+\gamma)(\alpha-\beta)(\beta-\gamma)(\gamma-\alpha)$$

Ans. Let $\Delta = \begin{vmatrix} \alpha & \alpha^2 & \beta+\gamma \\ \beta & \beta^2 & \gamma+\alpha \\ \gamma & \gamma^2 & \alpha+\beta \end{vmatrix}$

Applying $C_1 \to C_1 + C_3$

$$= \begin{vmatrix} \alpha+\beta+\gamma & \alpha^2 & \beta+\gamma \\ \alpha+\beta+\gamma & \beta^2 & \gamma+\alpha \\ \alpha+\beta+\gamma & \gamma^2 & \alpha+\beta \end{vmatrix}$$

$$= (\alpha+\beta+\gamma)\begin{vmatrix} 1 & \alpha^2 & \beta+\gamma \\ 1 & \beta^2 & \gamma+\alpha \\ 1 & \gamma^2 & \alpha+\beta \end{vmatrix}$$

Applying $R_1 \to R_1 - R_3$, $R_2 \to R_2 - R_3$

$$= (\alpha+\beta+\gamma)\begin{vmatrix} 0 & \alpha^2-\gamma^2 & \gamma-\alpha \\ 0 & \beta^2-\gamma^2 & \gamma-\beta \\ 1 & \gamma^2 & \alpha+\beta \end{vmatrix}$$

$$= (\alpha+\beta+\gamma)\begin{vmatrix} 0 & (\gamma^2-\alpha^2) & \gamma-\alpha \\ 0 & \beta^2-\gamma^2 & -(\beta-\gamma) \\ 1 & \gamma^2 & \alpha+\beta \end{vmatrix}$$

Taking $(\gamma-\alpha)$ and $(\beta-\gamma)$ common from R_1 & R_2 respectively.

$$= (\alpha+\beta+\gamma)(\gamma-\alpha)(\beta-\gamma)\begin{vmatrix} 0 & -(\gamma+\alpha) & 1 \\ 0 & \beta+\gamma & -1 \\ 1 & \gamma^2 & \alpha+\beta \end{vmatrix}$$

Expanding along C_1

$$= (\alpha+\beta+\gamma)(\gamma-\alpha)(\beta-\gamma)\{0-0+1\{\gamma+\alpha-\beta-\gamma\}\}$$

$$= (\alpha+\beta+\gamma)(\gamma-\alpha)(\beta-\gamma)(\alpha-\beta)$$

$$= (\alpha+\beta+\gamma)(\alpha-\beta)(\beta-\gamma)(\gamma-\alpha)$$ proved

14. Find the relationship between a and b so that function f defined by $f(x)\begin{cases} ax+1 & \text{if } x \le 3 \\ bx+3 & \text{if } x > 3 \end{cases}$ is continuous at $x=3$ 4

Ans. $f(x) = \begin{cases} ax+1 & \text{if } x \le 3 \\ bx+3 & \text{if } x > 3 \end{cases}$

A/q $\because f(x)$ is continuous at $x = 3$

$\therefore \lim\limits_{x\to 3^+} f(x) = f(3) = \lim\limits_{x\to 3^-} f(x)$

$\Rightarrow \lim\limits_{x\to 3^+}(bx+3) = a\cdot 3+1 = \lim\limits_{x\to 3^-}(ax+1)$

$\Rightarrow \lim\limits_{h\to 0}\{b(3-h)+3\} = 3a+1 = \lim\limits_{h\to 0} a(3-h)+1$

$\Rightarrow b(3+0)+3 = 3a+1 = a(3-0)+1$

$\Rightarrow 3b+3 = 3a+1 = 3a+1$

$\Rightarrow 3b+3 = 3a+1$

$\Rightarrow 3a-3b = 2$

$\Rightarrow 3(a-b) = 2$

$\therefore a-b = \dfrac{2}{3}$ Ans.

15. Find $\dfrac{dy}{dx}$, if $y = (\sin x)^x$ 4

Ans. $y = (\sin x)^x$

Taking log on both sides

$\log y = \log(\sin x)^x$

$\Rightarrow \log y = x\log\sin x$

Differentiating both sides with respect to 'x'

$$\frac{d}{dx}(\log y) = \frac{d}{dx}(x\log\sin x)$$

$$\Rightarrow \frac{1}{y}\cdot\frac{dy}{dx} = x\cdot\frac{d}{dx}(\log\sin x) + \log\sin x\cdot\frac{d}{dx}(x)$$

$$\Rightarrow \frac{dy}{dx} = y\left[x\cdot\frac{1}{\sin x}\cdot\cos x + \log\sin x\cdot 1\right]$$

$$\therefore \frac{dy}{dx} = (\sin x)^x\left[x\cot x + \log\sin x\right]$$ Ans.

OR

If $y = \sin^{-1}x$, then show that $(1-x^2)\dfrac{d^2y}{dx^2} - x\dfrac{dy}{dx} = 0$

$y = \sin^{-1}x$ ...(i)

Differentiating both sides w.r.t. 'x'

$$\frac{dy}{dx} = \frac{d}{dx}(\sin^{-1}x)$$

$$\frac{dy}{dx} = \frac{1}{\sqrt{1-x^2}} \quad ...(ii)$$

Again differentiating both sides w.r.t. 'x'

$$\frac{d}{dx}\left(\frac{dy}{dx}\right) = \frac{d}{dx}\left(\frac{1}{\sqrt{1-x^2}}\right)$$

$$\Rightarrow \frac{d^2y}{dx^2} = \frac{\sqrt{1-x^2}\,\frac{d}{dx}(1) - 1\frac{d}{dx}\left(\sqrt{1-x^2}\right)}{\left(\sqrt{1-x^2}\right)^2}$$

$$\Rightarrow \frac{d^2y}{dx^2} = \frac{\sqrt{1-x^2}\cdot 0 - 1\cdot\frac{1}{2\sqrt{1-x^2}}\times(0-2x)}{(1-x^2)}$$

$$\Rightarrow (1-x^2)\frac{d^2y}{dx^2} = 0 + \frac{2x}{2\sqrt{1-x^2}}$$

$$\Rightarrow (1-x^2)\frac{d^2y}{dx^2} = x\cdot\frac{1}{\sqrt{1-x^2}}$$

$$\Rightarrow \left(1-x^2\right)\frac{d^2y}{dx^2} = x\cdot\frac{dy}{dx}\text{, by (i)}$$

$$\therefore \left(1-x^2\right)\frac{d^2y}{dx^2} - x\frac{dy}{dx} = 0 \text{ proved.}$$

