HALF YEARLY EXAMINATION

SEPTEMBER 2019

SET A

CLASS XI

Marking Scheme – MATHEMATICS [THEORY]

QNO	Answers	Marks (with split up)
1.	(b) 1	1 mk
2.	(a) $\sqrt{2}$	1 mk
3.	(c) 25	1 mk
4.	(b) $\{x: x \in R, x^2 + 1 = 0\}$	1 mk
5.	(c) $\{2, 4, 6\}$	1 mk
6.	(b) d	1 mk
7.	(b) $(0, \infty)$	1 mk
8.	(a) 2870	1 mk
9.	(c) $(A \cup B) - (A \cap B)$	1 mk
10.	(a) -1	1 mk
11.	$-\frac{1}{\sqrt{3}}$	1 mk
12.	$-\frac{19\pi}{72}\ radians$	1 mk
13.	5	1 mk
14.	$(-\infty, 1]$	1 mk
15.	$9^{\frac{1}{3}+\frac{1}{9}+\frac{1}{27}+\cdots} = 9^{\frac{\frac{1}{3}}{1-\frac{1}{3}}} = 9^{\frac{1}{2}} = 3$	1 mk
16.	$5x^2 + 2 = 22 \Longrightarrow 5x^2 = 20 \Longrightarrow x = \pm 2$	1 mk
17.	$\{1, 2\}$	1 mk
18.	New $\sigma^2 = (3)^2\sigma^2 = 9 \times 8 = 72.$	1 mk
19.	$\left\{x: x = \frac{n}{2n+1}, n \in N, 1 < n \le 6\right\}$	1 mk
20.	$\frac{75 + 70 + x}{3} \ge 60 \Longrightarrow x \ge 35$	1 mk
21.	$R = \{(2,4), (2,6), (2,18), (6,18), (9,18), (9,27)\}$	2 mks
22.	$\theta = 105° \times \frac{\pi}{180} = \frac{7\pi}{12} radians$ $\therefore \frac{7\pi}{12} = \frac{l}{30} \Longrightarrow l = \frac{7}{12} \times \frac{22}{7} \times 30 = 55\ m.$	1 mk ½ + ½
23.	$\bar{x} = \frac{n+1}{2}$; $\sigma^2 = \frac{n^2-1}{12}$ (OR) $\bar{x} = 16.5$; $\sigma^2 = 74.25$	1 + 1

