

# FIITJEE

## Solutions to JEE(Main) -2023

Test Date: 29<sup>th</sup> January 2023 (First Shift)

### PHYSICS, CHEMISTRY & MATHEMATICS

Paper - 1

Time Allotted: 3 Hours

Maximum Marks: 300

- Please read the instructions carefully. You are allotted 5 minutes specifically for this purpose.

#### **Important Instructions:**

1. The test is of 3 hours duration.
2. This test paper consists of 90 questions. Each subject (PCM) has 30 questions. The maximum marks are 300.
3. This question paper contains **Three Parts**. **Part-A** is Physics, **Part-B** is Chemistry and **Part-C** is Mathematics. Each part has only two sections: **Section-A** and **Section-B**.
4. **Section – A** : Attempt all questions.
5. **Section – B** : Do any 5 questions out of 10 Questions.
6. **Section-A (01 – 20)** contains 20 multiple choice questions which have **only one correct answer**. Each question carries **+4 marks** for correct answer and **–1 mark** for wrong answer.
7. **Section-B (1 – 10)** contains 10 Numerical based questions. The answer to each question is rounded off to the nearest integer value. Each question carries **+4 marks** for correct answer and **–1 mark** for wrong answer.

# PART - A (PHYSICS)

## SECTION - A

(One Options Correct Type)

This section contains **20 multiple choice questions**. Each question has **four choices** (A), (B), (C) and (D), out of which **ONLY ONE** option is correct.

- Q1.** Surface tension of a soap bubble is  $2.0 \times 10^{-2} \text{ Nm}^{-1}$ . Work done to increase the radius of soap bubble from 3.5 cm to 7cm will be :

Take  $\left[ \pi = \frac{22}{7} \right]$

- (A)  $0.72 \times 10^{-4} \text{ J}$  (B)  $5.76 \times 10^{-4} \text{ J}$   
(C)  $9.24 \times 10^{-4} \text{ J}$  (D)  $18.48 \times 10^{-4} \text{ J}$

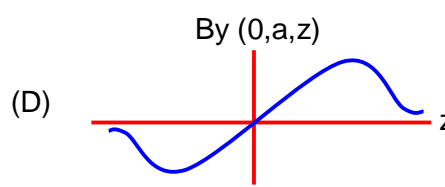
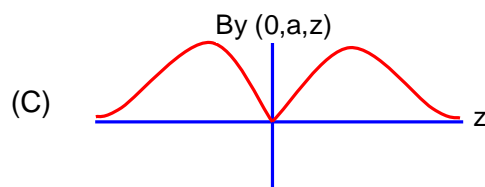
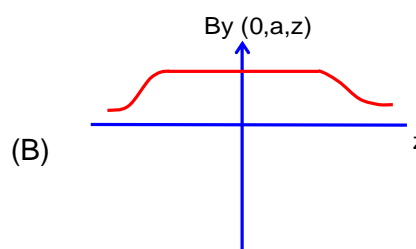
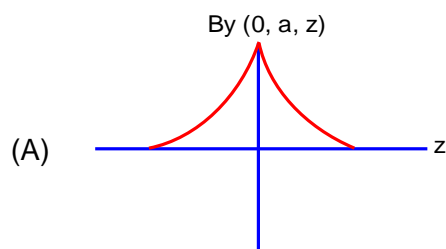
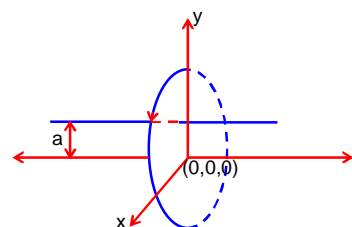
- Q2.** The threshold wavelength for photoelectric emission from a material is  $5500 \text{ \AA}$ . Photoelectrons will be emitted, when this material is illuminated with monochromatic radiation from a

- A. 75 W infra-red lamp  
B. 10 W infra-red lamp  
C. 75 W ultra-violet lamp  
D. 10 W ultra-violet lamp

Choose the correct answer from the options given below :

- (A) C and D only (B) C only  
(C) B and C only (D) A and D only

- Q3.** A single current carrying loop of wire carrying current  $I$  flowing in anticlockwise direction seen from +ve  $z$  direction and lying in  $xy$  plane is shown in figure. The plot of  $\hat{j}$  component of magnetic field ( $B_y$ ) at a distance 'a' (less than radius of the coil) and on  $yz$  plane vs  $z$  coordinate looks like



**Q4.** Match List I with List II :

**List – I (Physical Quantity)**

- A. Pressure gradient  
B. Energy density  
C. Electric Field  
D. Latent heat

**List – II (Dimensional Formula)**

- I.  $[M^0 L^2 T^{-2}]$   
II.  $[M^1 L^{-1} T^{-2}]$   
III.  $[M^1 L^{-2} T^{-2}]$   
IV.  $[M^1 L^1 T^{-3} A^{-1}]$

Choose the correct answer from the options given below :

- (A) A – III, B – II, C – I, D – IV  
(B) A – II, B – III, C – IV, D – I  
(C) A – III, B – II, C – IV, D – I  
(D) A – II, B – III, C – I, D – IV

**Q5.** A bicycle tyre is filled with air having pressure of 270 kPa at 27°C. The approximate pressure of the air in the tyre when the temperature increases to 36°C is

- (A) 270 kPa  
(B) 278 kPa  
(C) 360 kPa  
(D) 262 kPa

**Q6.** Which of the following are true ?

- A. Speed of light in vacuum is dependent on the direction of propagation.  
B. Speed of light in a medium is independent of the wavelength of light.  
C. The speed of light is independent of the motion of the source.  
D. The speed of light in a medium is independent of intensity.

Choose the correct answer from the options given below :

- (A) B and C only  
(B) C and D only  
(C) B and D only  
(D) A and C only

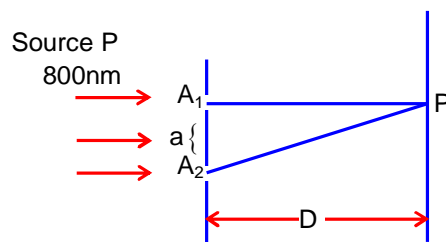
**Q7.** Ratio of thermal energy released in two resistors R and 3R connected in parallel in a electric circuit is :

- (A) 1 : 1  
(B) 1 : 27  
(C) 1 : 3  
(D) 3 : 1

**Q8.** In a Young's double slit experiment, two slits are illuminated with a light of wavelength 800nm. The line joining  $A_1P$  is perpendicular to  $A_1A_2$  as shown in the figure. If the first minimum is detected at P, the value of slits separation 'a' will be:

The distance of screen from slits  $D = 5$  cm

- (A) 0.1 mm  
(B) 0.2 mm  
(C) 0.4 mm  
(D) 0.5 mm



**Q9.** Two particle of equal mass 'm' move in a circle of radius 'r' under the action of their mutual gravitational attraction. The speed of each particle will be :

- (A)  $\sqrt{\frac{Gm}{2r}}$   
(B)  $\sqrt{\frac{Gm}{r}}$   
(C)  $\sqrt{\frac{4Gm}{r}}$   
(D)  $\sqrt{\frac{Gm}{4r}}$

- Q10.** If a radioactive element having half-life of 30 min is undergoing beta decay, the fraction of radioactive element remains undecayed after 90 min. will be

(A)  $\frac{1}{16}$  (B)  $\frac{1}{8}$   
(C)  $\frac{1}{2}$  (D)  $\frac{1}{4}$

- Q11.** A stone is projected at angle  $30^\circ$  to the horizontal. The ratio of kinetic energy of the stone at point of projection to its kinetic energy at the highest point of flight will be –

(A) 4 : 3 (B) 1 : 2  
(C) 1 : 4 (D) 4 : 1

- Q12.** Which one of the following statement is not correct in the case of light emitting diodes ?

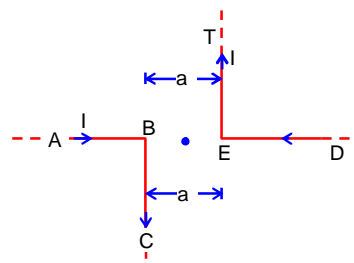
A. It is a heavily doped p-n junction..  
B. It emits light only when it is forward biased.  
C. It emits light only when it is reverse biased.  
D. The energy of the light emitted is equal to or slightly less than the energy gap of the semiconductor used.