16. Find the intervals in which the function f given by $f(x) = 4x^3 - 6x^2 - 72x + 30$ is

(a) strictly increasing, (b) strictly decreasing 4

Ans. $f(x) = 4x^3 - 6x^2 - 72x + 30$ ...(i)

$$\Rightarrow f'(x) = 12x^2 - 12x - 72.1 + 0$$

$$\Rightarrow f'(x) = 12\left(x^2 - x - 6\right)$$

$$= 12\left\{x^2 - 3x + 2x - 6\right\}$$

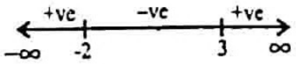
$$= 12\left\{x(x-3) + 2(x-3)\right\}$$

$$f'(x) = 12(x-3)(x+2)$$

(a) For f(x) to be strictly increasing, $f'(x) > 0$

$\Rightarrow$ $12(x-3)(x+2) > 0$, by (ii)

$\Rightarrow$ $(x-3)(x+2) > 0$



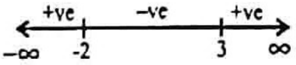
$\Rightarrow$ $x \in (-\infty, -2) \cup (3, \infty)$

$\therefore$ f(x) is strictly increasing in $(-\infty, -2) \cup (3, \infty)$ Ans.

(b) For f(x) to be strictly decreasing $f'(x) < 0$

$\Rightarrow$ $12(x-3)(x+2) < 0$

$\Rightarrow$ $(x-3)(x+2) < 0$



$\Rightarrow$ $x \in (-2, 3)$

$\therefore$ f(x) is strictly decreasing in $(-2, 3)$ Ans.

OR

An edge of a variable cube is increasing at the rate of 3cm/sec. How fast is the volume of the cube increasing when the edge is 10 cm long?

Ans. Let at any time 't', x be the edge of the cube and v be its volume.

A/q, $\frac{dx}{dt} = 3\text{cm/sec}$

$$\left[\frac{dv}{dt}\right]_{x=10cm} = ?$$

$\because$ $v = x^3$

$$\Rightarrow \frac{dv}{dt} = \frac{d}{dt}\left(x^3\right)$$

$$= 3x^2\frac{dx}{dt}$$

$$= 3x^2 \times 3$$

$$\frac{dv}{dt} = 9x^2$$

Put x = 10 cm

$$\therefore \left[\frac{dv}{dt}\right]_{x=10cm} = 9\times10^2 = 900\text{cm}^3/\text{sec} \text{ Ans.}$$

17. Find the value of $\int\frac{5x-2}{3x^2+2x+1}dx$ 4

Ans. Let $I = \int\frac{5x-2}{3x^2+2x+1}dx$ ...(i)

Let $5x - 2 = A.\frac{d}{dx}\left(3x^2 + 2x + 1\right) + B$

$\therefore$ $5x - 2 = A(6x+2) + B$ ...(ii)

Equating the coefficient of like terms

$5 = 6A \Rightarrow A = \frac{5}{6}$

and $-2 = 2A + B \Rightarrow B = -2 - 2A = -2 - 2\times\frac{5}{6}$

$\therefore$ $B = \frac{-6-5}{3}$; $B = \frac{-11}{3}$

Putting the value of A & B in equation (ii)

$5x - 2 = \frac{5}{6}(6x+2) - \frac{11}{3}$ ...(iii)

Using (iii) in (i), we get

$$I = \int\frac{\frac{5}{6}(6x+2)\frac{-11}{3}}{3x^2+2x+1}dx$$

$$= \frac{5}{6}\int\frac{(6x+2)}{3x^2+2x+1}dx \frac{-11}{3}\int\frac{1}{3x^2+2x+1}dx$$

Let $3x^2 + 2x + 1 = t$

$\Rightarrow (6x+2)dx = dt$

$$I = \frac{5}{6}\int\frac{dt}{t}\frac{-11}{9}\int\frac{1}{3\left(x^2+\frac{2}{3}x+\frac{1}{3}\right)}dx$$

$$= \frac{5}{6}\log|t| + c, \frac{-11}{9}\int\frac{dx}{x^2 + 2\cdot x\cdot\frac{1}{3} + \left(\frac{1}{3}\right)^2 - \left(\frac{1}{3}\right)^2 + \frac{1}{3}}$$

$$= \frac{5}{6}\log\left|3x^2+2x+1\right| + C, \frac{-11}{9}\int\frac{dx}{\left(x+\frac{1}{3}\right)^2 + \left(\frac{\sqrt{2}}{3}\right)^2}$$

$$= \frac{5}{6}\log\left|3x^2+2x+1\right|C_1\frac{-11}{9}\times\frac{1}{\sqrt{2}}\tan^{-1}\frac{\left(x+\frac{1}{3}\right)}{\frac{\sqrt{2}}{3}} + C_2$$

$$I = \frac{5}{6}\log\left|3x^2+2x+1\right|\frac{-11}{3\sqrt{2}}\tan^{-1}\left(\frac{3x+1}{\sqrt{2}}\right) + C \text{ Ans.}$$

18. Find the value of $\int\frac{2x-3}{\left(x^2-1\right)(2x+3)}dx$

Ans. Let $I = \int \frac{2x-3}{(x^2-1)(2x+3)}dx$

$$I = \int \frac{(2x-3)}{(x-1)(x+1)(2x+3)}dx \quad ...(i)$$

Let $\frac{2x-3}{(x-1)(x+1)(2x+3)} = \frac{A}{x-1} + \frac{B}{x+1} + \frac{C}{(2x+3)} \quad ...(ii)$

$\Rightarrow 2x-3 = A(x+1)(2x+3) + B(x-1)(2x+3) + C(x-1)(x+1) \quad ...(iii)$

Put $x = 1$ in (iii)

$$2\cdot1-3 = A(1+1)(2.1+3) + B.O + C.O$$

$$\Rightarrow -1 = A\times2\times5 \Rightarrow A = \frac{-1}{10}$$

Put $x = -1$ in (iii)

$$2\times-1-3 = A\cdot O + B(-1-1)(2\cdot(-1)+3) + C\cdot O$$

$$\Rightarrow -5 = B\times-2\times1 \Rightarrow B = \frac{5}{2}$$

Put $x = \frac{-3}{2}$ in (iii)

$$2\times\frac{-3}{2}-3 = A\cdot O + B\cdot O + C\cdot\left(\frac{-3}{2}-1\right)\left(\frac{-3}{2}+1\right)$$

$$\Rightarrow -6 = C\times\frac{-5}{2}\times\frac{-1}{2} \Rightarrow C = \frac{-6\times4}{5}$$

$$\Rightarrow C = -\frac{24}{5}$$

Putting the value of A,B and C in (ii), we get

$$\frac{2x-3}{(x-1)(x+1)(2x+3)} = \frac{-\frac{1}{10}}{x-1} + \frac{\frac{5}{2}}{x+1} + \frac{\left(\frac{-24}{5}\right)}{2x+3} \quad (iv)$$