24.	Finding P ={1, 2} and Q = {0, 1, 2} Obtaining $P \cup Q = \{0,1,2\}$ and $P \cap Q = \{1,2\}$ $\therefore (P \cup Q) \times (P \cap Q) = \{(0,1),(0,2),(1,1),(1,2),(2,1),(2,2)\}$	½ mk ½ mk 1 mk
25.	$D_f = R - \{-2\}$; $R_f = \{-1, 1\}$ (OR) $D_f = [1, \infty)$; $R_f = [0, \infty)$	1 + 1
26.	For correct values of C.F For calculating median (M = 12) For calculating $\sum f_i\|x_i - M\|$ = 36 For M. D(M) = 0.75	½ mk ½ mk ½ mk ½ mk
27.	For proving P(1) is true Assuming P(k) to be true To prove P(k+1) to be true Conclusion	1 mk ½ mk 2 mks ½ mk
28.	$(A \cup B)' = \{1, 9\}$; $A' \cap B' = \{1, 9\}$ $(A \cap B)' = \{1,3,4,5,6,7,8,9\}$; $A' \cup B' = \{1,3,4,5,6,7,8,9\}$	2 mks 2 mks
29.	$a + b = 6\sqrt{ab} \Rightarrow \frac{a+b}{2\sqrt{ab}} = \frac{3}{1}$ Applying componendo and dividendo to get $\frac{\sqrt{a}+\sqrt{b}}{\sqrt{a}-\sqrt{b}} = \frac{\sqrt{2}}{1}$ Again applying componendo and dividendo we get $\frac{\sqrt{a}}{\sqrt{b}} = \frac{\sqrt{2}+1}{\sqrt{2}-1}$ Squaring both sides to get $\frac{a}{b} = \frac{3+2\sqrt{2}}{3-2\sqrt{2}}$ (OR) Identifying $a_n = (1^2 + 2^2 + 3^2 + \cdots \ldots + n^2) = \frac{n(n+1)(2n+1)}{6} = \frac{1}{6}[2n^3 + 3n^2 + n]$ $\therefore S_n = \sum a_n = \sum \frac{1}{6}[2n^3 + 3n^2 + n] = \frac{1}{6}\left[2\sum n^3 + 3\sum n^2 + \sum n\right]$ $= \frac{1}{6}\left[2\left\{\frac{n(n+1)}{2}\right\}^2 + \frac{3n(n+1)(2n+1)}{6} + \frac{n(n+1)}{2}\right]$ Simplifying to get $\frac{n(n+1)^2(n+2)}{12}$	1 mk 1 mk 1 mk 1 mk 1 mk 2 mks 1 mk
30.	$\Rightarrow 2\sin 3x \cos 2x + \sin 3x = 0 \Rightarrow \sin 3x(2\cos 2x + 1 = 0)$ $\Rightarrow \sin 3x = 0$ or $\cos 2x = -\frac{1}{2} = cos\frac{2\pi}{3}$ $\Rightarrow 3x = n\pi$ or $2x = 2n\pi \pm \frac{2\pi}{3}$ $\Rightarrow x = \frac{n\pi}{3}, n \in Z$ or $x = n\pi \pm \frac{\pi}{3}, n \in Z.$	1 mk 1 mk 1 mk 1 mk
31.	Finding $\cos x = -\frac{4}{5}$ Showing that $\frac{x}{2}$ lies in II quadrant	1mk ½ mk (1+1+ ½)

	Solving to get $sin\frac{x}{2}=\frac{3}{\sqrt{10}}$, $cos\frac{x}{2}=-\frac{1}{\sqrt{10}}$ and $tan\frac{x}{2}=-3$ (OR) $\frac{cosx}{1-sinx}=\frac{cos^2\frac{x}{2}-sin^2\frac{x}{2}}{cos^2\frac{x}{2}+sin^2\frac{x}{2}-2sin\frac{x}{2}\ cos\frac{x}{2}}$ $=\frac{\left(cos\frac{x}{2}-sin\frac{x}{2}\right)\left(cos\frac{x}{2}+sin\frac{x}{2}\right)}{\left(cos\frac{x}{2}-sin\frac{x}{2}\right)^2}=\frac{cos\frac{x}{2}+sin\frac{x}{2}}{cos\frac{x}{2}-sin\frac{x}{2}}$ $=\frac{1+tan\frac{x}{2}}{1-tan\frac{x}{2}}=\frac{tan\frac{\pi}{4}+tan\frac{x}{2}}{1-tan\frac{\pi}{4}tan\frac{x}{2}}=tan\left(\frac{\pi}{4}+\frac{x}{2}\right)$	1 mk 1 ½ mk 1 ½ mk
32.	a, b, c are in A.P $\Rightarrow b=\frac{a+c}{2}$ ………..(1) b, c, d are in G.P $\Rightarrow c^2=bd$ …………….(2) $\frac{1}{c},\frac{1}{d},\frac{1}{e}$ are in A.P $\Rightarrow d=\frac{2ce}{c+e}$ ………….(3) Putting (1) and (3) in (2), we get $c^2=ae$	1 mk 1 mk 1 mk 1 mk
33.	Venn diagram with correct entries (i) 1 (ii) 23 (iii) 2	3 mks (1+1+1)
34.	Expanding the terms to get $\frac{1+\cos 2x}{2}+\frac{1+\cos 2\left(x+\frac{\pi}{3}\right)}{2}+\frac{1+\cos 2\left(x-\frac{\pi}{3}\right)}{2}$ $=\frac{1}{2}\left[3+cos2x+cos\left(2x+\frac{2\pi}{3}\right)+cos\left(2x-\frac{2\pi}{3}\right)\right]$ $=\frac{1}{2}\left[3+\cos 2x+2cos2x\ cos\frac{2\pi}{3}\right]=\frac{1}{2}\left[3+\cos 2x-2cos2x\ .\frac{1}{2}\right]=\frac{3}{2}$	½ mk each 1 mk 3 ½ mks
35.	Three lines with correct arrow directions Common solution region (OR) Let x litres of 30% sol be added to 600 litres of 12% sol, then, $\Rightarrow 15\%\ of\ (x+600)<30\%\ of\ x+12\%\ of\ 600<18\%\ of\ (x+600)$ $\Rightarrow 5(x+600)<10x+2400<6(x+600)$ $\Rightarrow 600<5x\ and\ 4x<1200$ $\Rightarrow 120<x<300$	1 ½ each 1 ½ mk 2 ½ mks 1 ½ mk 1 mk 1 mk
36	For correct entries in the table Calculating $\bar{x}=34$, $\sigma^2=214.71$, $\sigma=14.65$ (OR) Incorrect $\sum x_i=126$ $\Rightarrow$correct $\sum x_i=117$ ∴ correct $\bar{x}=6.5$ Incorrect $\sum x_i^2=1170$ $\Rightarrow$correct $\sum x_i^2=873$ ∴ correct $\sigma^2=6.25$, correct S.D=2.5	3 mks (1+1+1) 2 mks (2+2)mks
	SET – B	
27	$a+b=3\ \ldots\ldots(i)$ and $ab=p\ \ldots\ldots(ii)$ $c+d=12\ \ldots\ldots(iii)$ and $cd=q\ \ldots\ldots(iv)$ Also, $b=ar,\ c=ar^2,\ d=ar^3$ From (i)& (iii), we get $r^2=4$ From (ii)& (iv), we get $\frac{q}{p}=r^4=4^2$	½ mk ½ mk 1 mk 1 mk