Choose the correct answer from the options given below :

(A) C and D (B) C  
(C) B (D) A

- Q13.** The magnitude of magnetic induction at mid point O due to current arrangement as shown in Fig will be

(A)  $\frac{\mu_0 I}{4\pi a}$  (B) 0  
(C)  $\frac{\mu_0 I}{\pi a}$  (D)  $\frac{\mu_0 I}{2\pi a}$



- Q14.** In a cuboid of dimension  $2L \times 2L \times L$ , a charge  $q$  is placed at the centre of the surface 'S' having area of  $4L^2$ . The flux through the opposite surface to 'S' is given by

(A)  $\frac{q}{6\epsilon_0}$  (B)  $\frac{q}{2\epsilon_0}$   
(C)  $\frac{q}{3\epsilon_0}$  (D)  $\frac{q}{12\epsilon_0}$

- Q15.** A person observes two moving trains, 'A' reaching the station and 'B' leaving the station with equal speed of 30 m/s. If both trains emit sounds with frequency 300 Hz. (Speed of sound : 330 m/s) approximate difference of frequencies heard by the person will be :

(A) 10 Hz (B) 55 Hz  
(C) 80 Hz (D) 33 Hz

- Q16.** Given below are two statements: One is labeled as **Assertion A** and the other is labeled as **Reason R**.

**Assertion A :** If  $dQ$  and  $dW$  represent the heat supplied to the system and the work done on the system respectively. Then according to the first law of thermodynamics  $dQ = dU - dW$ .

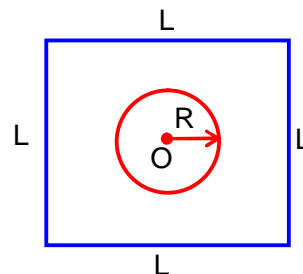
**Reason R :** First law of thermodynamics is based on law of conservation of energy.

In the light of the above statements, choose the correct answer from the options given below :

(A) A is not correct but R is correct  
(B) Both A and R are correct but R is not the correct explanation of A  
(C) Both A and R are correct and R is the correct explanation of A  
(D) A is correct but R is not correct

- Q17.** A car is moving on a horizontal curved road with radius 50 m. The approximate maximum speed of car will be, if friction between tyres and road is 0.34. [ take  $g = 10\text{ms}^{-2}$  ]  
 (A)  $22.4\text{ ms}^{-1}$  (B)  $13\text{ms}^{-1}$   
 (C)  $3.4\text{ ms}^{-1}$  (D)  $17\text{ ms}^{-1}$

- Q18.** Find the mutual inductance in the arrangement, when a small circular loop of wire of radius 'R' is placed inside a large square loop of wire of side L ( $L \gg R$ ). The loops are coplanar and their centres coincide :



- (A)  $M = \frac{\sqrt{2}\mu_0 R^2}{L}$   
 (B)  $M = \frac{\sqrt{2}\mu_0 R}{L^2}$   
 (C)  $M = \frac{2\sqrt{2}\mu_0 R^2}{L}$   
 (D)  $M = \frac{2\sqrt{2}\mu_0 R}{L^2}$

- Q19.** If the height of transmitting and receiving antennas are 80 m each, the maximum line of sight distance will be :

Given : Earth's radius =  $6.4 \times 10^6\text{ m}$

- (A) 32 km (B) 36 km  
 (C) 64 km (D) 28 km

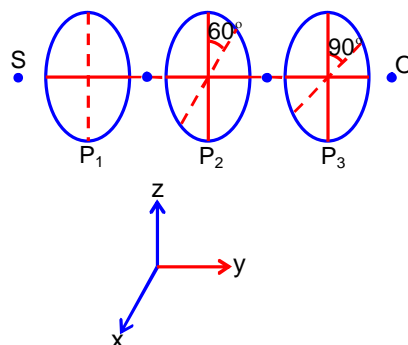
- Q20.** A block of mass m slides down the plane inclined at angle  $30^\circ$  with an acceleration  $\frac{g}{4}$ . The value of coefficient of kinetic friction will be :

- (A)  $\frac{2\sqrt{3}+1}{2}$  (B)  $\frac{2\sqrt{3}-1}{2}$   
 (C)  $\frac{1}{2\sqrt{3}}$  (D)  $\frac{\sqrt{3}}{2}$

**SECTION - B****(Numerical Answer Type)**

This section contains **10** Numerical based questions. The answer to each question is rounded off to the nearest integer value.

- Q1.** A 0.4 kg mass takes 8s to reach ground when dropped from a certain height 'P' above surface of earth. The loss of potential energy in the last second of fall is \_\_\_\_\_. J.  
(Take  $g = 10 \text{ m/s}^2$ )
- Q2.** A point charge  $q_1 = 4q_0$  is placed at origin. Another point charge  $q_2 = -q_0$  is placed at  $x = 12\text{cm}$ . Charge of proton is  $q_0$ . The proton is placed on x axis so that the electrostatic force on the proton is zero. In this situation, the position of the proton from origin is \_\_\_\_\_ cm.
- Q3.** In a meter bridge experiment the balance point is obtained if the gaps are closed by  $2\Omega$  and  $3\Omega$ . A shunt of  $X\Omega$  is added to  $3\Omega$  resistor to shift the balancing point by 22.5cm. The value of X is \_\_\_\_\_.
- Q4.** A certain elastic conducting material is stretched into a circular loop. It is placed with its plane perpendicular to a uniform magnetic field  $B = 0.8 \text{ T}$ . When released the radius of the loop starts shrinking at a constant rate of  $2\text{cm/s}$ . The induced emf in the loop at an instant when the radius of the loop is 10cm will be \_\_\_\_\_ mV.
- Q5.** A body cools from  $60^\circ\text{C}$  to  $40^\circ\text{C}$  in 6 minutes. If temperature of surroundings is  $10^\circ\text{C}$ . Then, after the next 6 minutes, its temperature will be \_\_\_\_\_  $^\circ\text{C}$ .
- Q6.** A tennis ball is dropped on to the floor from a height of 9.8m. It rebounds to a height 5.0m. Ball comes in contact with the floor for 0.2s. The average acceleration during contact is \_\_\_\_\_  $\text{ms}^{-2}$ .  
(Given  $g = 10 \text{ ms}^{-2}$ )
- Q7.** Two simple harmonic waves having equal amplitudes of 8cm and equal frequency of 10Hz are moving along the same direction. The resultant amplitude is also 8cm. The phase difference between the individual waves is \_\_\_\_\_ degree.
- Q8.** A solid sphere of mass 2kg is making pure rolling on a horizontal surface with kinetic energy 2240J. The velocity of centre of mass of the sphere will be \_\_\_\_\_  $\text{ms}^{-1}$ .
- Q9.** A radioactive element  ${}^{242}_{92}\text{X}$  emits two  $\alpha$  - particles, one electron and two positrons. The product nucleus is represented by  ${}^{234}_P\text{Y}$ , The value of P is \_\_\_\_\_.
- Q10.** As shown in the figure, three identical polaroids  $P_1$ ,  $P_2$  and  $P_3$  are placed one after another. The pass axis of  $P_2$  and  $P_3$  are inclined at angle of  $60^\circ$  and  $90^\circ$  with respect to axis of  $P_1$ . The source S has an intensity of  $256 \frac{\text{W}}{\text{m}^2}$ . The intensity of light at point O is \_\_\_\_\_  $\frac{\text{W}}{\text{m}^2}$ .

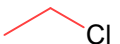
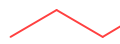
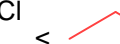


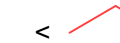

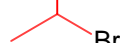

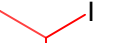



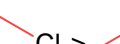



# PART – B (CHEMISTRY)

## SECTION - A

(One Options Correct Type)

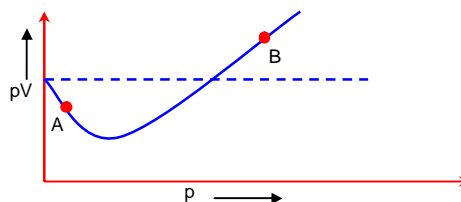
This section contains **20 multiple choice questions**. Each question has **four choices** (A), (B), (C) and (D), out of which **ONLY ONE** option is correct.