Using (iv), in (i), we get

$$I = \int\left[\frac{\frac{-1}{10}}{(x-1)} + \frac{\frac{5}{2}}{x+1} + \frac{\left(\frac{-24}{5}\right)}{2x+3}\right]dx$$

$$= \frac{-1}{10}\int\frac{dx}{x-1} + \frac{5}{2}\int\frac{dx}{x+1} - \frac{24}{5}\int\frac{dx}{2x+3}$$

$$I = \frac{-1}{10}\log|x-1| + \frac{5}{2}\log|x+1| - \frac{24}{5}\frac{\log|2x+3|}{2} + C$$

$$I = \frac{-1}{10}\log|x-1| + \frac{5}{2}\log|x+1| + \left(\frac{-12}{5}\right)\log|2x+3| + C \text{ Ans.}$$

OR

Evaluate $\int_a^b x\,dx$ as a limit of a sum.

Ans. $I = \int_a^b x\,dx \quad ...(i)$

Here $a = a$, $b = b$, $nh = b-a$, $f(x) = x$ ...(ii)

Integrating (i) as the limit of sum

$$I = \lim_{h\to0} h\left[f(a) + f(a+h) + f(a+2h) + ... + f\{a+(n-1)h\}\right]$$

$$= \lim_{h\to0} h\left[a + (a+h) + (a+2h) + ... + \{a+(n-1)h\}\right] \text{ by (ii)}$$

$$= \lim_{h\to0} h\left[(a+a+a+...+a) + \{h+2h+...+(n-1)\}h\right]$$

$$= \lim_{h\to0} h\left[na + h(1+2+...+(n-1))\right]$$

$$= \lim_{h\to0}\left[anh + h^2.\frac{n(n-1)}{2}\right]$$

$$= \lim_{h\to0}\left[a.nh + \frac{nh(nh-h)}{2}\right]$$

$$= \lim_{h\to0}\left[a\cdot(b-a) + \frac{(b-a)(b-a-h)}{2}\right]$$

$$= a(b-a) + \frac{(b-a)(b-a-0)}{2}$$

$$= \frac{2ab-2a^2+(b-a)^2}{2} = \frac{2ab^2-2a^2+b^2+a^2-2ah}{2}$$

$$= \frac{b^2-a^2}{2} \text{ Ans.}$$

19. Evaluate $\int_0^{\pi/2}\frac{\sin x-\cos x}{1+\sin x.\cos x}dx$

Ans. Let $I = \int_0^{\frac{\pi}{2}}\frac{\sin x-\cos x}{1+\sin x.\cos x}dx \quad ...(i)$

$$= \int_0^{\frac{\pi}{2}}\frac{\sin\left(\frac{\pi}{2}-x\right)-\cos\left(\frac{\pi}{2}-x\right)}{1+\sin\left(\frac{\pi}{2}-x\right)\cos\left(\frac{\pi}{2}-x\right)}dx$$

$$\left[\because \int_0^a f(x)dx = \int_0^a f(a-x)dx\right]$$

$$I = \int_0^{\frac{\pi}{2}}\frac{\cos x-\sin x}{1+\cos x.\sin x}dx \quad ...(ii)$$

Adding (i) and (ii), we get

$$I+I = \int_0^{\frac{\pi}{2}}\frac{\sin x-\cos x}{1+\text{cox}\cos x}dx + \int_0^{\frac{\pi}{2}}\frac{\cos-\sin x}{1+\cos x.\sin x}dx$$

$$\Rightarrow 2I = \int_0^{\frac{\pi}{2}}\left[\frac{\sin x-\cos x}{1+\sin x\cos x} + \frac{\cos x-\sin x}{1+\cos x-\sin x}\right]dx$$

$$= \int_0^{\frac{\pi}{2}}\left(\frac{\sin x-\cos x+\cos x-\sin x}{1+\sin x\cos x}\right)dx$$

$$= \int_0^{\frac{\pi}{2}}\frac{0}{1+\sin x\cos x}.dx$$

$$2I = 0$$

$$\therefore I = \frac{0}{2} = 0 \text{ Ans.}$$

20. **If $\bar{a}=\hat{i}+\hat{j}+\hat{k}, \bar{b}=2\hat{i}+5\hat{j}, \bar{c}=\hat{i}-6\hat{j}-\hat{k}$, then find** **4**

(i) $2\bar{a}-\bar{b}$ (ii) $\bar{a}\cdot\bar{c}$

(iii) $\bar{b}\times\bar{c}$ (iv) b^2

Ans. We have, $\bar{a}=\hat{i}+\hat{j}+\hat{k}$ ...(i)

$\bar{b}=2\hat{i}+5\hat{j}$ ...(ii)

$\bar{c}=\hat{i}-6\hat{j}-\hat{k}$ ...(iii)

(i) $2\bar{a}-\bar{b}=2(\hat{i}+\hat{j}+\hat{k})-(2\hat{i}+5\hat{j})$

$=2\hat{i}+2\hat{j}+2\hat{k}=-2\hat{i}-5\hat{j}$

$\therefore \quad 2\bar{a}-\bar{b}=-3\hat{j}+2\hat{k}$

(ii) $\bar{a}.\bar{c}=(\hat{i}+\hat{j}+\hat{k})\cdot(\hat{i}-6\hat{j}-\hat{k})$

$=1\times1+1\times(-6)+1\times(-1)$

$=1-6-1$

$\bar{a}.\bar{c}=-6$ Ans.