	$\frac{q}{p} = \frac{16}{1} \Rightarrow \frac{q+p}{q-p} = \frac{16+1}{16-1} = \frac{17}{15}$	1 mk
32	Same as Q.no 27 of Set – A	
36	Venn diagram with correct entries (i) 2 (ii) 3 (iii) 9	3 mks (1+1+1)
	SET – C	
30	$a + ar + ar^2 = 56 \Rightarrow a(1 + r + r^2) = 56$ Now $a - 1, ar - 7, ar^2 - 21$ are in A.P $\Rightarrow 2(ar - 7) = (a - 1) + (ar^2 - 21)$ Solving to get $r = 2, \frac{1}{2}$ and $a = 8, 32$ Hence the numbers are 8, 16, 32 or 32, 16, 8.	½ mk ½ mk 1 mk 1 ½ mk ½ mk
32	$\Rightarrow 2\sin 4x \cos 2x + \sin 4x = 0 \Rightarrow \sin 4x(2\cos 2x + 1 = 0)$ $\Rightarrow \sin 4x = 0$ or $\cos 2x = -\frac{1}{2} = cos\frac{2\pi}{3}$ $\Rightarrow 4x = n\pi$ or $2x = 2n\pi \pm \frac{2\pi}{3}$ $\Rightarrow x = \frac{n\pi}{4}, n \in Z$ or $x = n\pi \pm \frac{\pi}{3}, n \in Z.$	1 mk 1 mk 1 mk 1 mk
33	$\frac{\sqrt{3}}{2} sin10° \, sin50° \, sin70° \Rightarrow \frac{\sqrt{3}}{4}(2\, sin10° \, sin50°).sin70°$ $\Rightarrow \frac{\sqrt{3}}{4}(cos40° - cos60°).sin70°$ $\Rightarrow \frac{\sqrt{3}}{4}\left[\frac{1}{2}(2\, sin70° \, cos40°) - \frac{1}{2}sin70°\right]$ $\Rightarrow \frac{\sqrt{3}}{8}\left[sin70° + \frac{1}{2} - sin70°\right] = \frac{\sqrt{3}}{16}$	1 mk 1 ½ mk 1 ½ mk 1 +1