- Q1.** During the borax bead test with  $\text{CuSO}_4$ , a blue green colour of the bead was observed in oxidising flame due to the formation of  
 (A) Cu (B)  $\text{Cu}(\text{BO}_2)_2$   
 (C) CuO (D)  $\text{Cu}_3\text{B}_2$
- Q2.** Identify the correct order for the given property for following compounds.  
 (A) Boiling Point;   $\text{Cl}$  <   $\text{Cl}$  <   $\text{Cl}$   
 (B) Density;   $\text{Br}$  <   $\text{Cl}$  <   $\text{I}$   
 (C) Boiling Point;   $\text{Br}$  <   $\text{Br}$  <   $\text{Br}$   
 (D) Density;   $\text{I}$  <   $\text{Br}$  <   $\text{Cl}$   
 (E) Boiling Point;   $\text{Cl}$  >   $\text{Cl}$  >   $\text{Cl}$   
 Choose the correct answer from the option given below:  
 (A) (A), (C) and (E) only (B) (B), (C) and (D) only  
 (C) (A), (C) and (D) only (D) (A), (B) and (E) only
- Q3.** Which of the following salt solutions would coagulate the colloid solution formed when  $\text{FeCl}_3$  is added to  $\text{NaOH}$  solution, at the fastest rate?  
 (A) 10mL of  $0.2 \text{ mol dm}^{-3} \text{AlCl}_3$  (B) 10mL of  $0.1 \text{ mol dm}^{-3} \text{Na}_2\text{SO}_4$   
 (C) 10mL of  $0.15 \text{ mol dm}^{-3} \text{CaCl}_2$  (D) 10mL of  $0.1 \text{ mol dm}^{-3} \text{Ca}_3(\text{PO}_4)_2$
- Q4.** Compound that will give positive Lassaigne's test for both nitrogen and halogen is:  
 (A)  $\text{NH}_4\text{Cl}$  (B)  $\text{NH}_2\text{OH} \cdot \text{HCl}$   
 (C)  $\text{CH}_3\text{NH}_2 \cdot \text{HCl}$  (D)  $\text{N}_2\text{H}_4 \cdot \text{HCl}$
- Q5.** Which of the given compounds can enhance the efficiency of hydrogen storage tank?  
 (A)  $\text{NaNi}_5$  (B) Di-isobutylaluminium hydride  
 (C)  $\text{SiH}_4$  (D)  $\text{Li/P}_4$
- Q6.** The bond dissociation energy is highest for  
 (A)  $\text{F}_2$  (B)  $\text{Cl}_2$   
 (C)  $\text{I}_2$  (D)  $\text{Br}_2$
- Q7.** Correct statement about smog is:  
 (A)  $\text{NO}_2$  is present in classical smog  
 (B) Both  $\text{NO}_2$  and  $\text{SO}_2$  are present in classical smog  
 (C) Photochemical smog has high concentration of oxidizing agents  
 (D) Classical smog also has high concentration of oxidizing agents

- Q8.** "A" obtained by Ostwald's method involving air oxidation of  $\text{NH}_3$ , upon further air oxidation produces "B". "B" on hydration forms an oxoacid of Nitrogen along with evolution of "A". The oxoacid also produces "A" and gives positive brown ring test. Identify A and B respectively.

(A)  $\text{N}_2\text{O}_3$ ,  $\text{NO}_2$  (B)  $\text{NO}$ ,  $\text{NO}_2$   
(C)  $\text{NO}_2$ ,  $\text{N}_2\text{O}_5$  (D)  $\text{NO}_2$ ,  $\text{N}_2\text{O}_4$

- Q9.** For 1 mol of gas, the plot of  $pV$  vs.  $P$  is shown below.  $p$  is the pressure and  $V$  is the volume of What is the value of compressibility factor at point A?



(A)  $1 + \frac{a}{RTV}$   
(B)  $1 - \frac{b}{V}$   
(C)  $1 + \frac{b}{V}$   
(D)  $1 - \frac{a}{RTV}$

- Q10.** The reaction representing the Mond process for metal refining is\_\_\_\_\_.

(A)  $\text{Zr} + 2\text{I}_2 \xrightarrow{\Delta} \text{ZrI}_4$   
(B)  $\text{Ni} + 4\text{CO} \xrightarrow{\Delta} \text{Ni(CO)}_4$   
(C)  $\text{ZnO} + \text{C} \xrightarrow{\Delta} \text{Zn} + \text{CO}$   
(D)  $2\text{K}[\text{Au(CN)}_2] + \text{Zn} \xrightarrow{\Delta} \text{K}_2[\text{Zn(CN)}_4] + 2\text{Au}$

- Q11.** The standard electrode potential ( $\text{M}^{3+} / \text{M}^{2+}$ ) for V, Cr, Mn & Co are -0.26V, -0.41V, +1.57V and +1.97V, respectively. The metal ions which can liberate  $\text{H}_2$  from a dilute acid are

(A)  $\text{Mn}^{2+}$  and  $\text{Co}^{2+}$  (B)  $\text{Cr}^{2+}$  and  $\text{Co}^{2+}$   
(C)  $\text{V}^{2+}$  and  $\text{Cr}^{2+}$  (D)  $\text{V}^{2+}$  and  $\text{Mn}^{2+}$

- Q12.** Chiral complex from the following is:

Here en= ethylene diamine

(A) cis-  $[\text{PtCl}_2(\text{NH}_3)_2]$  (B) cis-  $[\text{PtCl}_2(\text{en})_2]^{2+}$   
(C) trans-  $[\text{PtCl}_2(\text{en})_2]^{2+}$  (D) trans-  $[\text{Co}(\text{NH}_3)_4\text{Cl}_2]^+$

- Q13.** Match List I with List II

| List-I<br>(Reaction) |                         | List-II<br>(Reagents) |   |
|----------------------|-------------------------|-----------------------|---|
| A.                   | Hoffmann Degradation    | I.                    | Conc.KOH, $\Delta$                                  |
| B.                   | Clemenson reduction     | II.                   | $\text{CHCl}_3, \text{NaOH} / \text{H}_3\text{O}^+$ |
| C.                   | Cannizaro reaction      | III.                  | $\text{Br}_2, \text{NaOH}$                          |
| D.                   | Reimer-Tiemann Reaction | IV.                   | $\text{Zn-Hg/HCl}$                                  |

Choose the correct answer from the options given below:

(A) A- II, B-I, C-III, D- IV (B) A-III, B-IV, C-I, D-II  
(C) A-III, B-IV, C-II, D-I (D) A-II, B-IV, C-I, D-III

- Q14.** Number of cyclic tripeptides formed with 2 amino acids A and B is

(A) 4 (B) 2  
(C) 5 (D) 3



**Q15.** The increasing order of pKa for the following phenols is

- A. 2, 4- Dinitrophenol  
 B. 4-Nitrophenol  
 C. 2,4,5- Trimethylphenol  
 D. Phenol  
 E. 3-Chlorophenol

Choose the correct answer from the option given below:

- (A) C, D, E, B, A  
 (B) A, B, E, D, C  
 (C) A, E, B, D, C  
 (D) C, E, D, B, A

**Q16.** Match List I with List II

| List-I<br>(Antimicrobials) |                            | List-II<br>(Names) |                 |
|----------------------------|----------------------------|--------------------|-----------------|
| A.                         | Narrow Spectrum Antibiotic | I.                 | Furacin         |
| B.                         | Antiseptic                 | II.                | Sulphur dioxide |
| C.                         | Disinfectants              | III.               | PenicillinG     |
| D.                         | Broad spectrum antibiotic  | IV.                | Chloramphenicol |

Choose the correct answer from the options given below:

- (A) A- II, B-I, C-IV, D- III  
 (B) A-III, B-I, C-IV, D-II  
 (C) A-III, B-I, C-II, D-IV  
 (D) A-I, B-II, C-IV, D-III

**Q17.** The shortest wavelength of hydrogen atom in Lyman series is  $\lambda$ . The longest wavelength in Balmer series of  $\text{He}^+$  is

- (A)  $\frac{5\lambda}{9}$   
 (B)  $\frac{9\lambda}{5}$   
 (C)  $\frac{5}{9\lambda}$   
 (D)  $\frac{36\lambda}{5}$

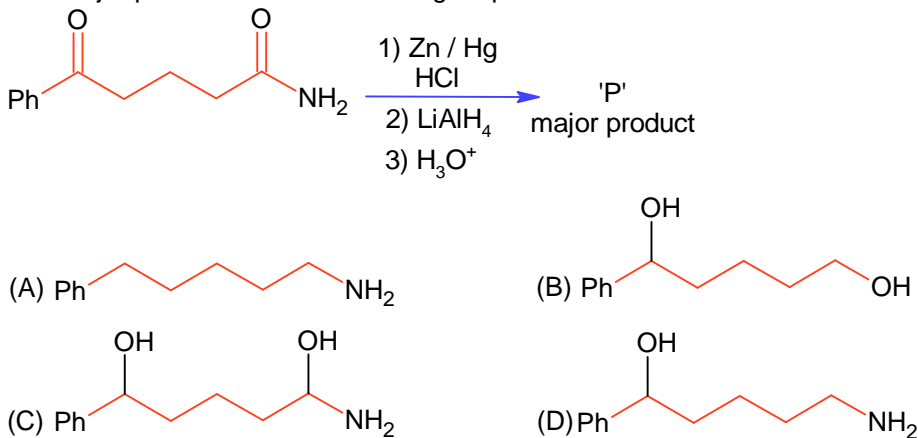
**Q18.** The correct order of hydration enthalpies is:

- A.  $\text{K}^+$   
 B.  $\text{Rb}^+$   
 C.  $\text{Mg}^{2+}$   
 D.  $\text{Cs}^+$   
 E.  $\text{Ca}^{2+}$

Choose the correct answer from the options given below:

- (A)  $\text{E} > \text{C} > \text{A} > \text{B} > \text{D}$   
 (B)  $\text{C} > \text{E} > \text{A} > \text{D} > \text{B}$   
 (C)  $\text{C} > \text{A} > \text{E} > \text{B} > \text{D}$   
 (D)  $\text{C} > \text{E} > \text{A} > \text{B} > \text{D}$

**Q19.** The major product 'P' for the following sequence of reactions is:

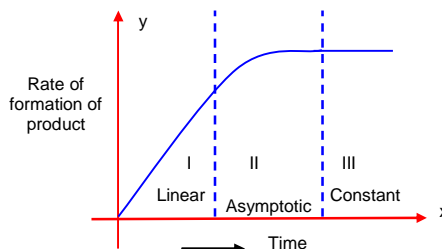


- Q20.** The magnetic behavior of  $\text{Li}_2\text{O}$ ,  $\text{Na}_2\text{O}_2$  and  $\text{KO}_2$ , respectively are
- (A) Diamagnetic, paramagnetic and diamagnetic
  - (B) Diamagnetic, diamagnetic and paramagnetic
  - (C) Paramagnetic, paramagnetic and diamagnetic
  - (D) Paramagnetic, diamagnetic and paramagnetic

**SECTION - B****(Numerical Answer Type)**

This section contains **10** Numerical based questions. The answer to each question is rounded off to the nearest integer value.

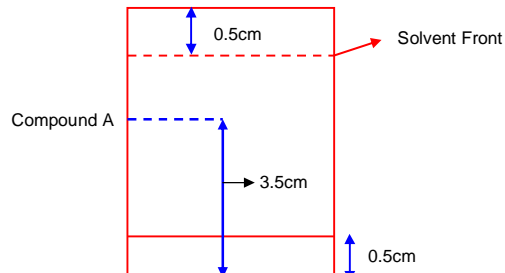
- Q1.** For certain chemical reaction  $X \rightarrow Y$ , the rate of



- (A) Over all order of this reaction is one  
 (B) Order of this reaction can't be determined  
 (C) In region I and III, the reaction is of first and zero order respectively  
 (D) In region-II, the reaction is of first order  
 (E) In region-II, the order of reaction is in the range of 0.1 to 0.9
- Q2.** The number of molecules or ions from the following, which do not have odd number of electrons are\_\_\_\_\_.
- A.  $\text{NO}_2$   
 B.  $\text{ICl}_4^-$   
 C.  $\text{BrF}_3$   
 D.  $\text{ClO}_2$   
 E.  $\text{NO}_2^+$   
 F.  $\text{NO}$
- Q3.** The sum of bridging carbonyls in  $\text{W}(\text{CO})_6$  and  $\text{Mn}_2(\text{CO})_{10}$  is\_\_\_\_\_.
- Q4.** Milimoles of calcium hydroxide required to produce 100 mL of aqueous solution of pH 12 is  $x \times 10^{-1}$ . The value of x is \_\_\_\_\_(Nearest integer)  
 Assume complete dissociation.
- Q5.** Solid Lead nitrate is dissolved in 1 litre of water. The solution was found to boil at  $100.15^\circ\text{C}$ . When 0.2 mol of  $\text{NaCl}$  is added to the resulting solution, it was observed that the solution froze at  $-0.8^\circ\text{C}$ . The solubility product of  $\text{PbCl}_2$  formed is  $\times 10^{-6}$  at 298 K. (Nearest integer)  
 Given:  $K_b = 0.5 \text{ K kg mol}^{-1}$  and  $K_f = 1.8 \text{ K kg mol}^{-1}$ . Assume molality to be equal to molarity in all cases.
- Q6.** Consider the following reaction approaching equilibrium at  $27^\circ\text{C}$  and 1 atm pressure
- $$\text{A} + \text{B} \xrightleftharpoons[K_b=10^2]{K_f=10^3} \text{C} + \text{D}$$
- The standard Gibb's energy change  $(\Delta_r G^0)$  at  $27^\circ\text{C}$  is  $(-)$  \_\_\_\_\_  $\text{kJ mol}^{-1}$  (Nearest integer)  
 (Given:  $R = 8.3 \text{ JK}^{-1}\text{mol}^{-1}$  and  $\ln 10 = 2.3$ )
- Q7.** Water decomposes at 2300K
- $$\text{H}_2\text{O}(\text{g}) \rightarrow \text{H}_2(\text{g}) + \frac{1}{2}\text{O}_2(\text{g})$$
- The percent of water decomposing at 2300 K and 1 bar is \_\_\_\_\_(Nearest integer)  
 Equilibrium constant for the reaction is  $2 \times 10^{-3}$  at 2300K

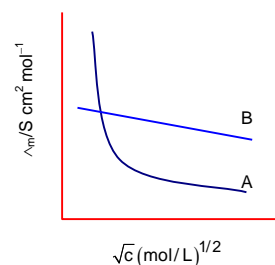
- Q8.** 17 mg of a hydrocarbon (M.F.  $C_{10}H_{16}$ ) takes up 8.40 mL of the  $H_2$  gas measured at  $0^\circ C$  and 760 mm of Hg. Ozonolysis of the same hydrocarbon yields
- $$CH_3-\overset{\overset{O}{\parallel}}{C}-CH_3, \quad H-\overset{\overset{O}{\parallel}}{C}-H, \quad H-\overset{\overset{O}{\parallel}}{C}-CH_2-CH_2-\overset{\overset{O}{\parallel}}{C}-\overset{\overset{O}{\parallel}}{C}-H$$
- The number of double bond/s present in the hydrocarbon is\_\_\_\_\_.

- Q9.** Following chromatogram was developed by



- Q10.** Following figure shows dependence of molar conductance of two electrolytes on concentration.  $\Lambda_m^0$  is the limiting molar conductivity. The number **incorrect** statement(s) from the following is\_\_\_\_\_.

- (A)  $\Lambda_m^0$  for electrolyte A is obtained by extrapolation
- (B) For electrolyte B,  $\Lambda_m$  vs  $\sqrt{C}$  graph is a straight line with intercept equal to  $\Lambda_m^0$
- (C) At infinite dilution, the value of degree of dissociation approaches zero for electrolyte B.
- (D)  $\Lambda_m^0$  for any electrolyte A or B can be calculated using  $\lambda^0$  for individual ions



# PART – C (MATHEMATICS)

## SECTION - A

(One Options Correct Type)

This section contains **20 multiple choice questions**. Each question has **four choices** (A), (B), (C) and (D), out of which **ONLY ONE** option is correct.