(iii) $\bar{b}\times\bar{c}=\begin{vmatrix}\hat{i} & \hat{j} & \hat{k}\\ 2 & 5 & 0\\ 1 & -6 & -1\end{vmatrix}$

$=(-5-0)\hat{i}-(-2-0)\hat{j}+(-12-5)\hat{k}$

$\bar{b}\times\bar{c}=-5\hat{j}+2\hat{j}-17\hat{k}$ Ans

(iv) $b^2=(|b|)^2$

$=\left(\sqrt{2^2+5^2+0^2}\right)^2$, by (ii)

$=\left(\sqrt{4+25}\right)^2 \therefore b^2=29$ Ans

21. **Find the angle between the following pair of lines**

$\frac{x}{2}=\frac{y}{2}=\frac{z}{1}$ **and** $\frac{x-5}{4}=\frac{y-2}{1}=\frac{z-3}{8}$.

Ans. Given line are $\frac{x}{2}=\frac{y}{2}=\frac{z}{1}$ [illegible]

and $\frac{x-5}{4}=\frac{y-2}{1}=\frac{z-3}{8}$ [illegible]

Here $a_1=2, b_1=2,$ [illegible]

$a_2=4, b_2=1,$ [illegible]

Let θ is the angle between [illegible]

$\therefore \cos\theta=\frac{a_1a_2+b_1b_2+c_1c_2}{\sqrt{a_1^2+b_1^2+c_1^2}\sqrt{a_2^2+b_2^2+c_2^2}}$

$=\frac{2\cdot4+2\cdot1+1\cdot8}{\sqrt{2^2+2^2+1^2}\sqrt{4^2+1^2+8^2}}$

$=\frac{18}{\sqrt{9}\times\sqrt{81}}=\frac{18}{3\times9}$

$\cos=\frac{2}{3}$

$\therefore \quad \theta=\cos^{-1}\left(\frac{2}{3}\right)$ Ans

OR

Find the equation of the plane that contains the point $(1,-1,2)$ and is perpendicular to each of the planes $2x+3y-2z=5$ and $x+2y-3z=8$.

Ans. The required equation of plane passing through $(1,-1,2)$ is $a(x-x_1)+b(y-y_1)+c(z-z_1)=0$

$\Rightarrow a(x-1)+b(y+1)+c(z-2)=0$ ...(i)

∵ it is perpendicular to each of the plane $2x+3y-2z=5$ and $x+2y-3z=8$

$\therefore \quad 2a+3b-2c=0$ ...(ii)

and $a+2b-3c=0$ ...(iii)

solving equation (ii) and equation (iii) by cross-multiplication

$\frac{a}{3\times(-3)-2\times(-2)}=\frac{b}{1\times(-2)-2\times(-3)}=\frac{c}{2\times2-1\times3}$

$\Rightarrow \frac{a}{-5}=\frac{b}{4}=\frac{c}{1}=k$ (let)

[illegible]

Putting the value of a, b and c in (i) we get

$-5k(x-1)+4k(y+1)+k(z-2)=0$

[illegible]

[illegible]

[illegible] Ans

22. **Let A and B be independent events with $P(A)=0.3$ and $P(B)=0.4$, then find**

(i) $P(A\cap B)$ (ii) $P(A\cup B)$

(iii) $P\left(\frac{A}{B}\right)$ (iv) $P\left(\frac{B}{A}\right)$ **4**

Ans. We have [illegible]

A and B are independent events

[illegible]

[illegible]

[illegible] Ans

[illegible]

[illegible]

[illegible]

[illegible] Ans

[illegible] Ans

[illegible]

$=\frac{0.12}{0.3}=\frac{2}{5}\ P\left(\frac{B}{A}\right)=\frac{2}{5}$ Ans

OR

Mother, father and son line up at random for a family picture. Find $P\left(\frac{E}{F}\right)$ where

$E \rightarrow$ Son on one end

$E \rightarrow$ Father in middle

Ans. Here $S = \{MFS, MSF, FSM, FMS, SFM, SMF\}$

$n(s) = 6$

$E \rightarrow$ son on one end $= \{MFS, FMS, SFM, SMF\}$

$n(E) = 4$

$F \rightarrow$ Father in middle $= \{MFS, SFM\}$

$n(F) = 2$

$E \cap F = \{MFS, SFM\}$

$n(f) = 2$

$$\therefore P\left(\frac{E}{F}\right) = \frac{n(E \cap F)}{n(F)}$$

$$= \frac{2}{2} = 1 \text{ Ans.}$$

Section - C

23. Solve the system of linear equations using matrix method:

$x - y + 2z = 1$

$2y - 3z = 1$

$3x - 2y + 4z = 2$

Ans. We have,

$$x - y + 2z = 1$$
$$2y - 3z = 1$$
$$3x - 2y + 4z = 2$$

$$\Rightarrow \begin{bmatrix} 1 & -1 & 2 \\ 0 & 2 & -3 \\ 3 & -2 & 4 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \\ 2 \end{bmatrix}$$

$\Rightarrow Ax = B$ (let) ...(i)

$$\text{Now } |A| = \begin{vmatrix} 1 & -1 & 2 \\ 0 & 2 & -3 \\ 3 & -2 & 4 \end{vmatrix}$$

$$= 1(8-6) + 1(0+9) + 2(0-6)$$

$$= 2 + 9 - 12$$

$\Rightarrow |A| = -1 \neq 0$ ...(ii)

$\therefore$ A^{-1} exists and solution is given by

$X = A^{-1}B$ ...(iii)

$$\text{Now, } C_{11} = M_{11} = \begin{vmatrix} 2 & -3 \\ -2 & 4 \end{vmatrix} = 8 - 6 = 2$$

$$C_{12} = M_{12} = -\begin{vmatrix} 0 & -3 \\ 3 & 4 \end{vmatrix} = -(0+9) = -9$$

$$C_{13} = M_{13} \begin{vmatrix} 0 & 2 \\ 3 & -2 \end{vmatrix} = 0 - 6 = -6$$

$$C_{21} = M_{21} = \begin{vmatrix} -1 & 2 \\ -2 & 4 \end{vmatrix} = -(-4+4) = 0$$

$$C_{22} = M_{22} = \begin{vmatrix} 1 & 2 \\ 3 & 4 \end{vmatrix} = 4 - 6 = -2$$

$$C_{23} = M_{23} = \begin{vmatrix} 1 & -1 \\ 3 & -2 \end{vmatrix} = -(-2+3) = -1$$

$$C_{31} = M_{31} = \begin{vmatrix} -1 & 2 \\ 2 & -3 \end{vmatrix} = (3-4) = -1$$

$$C_{32} = M_{32} = -\begin{vmatrix} 1 & 2 \\ 0 & -3 \end{vmatrix} = -(-3-0) = 3$$

$$C_{33} = M_{33} = \begin{vmatrix} 1 & -1 \\ 0 & 2 \end{vmatrix} = 2 - 0 = 2$$

$$A^{-1} = \frac{1}{|A|} \text{Adj}A = \frac{1}{|A|} \begin{bmatrix} c_{11} & c_{21} & c_{31} \\ c_{12} & c_{22} & c_{32} \\ c_{13} & c_{23} & c_{33} \end{bmatrix}$$

$$= \frac{1}{-1} \begin{bmatrix} 2 & 0 & -1 \\ -9 & -2 & 3 \\ -6 & -1 & 2 \end{bmatrix}$$

$$A^{-1} = \begin{bmatrix} -2 & 0 & 1 \\ 9 & 2 & -3 \\ 6 & 1 & -2 \end{bmatrix} \quad \text{...(iv)}$$