- Q1.** Let  $\alpha$  and  $\beta$  be real numbers. Consider a  $3 \times 3$  matrix  $A$  such that  $A^2 = 3A + \alpha I$ . If  $A^4 = 21A + \beta I$ , then  
 (A)  $\alpha = 1$  (B)  $\alpha = 4$   
 (C)  $\beta = -8$  (D)  $\beta = 8$
- Q2.** Let the tangents at the points  $A(4, -11)$  and  $B(8, -5)$  on the circle  $x^2 + y^2 - 3x + 10y - 15 = 0$ , intersect at the point  $C$ . Then the radius of the circle, whose centre is  $C$  and the line joining  $A$  and  $B$  is its tangent, is equal to  
 (A)  $\frac{3\sqrt{3}}{4}$  (B)  $\frac{2\sqrt{13}}{3}$   
 (C)  $\sqrt{13}$  (D)  $2\sqrt{13}$
- Q3.** Let  $y = f(x)$  be the solution of the differential equation  $y(x+1)dx - x^2dy = 0$ ,  $y(1) = e$ . Then  $\lim_{x \rightarrow 0^+} f(x)$  is equal to  
 (A)  $\frac{1}{e}$  (B)  $e^2$   
 (C)  $\frac{1}{e^2}$  (D)  $0$
- Q4.** If the vectors  $\vec{a} = \lambda\hat{i} + \mu\hat{j} + 4\hat{k}$ ,  $\vec{b} = -2\hat{i} + 4\hat{j} - 2\hat{k}$  and  $\vec{c} = 2\hat{i} + 3\hat{j} + \hat{k}$  are coplanar and the projection of  $\vec{a}$  on the vector  $\vec{b}$  is  $\sqrt{54}$  units, then the sum of all possible values of  $\lambda + \mu$  is equal to  
 (A)  $0$  (B)  $6$   
 (C)  $18$  (D)  $24$
- Q5.** Let  $A = \{(x, y) \in \mathbb{R}^2 : y \geq 0, 2x \leq y \leq \sqrt{4 - (x-1)^2}\}$  and  $B = \{(x, y) \in \mathbb{R} \times \mathbb{R} : 0 \leq y \leq \min\left\{2x, \sqrt{4 - (x-1)^2}\right\}\}$ , Then the ratio of the area of  $A$  to the area of  $B$  is  
 (A)  $\frac{\pi+1}{\pi-1}$  (B)  $\frac{\pi-1}{\pi+1}$   
 (C)  $\frac{\pi}{\pi+1}$  (D)  $\frac{\pi}{\pi-1}$

- Q6.** Consider the following system of equations  
 $\alpha x + 2y + z = 1$   
 $2\alpha x + 3y + z = 1$   
 $3x + \alpha y + 2z = \beta$   
 for some  $\alpha, \beta \in \mathbb{R}$ . Then which of the following is NOT correct.  
 (A) It has no solution for  $\alpha = 3$  and for all  $\beta \neq 2$   
 (B) It has no solution for  $\alpha = -1$  and for all  $\beta \in \mathbb{R}$ .  
 (C) It has a solution for all  $\alpha \neq -1$  and  $\beta = 2$   
 (D) It has no solution if  $\alpha = -1$  and  $\beta \neq 2$
- Q7.** The domain of  $f(x) = \frac{\log_{(x+1)}(x-2)}{e^{2\log_e x} - (2x+3)}$ ,  $x \in \mathbb{R}$  is  
 (A)  $(2, \infty) - \{3\}$  (B)  $\mathbb{R} - \{-1, 3\}$   
 (C)  $(-1, \infty) - \{3\}$  (D)  $\mathbb{R} - \{3\}$
- Q8.** Three rotten apples are mixed accidentally with seven good apples and four apples are drawn one by one without replacement. Let the random variable  $X$  denote the number of rotten apples. If  $\mu$  and  $\sigma^2$  represent mean and variance of  $X$ , respectively, then  $10(\mu^2 + \sigma^2)$  is equal to  
 (A) 30 (B) 20  
 (C) 25 (D) 250
- Q9.** Let  $f(\theta) = 3\left(\sin^4\left(\frac{3\pi}{2} - \theta\right) + \sin^4(3\pi + \theta)\right) - 2(1 - \sin^2 2\theta)$  and  $S = \left\{\theta \in [0, \pi] : f'(\theta) = -\frac{\sqrt{3}}{2}\right\}$ . If  $4\beta = \sum_{\theta \in S} \theta$ , then  $f(\beta)$  is equal to  
 (A)  $\frac{3}{2}$  (B)  $\frac{5}{4}$   
 (C)  $\frac{9}{8}$  (D)  $\frac{11}{8}$
- Q10.** Let  $B$  and  $C$  be the two points on the line  $y + x = 0$  such that  $B$  and  $C$  are symmetric with respect to the origin. Suppose  $A$  is a point on  $y - 2x = 2$  such that  $\triangle ABC$  is an equilateral triangle. Then, the area of the  $\triangle ABC$  is  
 (A)  $3\sqrt{3}$  (B)  $\frac{8}{\sqrt{3}}$   
 (C)  $2\sqrt{3}$  (D)  $\frac{10}{\sqrt{3}}$
- Q11.** Fifteen football players of a club-team are given 15 T-shirts with their names written on the backside. If the players pick up the T-shirts randomly, then the probability that at least 3 players pick the correct T-shirt is  
 (A)  $\frac{5}{24}$  (B)  $\frac{2}{15}$   
 (C)  $\frac{5}{36}$  (D)  $\frac{1}{6}$

**Q12.** Let  $f(x) = x + \frac{a}{\pi^2 - 4} \sin x + \frac{b}{\pi^2 - 4} \cos x$ ,  $x \in \mathbb{R}$  be a function which satisfies

$$f(x) = x + \int_0^{\pi/2} \sin(x+y)f(y)dy. \text{ Then } (a+b) \text{ is equal to}$$

- (A)  $-\pi(\pi+2)$  (B)  $-\pi(\pi-2)$   
(C)  $-2\pi(\pi-2)$  (D)  $-2\pi(\pi+2)$

**Q13.** A light ray emits from the origin making an angle  $30^\circ$  with the positive x-axis. After getting reflected by the line  $x+y=1$ , if this ray intersects x-axis at Q, then the abscissa of Q is

- (A)  $\frac{2}{\sqrt{3}-1}$  (B)  $\frac{2}{3+\sqrt{3}}$   
(C)  $\frac{\sqrt{3}}{2(\sqrt{3}+1)}$  (D)  $\frac{2}{3-\sqrt{3}}$

**Q14.** Let  $\Delta$  be the area of the region  $\{(x,y) \in \mathbb{R}^2 : x^2 + y^2 \leq 21, y^2 \leq 4x, x \geq 1\}$ . Then

$$\frac{1}{2} \left( \Delta - 21 \sin^{-1} \frac{2}{\sqrt{7}} \right) \text{ is equal to}$$

- (A)  $\sqrt{3} - \frac{2}{3}$  (B)  $\sqrt{3} - \frac{4}{3}$   
(C)  $2\sqrt{3} - \frac{2}{3}$  (D)  $2\sqrt{3} - \frac{1}{3}$

**Q15.** If p, q and r are three propositions, then which of the following combination of truth values of p, q and r makes the logical expression  $\{(p \vee q) \wedge ((\sim p) \vee r)\} \rightarrow ((\sim q) \vee r)$  false?

- (A)  $p = T, q = T, r = F$  (B)  $p = F, q = T, r = F$   
(C)  $p = T, q = F, r = T$  (D)  $p = T, q = F, r = F$

**Q16.** Let  $f : \mathbb{R} \rightarrow \mathbb{R}$  be a function such that  $f(x) = \frac{x^2 + 2x + 1}{x^2 + 1}$ . Then

- (A)  $f(x)$  is many-one in  $(-\infty, -1)$   
(B)  $f(x)$  is many-one in  $(1, \infty)$   
(C)  $f(x)$  is one-one in  $(-\infty, \infty)$   
(D)  $f(x)$  is one-one in  $[1, \infty)$  but not in  $(-\infty, \infty)$

**Q17.** Let  $[x]$  denote the greatest integer  $\leq x$ . Consider the function  $f(x) = \max\{x^2, 1 + [X]\}$ . Then the

$$\text{value of the integral } \int_0^2 f(x)dx \text{ is}$$

- (A)  $\frac{8+4\sqrt{2}}{3}$  (B)  $\frac{4+5\sqrt{2}}{3}$   
(C)  $\frac{1+5\sqrt{2}}{3}$  (D)  $\frac{5+4\sqrt{2}}{3}$

**Q18.** Let  $\lambda \neq 0$  be a real number. Let  $\alpha, \beta$  be the roots of the equation  $14x^2 - 31x + 3\lambda = 0$  and  $\alpha, \gamma$  be the roots of the equation  $35x^2 - 53x + 4\lambda = 0$ . Then  $\frac{3\alpha}{\beta}$  and  $\frac{4\alpha}{\gamma}$  are the roots of the equation

- (A)  $7x^2 + 245x - 250 = 0$  (B)  $49x^2 + 245x + 250 = 0$   
 (C)  $7x^2 - 245x + 250 = 0$  (D)  $49x^2 - 245x + 250 = 0$

**Q19.** Let  $x = 2$  be a root of the equation  $x^2 + px + q = 0$  and

$$f(x) = \begin{cases} \frac{1 - \cos(x^2 - 4px + q^2 + 8q + 16)}{(x - 2p)^4}, & x \neq 2p \\ 0, & x = 2p \end{cases} . \text{ Then } \lim_{x \rightarrow 2p^+} [f(x)], \text{ where } [.] \text{ denotes greatest}$$

integer function, is

- (A) 0 (B) -1  
(C) 2 (D) 1

**Q20.** For two non-zero complex numbers  $z_1$  and  $z_2$ , if  $\operatorname{Re}(z_1 z_2) = 0$  and  $\operatorname{Re}(z_1 + z_2) = 0$ , then which of the following are possible?

- (i)  $\operatorname{Im}(z_1) > 0$  and  $\operatorname{Im}(z_2) > 0$   
 (ii)  $\operatorname{Im}(z_1) < 0$  and  $\operatorname{Im}(z_2) > 0$   
 (iii)  $\operatorname{Im}(z_1) > 0$  and  $\operatorname{Im}(z_2) < 0$   
 (iv)  $\operatorname{Im}(z_1) < 0$  and  $\operatorname{Im}(z_2) < 0$

Choose the correct answer from the options given below :

- (A) (ii) and (iii) (B) (i) and (iii)  
 (C) (ii) and (iv) (D) (i) and (ii)



**SECTION - B****(Numerical Answer Type)**

This section contains **10** Numerical based questions. The answer to each question is rounded off to the nearest integer value.

- Q1.** Five digits numbers are formed using the digits 1, 2, 3, 5, 7 with repetitions and are written in descending order with serial numbers. For example, the number 77777 has serial number 1. Then the serial number of 35337 is.....
- Q2.** Suppose  $f$  is a function satisfying  $f(x+y) = f(x) + f(y)$  for all  $x, y \in \mathbb{N}$  and  $f(1) = \frac{1}{5}$ . If  $\sum_{n=1}^m \frac{f(n)}{n(n+1)(n+2)} = \frac{1}{12}$ , then  $m$  is equal to.....
- Q3.** Let  $f : \mathbb{R} \rightarrow \mathbb{R}$  be a differentiable function that satisfies the relation  $f(x+y) = f(x) + f(y) - 1, \forall x, y \in \mathbb{R}$ . If  $f'(0) = 2$ , then  $|f(-2)|$  is equal to.....
- Q4.** Let the equation of the plane  $P$  containing the line  $x+10 = \frac{8-y}{2} = z$  be  $ax+by+3z = 2(a+b)$  and the distance of the plane  $P$  from the point  $(1, 27, 7)$  be  $c$ . Then  $a^2 + b^2 + c^2$  is equal to.....
- Q5.** If the co-efficient of  $x^9$  in  $\left(\alpha x^3 + \frac{1}{\beta x}\right)^{11}$  and the co-efficient of  $x^{-9}$  in  $\left(\alpha x - \frac{1}{\beta x^3}\right)^{11}$  are equal, then  $(\alpha\beta)^2$  is equal to.....
- Q6.** Let the co-ordinates of one vertex of  $\triangle ABC$  be  $A(0, 2, \alpha)$  and the other two vertices lie on the line  $\frac{x+\alpha}{5} = \frac{y-1}{2} = \frac{z+4}{3}$ . For  $\alpha \in \mathbb{Z}$ , if the area of  $\triangle ABC$  is 21 sq. units and the line segment  $BC$  has length  $2\sqrt{21}$  units, then  $\alpha^2$  is equal to.....
- Q7.** If all the six digit numbers  $x_1x_2x_3x_4x_5x_6$  with  $0 < x_1 < x_2 < x_3 < x_4 < x_5 < x_6$  are arranged in the increasing order, then the sum of the digits in the 72<sup>th</sup> number is.....
- Q8.** Let  $\vec{a}, \vec{b}$  and  $\vec{c}$  be three non-zero non-coplanar vectors. Let the position vectors of four points  $A, B, C$  and  $D$  be  $\vec{a} - \vec{b} + \vec{c}, \lambda\vec{a} - 3\vec{b} + 4\vec{c}, -\vec{a} + 2\vec{b} - 3\vec{c}$  and  $2\vec{a} - 4\vec{b} + 6\vec{c}$  respectively. If  $\overline{AB}, \overline{AC}$  and  $\overline{AD}$  are coplanar, then  $\lambda$  is equal to.....
- Q9.** Let  $a_1, a_2, a_3, \dots$  be a GP of increasing positive numbers. If the product of fourth and sixth terms is 9 and the sum of fifth and seventh terms is 24, then  $a_1a_9 + a_2a_4a_9 + a_5 + a_7$  is equal to.....
- Q10.** Let the coefficients of three consecutive terms in the binomial expansion of  $(1+2x)^n$  be in the ratio 2 : 5 : 8. Then the coefficient of the term, which is in the middle of these three terms, is.....

# FIITJEE

## KEYS to JEE (Main)-2023

### PART - A (PHYSICS)

#### SECTION - A

|     |   |     |   |     |   |     |   |
|-----|---|-----|---|-----|---|-----|---|
| 1.  | D | 2.  | A | 3.  | D | 4.  | C |
| 5.  | B | 6.  | B | 7.  | D | 8.  | B |
| 9.  | D | 10. | B | 11. | A | 12. | B |
| 13. | C | 14. | A | 15. | B | 16. | C |
| 17. | B | 18. | C | 19. | C | 20. | C |

#### SECTION - B

|    |     |     |     |    |     |    |    |
|----|-----|-----|-----|----|-----|----|----|
| 1. | 300 | 2.  | 24  | 3. | 2   | 4. | 10 |
| 5. | 28  | 6.  | 120 | 7. | 120 | 8. | 40 |
| 9. | 87  | 10. | 24  |    |     |    |    |

### PART - B (CHEMISTRY)

#### SECTION - A

|     |   |     |   |     |   |     |   |
|-----|---|-----|---|-----|---|-----|---|
| 1.  | B | 2.  | A | 3.  | A | 4.  | C |
| 5.  | A | 6.  | B | 7.  | C | 8.  | B |
| 9.  | D | 10. | B | 11. | C | 12. | B |
| 13. | B | 14. | A | 15. | B | 16. | C |
| 17. | B | 18. | D | 19. | A | 20. | B |

#### SECTION - B

|    |    |     |   |    |   |    |   |
|----|----|-----|---|----|---|----|---|
| 1. | 2  | 2.  | 3 | 3. | 0 | 4. | 5 |
| 5. | 13 | 6.  | 6 | 7. | 2 | 8. | 3 |
| 9. | 6  | 10. | 2 |    |   |    |   |

## **PART – C (MATHEMATICS)**

### **SECTION - A**

- |     |   |     |   |     |      |     |   |
|-----|---|-----|---|-----|------|-----|---|
| 1.  | C | 2.  | B | 3.  | D    | 4.  | D |
| 5.  | B | 6.  | B | 7.  | A    | 8.  | B |
| 9.  | B | 10. | B | 11. | DROP | 12. | D |
| 13. | B | 14. | B | 15. | B    | 16. | D |
| 17. | D | 18. | D | 19. | A    | 20. | A |

### **SECTION - B**

- |    |      |     |      |    |    |    |     |
|----|------|-----|------|----|----|----|-----|
| 1. | 1436 | 2.  | 10   | 3. | 3  | 4. | 355 |
| 5. | 1    | 6.  | 9    | 7. | 32 | 8. | 2   |
| 9. | 60   | 10. | 1120 |    |    |    |     |

# FIITJEE

## Solutions to JEE (Main)-2023

### PART - A (PHYSICS)

#### SECTION - A

**Sol1.** W. done =  $T\Delta A \times \text{No of air liq surface}$

$$= 2T \cdot 4\pi(r_2^2 - r_1^2)$$

$$= 2 \times 2 \times 10^{-2} \times 4\pi \left[ 49 - \frac{49}{4} \right] \times 10^{-4}$$

$$= 18.47 \times 10^{-4} \text{ J}$$

$$\approx 18.48 \times 10^{-4} \text{ J}$$

**Sol2.**  $\lambda_0 = 5500 \text{ \AA}$ ,  $\phi_0 = \frac{12400}{5500} = 2.25 \text{ eV}$

Also  $\phi = 3.6 \times 10^{-19} \text{ J}$

$$\lambda_{\text{UV rays}} = 4000 \text{ \AA}$$

$$\lambda_{\text{visible}} = 5500 \text{ \AA}$$

$$\lambda_{\text{IR}} = 7000 \text{ \AA}$$

As we know that the photoelectric effect occurs only where the wavelength of the incidence wave is less than the threshold wavelength.

So, UV rays will be useful for emission.

Hence, both UV rays lamps can be used.