Using (i) and (iv) in (iii), we get

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} -2 & 0 & 1 \\ 9 & 2 & -3 \\ 6 & 1 & -2 \end{bmatrix} \begin{bmatrix} 1 \\ 1 \\ 2 \end{bmatrix}$$

$$= \begin{bmatrix} -2 \times 1 + 0 \times 1 + 1 \times 2 \\ 9 \times 1 + 2 \times 1 + (-3) \times 2 \\ 6 \times 1 + 1 \times 1 + (-2) \times 2 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 0 \\ 5 \\ 3 \end{bmatrix}$$

$\therefore x = 0, y = 5, z = 3$ Ans.

OR

Obtain the inverse of the matrix using elementary operations:

$$A = \begin{bmatrix} -1 & 1 & 2 \\ 1 & 2 & 3 \\ 3 & 1 & 1 \end{bmatrix}$$

Ans. We have, $A = \begin{bmatrix} -1 & 1 & 2 \\ 1 & 2 & 3 \\ 3 & 1 & 1 \end{bmatrix}$

We know that $A = I\,A$

$$\Rightarrow \begin{bmatrix}-1 & 1 & 2\\ 1 & 2 & 3\\ 3 & 1 & 1\end{bmatrix} = \begin{bmatrix}1 & 0 & 0\\ 0 & 1 & 0\\ 0 & 0 & 1\end{bmatrix} A$$

Applying $R_1 \to (-1).R_1$

$$\begin{bmatrix}1 & -1 & -2\\ 1 & 2 & 3\\ 3 & 1 & 1\end{bmatrix} = \begin{bmatrix}-1 & 0 & 0\\ 1 & 1 & 0\\ 3 & 0 & 1\end{bmatrix} A$$

Applying $R_2 \to R_2 - R_1, R_3 \to R_3 - 3R_1$

$$\begin{bmatrix}1 & -1 & -2\\ 0 & 3 & 5\\ 0 & 4 & 7\end{bmatrix} = \begin{bmatrix}-1 & 0 & 0\\ 1 & 1 & 0\\ 3 & 0 & 1\end{bmatrix}.A$$

Applying $R_2 - \frac{1}{3}R_2$

$$\begin{bmatrix}1 & -1 & -2\\ 0 & 1 & \frac{5}{3}\\ 0 & 4 & 7\end{bmatrix} = \begin{bmatrix}-1 & 0 & 0\\ \frac{1}{3} & \frac{1}{3} & 0\\ 3 & 0 & 1\end{bmatrix}.A$$

Applying $R_1 \to R_1 + R_2 ; R_3 \to R_3 - 4R_2$

$$\begin{bmatrix}1 & 0 & -2+\frac{5}{3}\\ 0 & 1 & \frac{5}{3}\\ 0 & 0 & 7-4\times\frac{5}{3}\end{bmatrix} = \begin{bmatrix}-1+\frac{1}{3} & \frac{1}{3} & 0\\ \frac{1}{3} & \frac{1}{3} & 0\\ 3-4\times\frac{1}{3} & 0-4\times\frac{1}{3} & 1\end{bmatrix}.A$$

$$\begin{bmatrix}1 & 0 & -\frac{1}{3}\\ 0 & 1 & \frac{5}{3}\\ 0 & 0 & \frac{1}{3}\end{bmatrix} = \begin{bmatrix}\frac{-2}{3} & \frac{1}{3} & 0\\ \frac{1}{3} & \frac{1}{3} & 0\\ \frac{5}{3} & \frac{-4}{3} & 1\end{bmatrix}.A$$

Applying $R_3 \to 3R_1$

$$\begin{bmatrix}1 & 0 & \frac{-1}{3}\\ 0 & 1 & \frac{5}{3}\\ 0 & 0 & 1\end{bmatrix} = \begin{bmatrix}\frac{-2}{3} & \frac{1}{3} & 0\\ \frac{1}{3} & \frac{1}{3} & 0\\ 5 & -4 & 3\end{bmatrix}.A$$

Applying $R_1 \to R_1 + \frac{1}{3}R_3; R_2 \to R_2 - \frac{5}{3}R_3$

$$\begin{bmatrix}1 & 0 & 0\\ 0 & 1 & 0\\ 0 & 0 & 1\end{bmatrix} = \begin{bmatrix}\frac{-2}{3} + \frac{1}{3}5 & \frac{1}{3} + \frac{1}{3}(-4) & 0 + \frac{1}{3}3\\ \frac{1}{3} - \frac{5}{3}5 & \frac{1}{3} - \frac{5}{3}(-4) & 0 - \frac{5}{3}3\\ 5 & -4 & 3\end{bmatrix} A$$

$$\begin{bmatrix}1 & 0 & 0\\ 0 & 1 & 0\\ 0 & 0 & 1\end{bmatrix} = \begin{bmatrix}1 & -1 & 1\\ -8 & 7 & -5\\ 5 & -4 & 3\end{bmatrix} A$$

$I = A^{-1}.A$

Hence $A^{-1} = \begin{bmatrix}1 & -1 & 1\\ -8 & 7 & -5\\ 5 & -4 & 3\end{bmatrix}$ Ans.

24. **Find the local maximum and local minimum values of the function f given by $f(x) = 3x^4 + 4x^3 - 12x^2 + 12$** 6

Ans. $f(x) = 3x^4 + 4x^3 - 12x^2 + 12$ ...(i)

$\Rightarrow f'(x) = 12x^3 + 12x^2 - 24x + 0$ ...(ii)

$\Rightarrow f''(x) = 36x^2 + 24x - 24$ ...(iii)

For the local maxima or local minima $f'(x) = 0$

$\Rightarrow 12x^3 + 12x^2 - 24x = 0$

$\Rightarrow 12x(x^2 + x - 2) = 0$

$\Rightarrow x(x^2 + 2x - x - 2) = \frac{0}{12}$

$\Rightarrow x\{x(x+2) - (x+2)\} = 0$

$\Rightarrow x(x+2)(x-1) = 0$

$\Rightarrow x = 0, -2, 1$

At $x = 0$

(iii) $\Rightarrow f''(0) = 36 \cdot 0^2 + 24 \cdot 0 - 24$

$\Rightarrow f''(0) = -24 < 0$

This $\Rightarrow x = 0$ is the point of local maxima

$\therefore$ Local maximum value $= f(0)$

$= 3\times0^4 + 4\times0^3 - 12\times0^2 + 12$, by (i)

$= 12$ Ans.

At $x = -2$

(iii) $\Rightarrow f''(-2) = 36\times(-2)^2 + 24\times(-2) - 24$

$= 144 - 48 - 24 = 144 - 72$

$= 72 > 0$

This $\Rightarrow x = -2$ is the point of local minima.

Local minimum value $= f(-2)$

$= 3\times(-2)^4 + 4\times(-2)^3 - 12\times(-2)^2 + 12$, by (i)

$= 48 - 32 - 48 + 12 = -20$ Ans.

At $x = 1$

$(iii) \Rightarrow f''(1) = 36\times1^2 + 24\times1 - 24$

$= 36 > 0$

This $\Rightarrow x = 1$ is the point of local minima

$\therefore$ Local minimum value $= f(1)$

$= 3\times1^4 + 4\times1^3 - 12\times1^2 + 12$

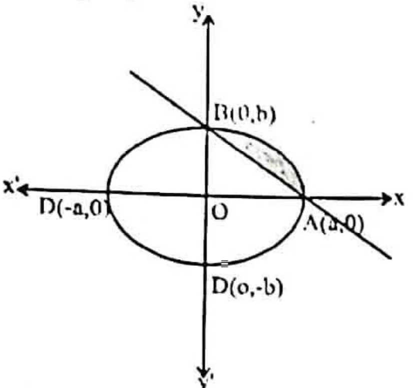
$= 7$ Ans.

25. Find the area of the smaller region bounded by the ellipse $\frac{x^2}{d^2}+\frac{y^2}{b^2}=1$ and the line $\frac{x}{a}+\frac{y}{b}=1$. 6

Ans. Given ellipe is

$$\frac{x^2}{a^2}+\frac{y^2}{b^2}=1 \qquad ...(i)$$

Given line is $\frac{x}{a}+\frac{y}{b}=1 \quad ...(ii)$



Required Area

= Area of shaded region ABEB

= Area of region BOAEBO – Area of region BOAB

$$= \int_0^a y_{\text{ellipse}}dx - \int_0^a y_{\text{line}}dx$$

$$= \int_0^a \frac{b}{a}\sqrt{a^2-x^2}dx - \int_0^a -\frac{b}{a}(a-x)dx$$

$$= \frac{b}{a}\left[\frac{x}{2}\sqrt{a^2-x^2}+\frac{a^2}{2}\sin^{-1}\frac{x}{a}\right]_0^a - \frac{b}{a}\left[ax-\frac{x^2}{2}\right]_0^a$$

From (i), $\frac{y^2}{b^2} = 1-\frac{x^2}{a^2} = \frac{a^2-x^2}{a^2}$

$y = \frac{b}{a}\sqrt{a^2-x^2}$

From (ii), $\frac{y}{b} = 1\frac{1-x}{a} = \frac{a-x}{a}$

$y = \frac{b}{a}(a-x)$

$$= \frac{b}{a}\left[\left(\frac{a}{2}\sqrt{a^2-a^2}+\frac{a^2}{2}\sin^{-1}\frac{a^1}{a}\right)-\left(\frac{0}{2}\sqrt{a^2-0^2}+\frac{a^2}{2}\sin^{-1}\frac{0}{a}\right)\right]$$

$$-\frac{b}{a}\left[\left(a\cdot a-\frac{a^2}{a}\right)-\left(a\cdot0-\frac{0^2}{2}\right)\right]$$

$$= \frac{b}{a}\left[0+\frac{a^2}{2}\times\frac{\pi}{2}-0-0\right]-\frac{b}{a}\left[\frac{a^2}{2}-0\right]$$

$$= \frac{b}{a}\times\frac{\pi a^2}{4} - \frac{b}{a}\times\frac{a^2}{2}$$

$\therefore$ Requried Area $= \left(\frac{\pi ab}{4}-\frac{ab}{2}\right)$ sq. unit. **Ans.**

OR

Prove that $\int_0^{\pi/2}\log(\sin x)dx = -\frac{\pi}{2}\log 2$.

Ans. Let $I = \int_0^{\frac{\pi}{2}} \log\sin x\,dx \qquad ...(i)$

$$\Rightarrow I = \int_0^{\frac{\pi}{2}} \log\sin\left(\frac{\pi}{2}-x\right)dx$$

$$I = \int_0^{\frac{\pi}{2}} \log\cos x\,dx \qquad ...(ii)$$

Adding equation (i) and equation (ii), we get

$$I+I = \int_0^{\frac{\pi}{2}} (\log\sin x + \log\cos x)d$$

$$\Rightarrow 2I = \int_0^{\frac{\pi}{2}} \log\sin x\cos x\,dx$$

$$\Rightarrow 2I = \int_0^{\frac{\pi}{2}} \log\left(\frac{2\sin x\cos x}{2}\right)dx = \int_0^{\frac{\pi}{2}}(\log\sin 2x - \log x)dx$$

$$\Rightarrow 2I = \int_0^{\frac{\pi}{2}} \log\sin 2x\,dx - \log 2\int_0^{\frac{\pi}{2}} dx$$

Let $2x = t$

$\Rightarrow 2dx = dt \Rightarrow dx = \frac{dt}{2}$

When $x = 0,\ t = 2.0 = 0$

When $x = \frac{\pi}{2},\ t = 2.\frac{\pi}{2} = \pi$

$$2I = \int_0^{\pi} \log\sin t.\frac{dt}{2} - \log 2[dx]_0^{\frac{\pi}{2}}$$

$$\Rightarrow 2I = \frac{1}{2}\int_0^{\pi} \log\sin x\,dx - \log 2\left(\frac{\pi}{2}-0\right) \quad ...(iii)$$