**Sol3.** As we know that the magnetic field is zero at the centre of the loop & it is max. at dist  $\pm \frac{R}{\sqrt{2}}$

**Sol4.** (A) Pressure gradient  $\frac{\text{Pressure}}{\text{Length}} = \frac{\text{Force}}{\text{Area} \times \text{length}} = \frac{MLT^{-2}}{L^3} = [ML^{-2}T^{-2}]$

(B) Energy density =  $\frac{\text{Energy}}{\text{Vol}} = \frac{ML^2T^{-2}}{L^3} = [ML^{-1}T^{-2}]$

(C) Electric field =  $\frac{\text{Force}}{\text{Charge}} = \frac{MLT^{-2}}{AT} = [ML^{-3}A^{-1}]$

(D) Latent heat =  $\frac{\text{Heat}}{\text{mass}} = \frac{ML^2T^{-2}}{M} = [L^2T^{-2}]$

**Sol5.**  $PV = nRT$

$\therefore n \rightarrow \text{const}, v = \text{const}$

$P \propto T$

$$\therefore \frac{P_1}{P_2} = \frac{T_1}{T_2} \Rightarrow \frac{270 \text{ kPa}}{P_2} = \frac{300}{309}$$

$$\Rightarrow P_2 = \frac{103}{100} \times 270 \text{ kPa} = 278 \text{ kPa}$$

**Sol6.** Velocity of light depends on Refractive index of medium and independent of intensity and source.

**Sol7.** In parallel, pot. difference is same.

$$\text{Hence, } H = \frac{v^2}{R} t$$

$$\Rightarrow H \propto \frac{1}{R}$$

$$\therefore \frac{H_1}{H_2} = \frac{R_2}{R_1} = \frac{3R}{R} = \frac{3}{1}$$

**Sol8.**  $\frac{\beta}{2} = \frac{a}{2} \Rightarrow \beta = a$

Also  $\beta = \frac{\lambda D}{d}$

$$\therefore a = \frac{\lambda D}{d}$$

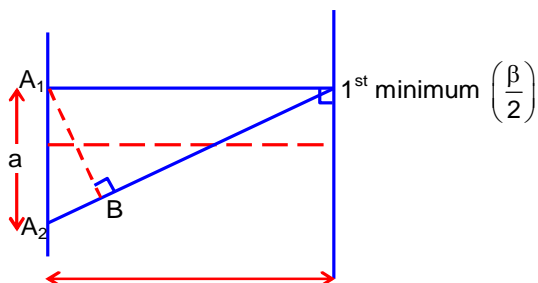
or  $\lambda D = a^2$

$$800 \times 10^{-9} \times 5 \times 10^{-2} = a^2$$

$$4 \times 10^{-8} = a^2$$

$$2 \times 10^{-4} = a$$

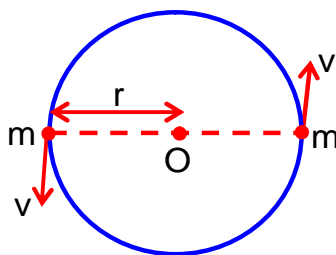
$$0.2 \text{ mm} = a$$



**Sol9.**  $F_G = F_C$

$$\frac{Gm^2}{(2r)^2} = \frac{mv^2}{r}$$

$$\Rightarrow v = \sqrt{\frac{Gm}{4r}}$$



**Sol10.**  $\left. \begin{array}{l} t_{1/2} = 30 \text{ min} \\ t = 90 \text{ min} \end{array} \right\} n = \frac{t}{t_{1/2}} = \frac{90 \text{ min}}{30 \text{ min}} = 3$

$$\therefore N = \frac{N_0}{2^n}$$

$$= \frac{N_0}{2^3}$$

$$= \frac{N_0}{8}$$

$$\therefore \frac{N}{N_0} = \frac{1}{8}$$

**Sol11.**  $\frac{kE_{\text{initial}}}{kE_{\text{max height}}} = \frac{\frac{1}{2}mu^2}{\frac{1}{2}m(u^2 \cos^2 \theta)} = \frac{1}{(\cos 30^\circ)^2} = \left(\frac{2}{\sqrt{3}}\right)^2 = \frac{4}{3}$

**Sol12.** Light emitting diode (LED) only used in forward bias.

**Sol13.**  $B_{AB} = B_{ED} = 0$  [ $\theta = 0^\circ$  ie, point along wire]

$$\vec{B}_{BC} = \frac{\mu_0 i}{4\pi \left(\frac{a}{2}\right)} \odot \quad \& \quad \vec{B}_{TE} = \frac{\mu_0 i}{4\pi \left(\frac{a}{2}\right)} \odot$$

$$\therefore B_{\text{net}} \text{ at } 0 = \frac{\mu_0 i}{\pi a} \odot$$

**Sol14.**  $\phi_s = \frac{1}{6} \times \phi_{\text{Total}}$

$$= \frac{1}{6} \times \frac{q}{\epsilon_0}$$

$$\phi_s = \frac{q}{6\epsilon_0}$$

**Sol15.**  $f_0 = 300\text{Hz}$        $V_0 = 0$        $f_0 = 300\text{Hz}$

$$\boxed{A} \rightarrow 30\text{m/s} \quad \overset{o}{\approx} \quad \text{(Observer)} \quad \boxed{B} \rightarrow 30\text{m/s}$$

$$f_A = f_0 \left( \frac{V}{V - V_A} \right) = 300 \left( \frac{300}{300 - 30} \right) = 330\text{Hz}$$

$$f_B = f_0 \left( \frac{V}{V + V_B} \right) = 300 \left( \frac{300}{300 + 30} \right) = 275\text{Hz}$$

$$\therefore \Delta f = f_A - f_B = 55\text{Hz}$$

**Sol16.** First law of thermodynamics is based on the conservation of energy.

According to this :

$$d\theta = du + dw$$

Here,  $dw \rightarrow$  work done on the system, so, vol. decreases.

So,  $dw < 0$

$$\text{or, } d\theta = dv - dw$$

**Sol17.**  $v = \sqrt{\mu g R}$

$$= \sqrt{0.34 \times 10 \times 50}$$

$$= \sqrt{170}$$

$$\approx 13\text{m/s}$$

**Sol18.**  $\therefore \phi = MI$

$$\Rightarrow \phi_2 = MI_1$$

$$\Rightarrow B_1 A_2 = MI_1$$

$$\Rightarrow M = \frac{B_1 A_2}{I_1}$$

Where  $B_1 = B$  due to square frame

$A_2 =$  Area of circle

$I_1 =$  Current in square frame

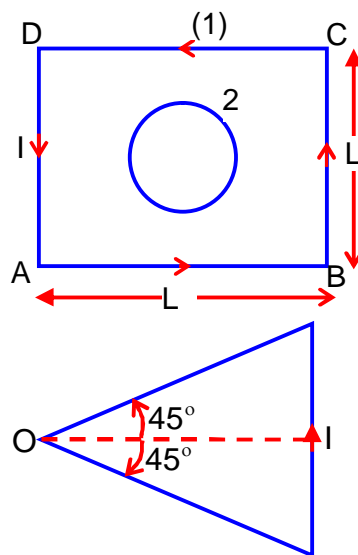
$$B_1 = 4 \times B_{AB}$$

$$= 4 \left[ \frac{\mu_0 I_1}{4\pi \frac{1}{2}} (\sin 45^\circ + \sin 45^\circ) \right]$$

$$= \frac{2\mu_0 I_1}{\pi L} \left( \frac{1}{\sqrt{2}} + \frac{1}{\sqrt{2}} \right)$$

$$B_1 = \frac{2\sqrt{2}\mu_0 I_1}{\pi L}$$

$$\begin{aligned} \text{As } M &= \frac{B_1 \cdot A_2}{I_1} \\ &= \frac{(2\sqrt{2}\mu_0 I_1)}{\pi L} \times \frac{\pi R^2}{I_1} \\ &= \frac{2\sqrt{2}\mu_0 R^2}{L} \end{aligned}$$



**Sol19.**  $h_T = h_R = 80\text{m}$  (Say  $h$ )

$$\therefore d_T = \sqrt{2Rh_T} \text{ \& } d_R = \sqrt{2R \cdot h_R}$$

$$\begin{aligned} \text{Max. line of sight} &= d_T + d_R \\ &= \sqrt{2Rh} + \sqrt{2Rh} \\ &= 2\sqrt{2Rh} \\ &= 2\sqrt{2 \times 604 \times 10^6 \times 80} \\ &= 2 \times 8 \times 4 \times 10^3 \\ &= 64000\text{m} \\ &= 64\text{Km} \end{aligned}$$