Here $2a = \pi \Rightarrow a = \frac{\pi}{2}$

$f(x) = \log\sin x$

$\therefore f(2a-x) = f(\pi-x) = \log\sin(\pi-x) = \log\sin x$

$\therefore f(2a-x) = f(x)$

This $\Rightarrow \int_0^{2a} f(x)dx = 2\int_0^{a} f(x)dx$

$$\Rightarrow \int_0^{\pi} \log\sin x dx = 2\int_0^{\frac{\pi}{2}} \log\sin x dx \quad ...(iv)$$

Using (iv) in (iii), we get

$$2I = \frac{1}{2}\times 2\int_0^{\frac{\pi}{2}} \log\sin x dx - \frac{\pi}{2}\log 2$$

$$2I = I - \frac{\pi}{2}\log 2, \text{ by (i)}$$

$$\Rightarrow 2I - I = -\frac{\pi}{2}\log 2$$

$$\Rightarrow I = -\frac{\pi}{2}\log 2$$

$$\therefore \int_0^{\frac{\pi}{2}} \log\sin x dx = -\frac{\pi}{2}\log 2 \text{ proved}$$

26. Solve the differential equation

$2xy + y^2 - 2x^2\frac{dy}{dx} = 0, y = 2 \text{ at } x = 1$ 6

Ans. $2xy + y^2 - 2x^2\frac{dy}{dx} = 0,\ \ y = 2 \text{ at } x = 1$

$$\Rightarrow 2x^2\frac{dy}{dx} = 2xy + y^2$$

$$\Rightarrow \frac{dy}{dx} = \frac{2xy + y^2}{2x^2} \quad ...(i)$$

Which is a homogeneous differential equation.

$\therefore$ Put $y = vx$ ...(ii)

$$\Rightarrow \frac{dy}{dx} = v.1 + n\frac{dv}{dx} = v + x\frac{dv}{dx} \ ...(iii)$$

Using (ii) and (iii) in (i), we get

$$v + x\frac{dv}{dx} = \frac{2x.vx + v^2x^2}{2x^2}$$

$$\Rightarrow x\frac{dv}{dx} = \frac{x^2(2v + v^2)}{2x^2} - v$$

$$\Rightarrow x\frac{dv}{dx} = \frac{2v + v^2 - 2v}{2}$$

$$\Rightarrow 2\int\frac{dv}{dx} = \int\frac{dx}{x}$$

$$\Rightarrow 2\frac{v^{-2+1}}{-2+1} = \log x + C$$

$$\Rightarrow \frac{-2}{v} = \log x + C$$

$$\Rightarrow \frac{-2}{y/x} = \log x + C \text{, by (ii)}$$

$$\Rightarrow -\frac{2x}{y} = \log x + C \quad ...(iv)$$

Put $y = 2$ and $x = 1$ in equation (iv)

$$\frac{-2\times 1}{2} = \log 1 + C$$

$$\Rightarrow -1 = 0 + C \Rightarrow C = -1$$

Put $c = -1$ in equation (iv)

$$\frac{-2x}{y} = \log - 1 \Rightarrow -2x = y\log x - y$$

$$\Rightarrow y - 2x - y\log x = 0$$

Which is the required solution.

OR

Solve the differential equation $x.\log x\frac{dy}{dx} + y = \frac{2}{x}\log x$

Ans. $x\log\frac{dy}{dx} + y = \frac{2}{x}\log x$

Dividing both sides by x . log x

$$\Rightarrow \frac{dy}{dx} + \frac{1}{x\log x}.y = \frac{1}{x\log x}.\frac{2}{x}\log x$$

$$\Rightarrow \frac{dy}{dx} + \frac{1}{x.\log x}.y = \frac{2}{x^2} \quad ...(i)$$

Which is a linear differential equation in y.

Here $P = \frac{1}{x\log x}, Q = \frac{2}{x^2}$

$\therefore$ I.F.$= e^{\int Pdx} = e^{\int\frac{1}{x\log x}dx}$ Let $\log = t, \ \frac{1}{x}dx = dt$

$$= e^{\int\frac{dt}{t}}$$

$$= e^{\log t} = t$$

$\Rightarrow$ I. F $= \log x$

$\therefore$ Solution of Differential equation (i) is

$$y\times \text{I.F.} = \int Q\times \text{I.F.}dx$$

$$\Rightarrow y\times\log x = \int\frac{2}{x^2}\times\log x dx$$

$$\Rightarrow y.\log x = 2\left[\log x\int\frac{1}{x^2}dx - \int\left\{\frac{d}{dx}(\log)\right\}\left\{\int\frac{1}{x^2}dx\right\}dx\right]$$

$$= 2\left[\log x\times\frac{x^{-2+1}}{-2+1} - \int\frac{1}{x}\times\frac{x^{-2+1}}{-2+1}dx\right]$$

$$= 2\left[\log x\times\frac{-1}{x} - \int\frac{1}{x}\times\frac{-1}{x}dx\right]$$

$$= -2\frac{\log x}{x} + 2\frac{x^{-2+1}}{-2+1} + C$$

$$\Rightarrow y\log x = -2\frac{\log x}{x} - \frac{2}{x} + C$$

Which is the required solution.

27. Find the shortest distance between the following pairs of parallel lines whose equations are $\vec{r} = (\hat{i} + \hat{j}) + \lambda(2\hat{i} - \hat{j} + \hat{k})$ and $\vec{r} = (2\hat{i} + \hat{j} - \hat{k}) + \mu(4\hat{i} - 2\hat{j} + 2\hat{k})$

Ans. Gvien pairs of parallel line are

$$\vec{r} = (\hat{i} + \hat{j}) + \lambda(2\hat{i} - \hat{j} + \hat{k}) \quad \text{...(i)}$$

$$\vec{r} = (2\hat{i} + \hat{j} - \hat{k}) + \lambda(4\hat{i} - 2\hat{j} + 2\hat{k})$$

$$\Rightarrow \vec{r} = (2\hat{i} + \hat{j} - \hat{k}) + 2\lambda(2\hat{i} - \hat{j} + \hat{k}) \quad \text{...(ii)}$$

Here, $\vec{a}_1 = \hat{i} + \hat{j}$, $\vec{b} = 2\hat{i} - \hat{j} + \hat{k}$

$\vec{a}_2 = 2\hat{i} + \hat{j} - \hat{k}$

Now $\vec{a}_2 - \vec{a}_1 = \hat{i} - \hat{k}$

$$\therefore \vec{b} \times (\vec{a}_2 - \vec{a}_1) = (2\hat{i} - j + \hat{k}) \times (\hat{i} - \hat{k})$$

$$= \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & -1 & 1 \\ 1 & 0 & -1 \end{vmatrix}$$

$$= (1-0)\hat{i} - (-2-1)\hat{j} + (0+1)\hat{k}$$

$$\vec{b} \times (\vec{a}_2 - \vec{a}_1) = \hat{i} + 3\hat{j} + \hat{k}$$

Also $|\vec{b}| = \sqrt{2^2 + (-1)^2 + 1^2} = \sqrt{6}$

$\therefore$ Required shortest Distance (S.D.)

$$= \left|\frac{\vec{b} \times (\vec{a}_2 - \vec{a}_1)}{|\vec{b}|}\right|$$

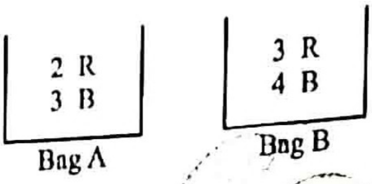
$$= \left|\frac{\hat{i} + 3\hat{j} + \hat{k}}{\sqrt{6}}\right|$$

$$= \frac{|\hat{i} + 3\hat{j} + \hat{k}|}{\sqrt{6}} = \frac{\sqrt{1^2 + 3^2 + 1^2}}{\sqrt{6}}$$

$$= \frac{\sqrt{11}}{\sqrt{6}} \times \frac{\sqrt{6}}{\sqrt{6}} = \frac{\sqrt{66}}{6} \text{ unit}$$ Ans.

28. Bag A contains 2 red and 3 black balls while another Bag B contains 3 red and 4 black balls. One ball is drawn at random from one of the bags and it is found to be red. Find the probability that it was drawn from Bag B. **6**

Ans.



Let E_1 = event of getting Bag A

E_2 = event of getting Bag B

$$\therefore P(E_1) = \frac{1}{2}, P(E_2) = \frac{1}{2}$$

Let E = event of gettingared ball

$$\therefore P\left(\frac{E}{E_1}\right) = P(\text{getting a red ball from bag A}) = \frac{2}{5}$$

$$P\left(\frac{E}{E_2}\right) = P(\text{getting a red ball from bag B}) = \frac{3}{7}$$

$$\text{Required probability} = P\left(\frac{E_2}{E}\right)$$

$$= \frac{P(E_2).P\left(\frac{E}{E_2}\right)}{P(E_1).P\left(\frac{E}{E_1}\right) + P(E_2).P\left(\frac{E}{E_2}\right)}$$ {From baye's theorem}

$$= \frac{\frac{1}{2} \times \frac{3}{7}}{\frac{1}{2} \times \frac{2}{5} + \frac{1}{2} \times \frac{3}{7}}$$

$$= \frac{\frac{1}{2} \times \frac{3}{7}}{\frac{1}{2}\left(\frac{2}{5} + \frac{3}{7}\right)} = \frac{\frac{3}{7}}{\frac{14+15}{35}}$$

$$= \frac{3}{7} \times \frac{35}{29} = \frac{15}{29}$$ Ans.

OR

Two cards are drawn successively with replacement from a well-shuffled deck of 52 cards. Find the probability distribution of the number of aces.

Ans. Let x = number of aces when two cards are drawn successively with replacement from a deck of 52 cards

$\therefore x = 0, 1, 2$

Let E = event of getting an ace card.

$P(x=0)=P(\bar{E}\,\bar{E})$

$=P(\bar{E}).P(\bar{E})$

$=\frac{48}{52}\times\frac{48}{52}$

$\therefore P(x=0)=\frac{144}{169}$

$P(x=1)=P(E,\bar{E}\text{ or }\bar{E}\,E)$

$=P(E).P(\bar{E})+P(\bar{E}).P(E)$

$=\frac{4}{52}\times\frac{48}{52}+\frac{48}{52}\times\frac{4}{52}$

$=\frac{12}{169}+\frac{12}{169}$

$=\frac{24}{169}$

$P(x=2)=P(E\,E)$

$=P(E).P(E)$

$=\frac{4}{52}\times\frac{4}{52}$

$=\frac{1}{169}$

∴ Required probability Distribution is

x	0	1	2
p(x)	$\frac{144}{169}$	$\frac{24}{169}$	$\frac{1}{169}$

Ans.

29. **Solve the following LPP graphically:**

Maximize $Z=3x+2y$

Subject to constrainst $x+y\le 4$

$x-y\le 2$

$x,y\ge 0$

Ans. Maximize $Z=3x+2y$

Subject to constrainst

$x+y\le 4$

$x-y\le 2$

$x,y\ge 0$

Table for $x+y=4$

x	0	4
y	4	0

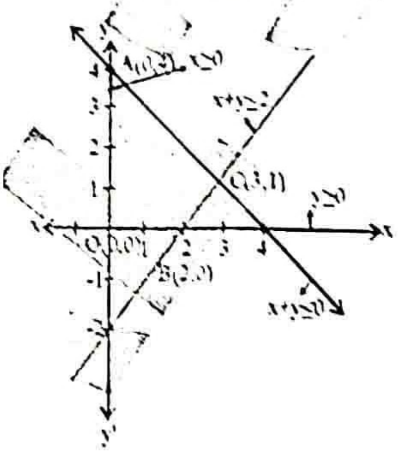
Table for $x-y=2$

x	0	2
y	-2	0

$x=0$ is Y-axis

$y=0$ is X-axis

Scale : Let 10mm = 1unit



From graph, the shaded region AOBCA is the solution set.

Clerly, co-ordinate of point A, O, B and C are A (0, 4), O (0, 0), B (2, 0) and C (3, 1)

At the point A (0, 4) $Z=3\times0+2\times4=8$

At the point O (0, 0) $Z=3\times0+2\times0=0$

At the point B (2, 0) $Z=3\times2+2\times0=6$

At the point C (3, 1) $Z=3\times3+2\times1=11$

∵ Z is maximum at the point C(3, 1)

∴ $x=3$ and $y=1$ is the required solution.

Also, maximize z = 11 Ans.

* * *