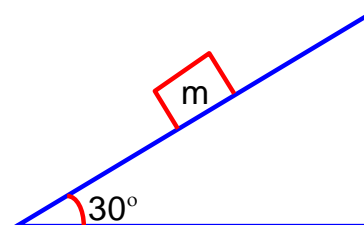
**Sol20.**  $a = g \sin \theta - \mu g \cos 30^\circ$

$$\Rightarrow \frac{g}{4} = g \sin 30^\circ - \mu g \cos 30^\circ$$

$$\Rightarrow \frac{g}{4} = \frac{g}{2} - \mu g \frac{\sqrt{3}}{2}$$

$$\Rightarrow \mu g \frac{\sqrt{3}}{2} = \frac{g}{4}$$

$$\Rightarrow \mu = \frac{1}{2\sqrt{3}}$$



SECTION – B

**Sol1.**  $h = \frac{1}{2} \times 10 \times (8)^2 = 320 \text{ m}$

$$S_{8\text{th}} = 0 + \frac{10}{2}(2 \times 8 - 1) = 75 \text{ m} \equiv \Delta h$$

$$\Delta U = mg(\Delta h) = 0.4 \times 10 \times 75 = 300 \text{ J}$$

**Sol2.**  $\vec{F}_{\text{on proton}} = 0$

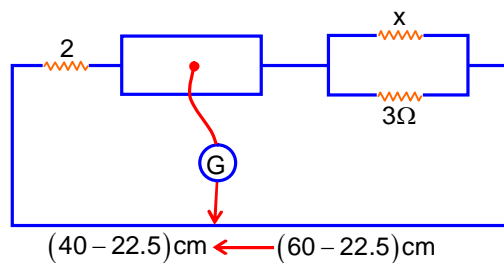
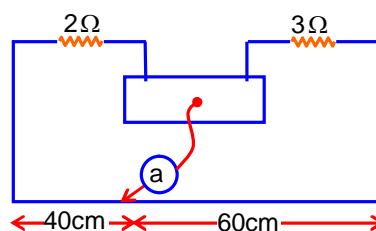
$$\Rightarrow \frac{kq_0q_0}{(x-12)^2} = \frac{k4q_0q_0}{x^2}$$

$$\Rightarrow \frac{x}{x+2} = 2 \Rightarrow x = 24 \text{ cm}$$

**Sol3.**  $\frac{2}{3} = \frac{\ell}{100 - \ell} \Rightarrow \ell = 40 \text{ cm}$

$$\frac{2}{\left(\frac{3x}{3+x}\right)} = \frac{40 + 22.5}{60 - 22.5}$$

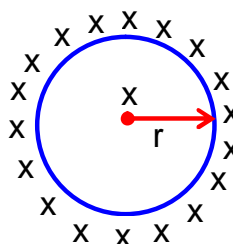
On solving we get  $x = 2 \Omega$



**Sol4.**  $\left. \begin{array}{l} \vec{B} = 0.8 \text{ T} \\ \frac{dr}{dt} = 2 \text{ cm/sec} \end{array} \right\} \text{ (Given)}$

$$\therefore |\varepsilon| = \left| \frac{d\phi}{dt} \right| = \frac{d(B \cdot A)}{dt}$$

$$\begin{aligned} \text{Or } \varepsilon &= \frac{Bd(\pi r)^2}{dt} = 2\pi r B \left( \frac{dr}{dt} \right) \\ &= 2\pi \times 0.1 \times 0.8 \times 0.02 \\ &= 32\pi \times 10^{-4} \\ &\approx 10 \text{ mV} \end{aligned}$$



**Sol5.**  $60^\circ\text{C} \xrightarrow{6\text{min}} 40^\circ\text{C} \xrightarrow{6\text{min}} T^\circ\text{C}$

$$\frac{\Delta T}{\Delta t} = k(T - T_0)$$



$$\frac{60-40}{6\text{min}} = k \left[ \frac{60+40}{2} - 10 \right]$$

$$\frac{20}{6 \times 40} = k$$

Again,  $\frac{40-7}{6} = k[T - T_0]$

$$\Rightarrow \frac{40-T}{6} = \frac{20}{6 \times 40} \left[ \frac{40+T}{2} - 10 \right]$$

$$\Rightarrow 80 - 2T = \frac{40+T-20}{2}$$

$$\Rightarrow 160 - 4T = 40 + T - 20$$

$$\Rightarrow 140 = 5T$$

$$28^\circ\text{C} = T$$

**Sol6.**  $V_1 = \sqrt{2gh} = \sqrt{2 \times 10 \times 908} = \sqrt{196} \text{ m/s} = 14 \text{ m/s}$

$$V_2 = \sqrt{2gh} = \sqrt{2 \times 10 \times 5} = 10 \text{ m/s}$$

$$\langle a \rangle = \frac{\Delta V}{\Delta t} = \frac{10 - (-14)}{0.2} = \frac{24}{0.2} = 120 \text{ m/s}^2$$

**Sol7.**  $A_1 = A$  ;  $A_2 = A$  &  $A_{\text{eq}} = A$

As we know that

$$A_{\text{eq}} = \sqrt{A_1^2 + A_2^2 + 2A_1 A_2 \cos \phi}$$

$$\Rightarrow A_{\text{eq}}^2 = A_1^2 + A_2^2 + 2A_1 A_2 \cos \phi$$

$$\Rightarrow A^2 = A^2 + A^2 + 2A A \cos \phi$$

$$\Rightarrow -A^2 = 2A^2 \cos \phi$$

$$\Rightarrow \frac{-1}{2} = \cos \phi$$

$$\Rightarrow \phi = 120^\circ$$

**Sol8.**  $m = 2 \text{ kg}$

$$kE = 2240 \text{ J}$$

$$kE = kE_T + kE_R$$

$$= \frac{1}{2}mv_0^2 + \frac{1}{2}I\omega^2$$

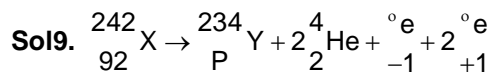
$$= \frac{1}{2}mv_0^2 + \frac{1}{2} \cdot \frac{2}{5} mR^2 \left( \frac{v_0}{R} \right)^2$$

$$\Rightarrow \frac{1}{2}mv_0^2 + \frac{1}{5}mv_0^2$$

$$\Rightarrow \frac{7}{10}mv_0^2$$

So,  $2240 \text{ J} = \frac{7}{10} \times 2 \times v_0^2$

$$V_0^2 = 1600 \Rightarrow V_0 = 40 \text{ m/s}$$



Using charge conservation :-

$$92 = P + 2(2) + (-1) + 2(1)$$

$$92 = P + 5$$

$$P = 87$$

**Sol10.** Intensity of source  $256 \text{ w/m}^2$

Intensity after passing  $P_1$  is  $I_1 = \frac{I_0}{2} = 128 \text{ w / m}^2$

Intensity after passing  $P_2$  is  $I_2 = I_1 \cos^2 \theta$

$$= 128.(\cos 60^\circ)^2$$

$$= 32 \text{ w / m}^2$$

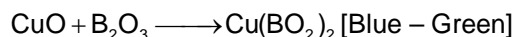
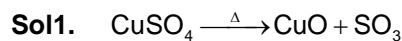
Intensity after passing  $P_3$  is  $I_3 = I_2 \cos^2 \theta$

$$= 32 \cos^2 30^\circ$$

$$= 32 \times \left(\frac{\sqrt{3}}{2}\right)^2 = 24 \text{ w / m}^2$$

## PART – B (CHEMISTRY)

### SECTION – A



**Sol2.** Increase in carbon increases boiling points . Boiling point  $\propto$  molar mass  $\propto$  Vander Wall force of attraction.

**Sol3.** More concentration of  $\text{Al}^{3+}$  is required to coagulate this negatively charged colloid.

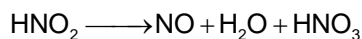
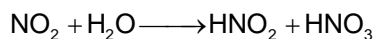
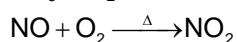
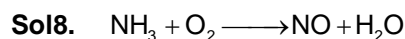
**Sol4.** Lassaigne test for both nitrogen and halogen is given by compound having C, N & X atoms.

**Sol5.** Tanks of metal alloys are used for storage of hydrogen gas.

**Sol6.**

|              |              |              |              |            |
|--------------|--------------|--------------|--------------|------------|
| Halogen:     | F–F          | Cl–Cl        | Br–Br        | I–I        |
| Bond energy: | 158.8KJ/mole | 242.6KJ/mole | 192.8KJ/mole | 151KJ/mole |

**Sol7.** Photochemical smog has high concentration of oxidizing agent like oxides of sulphur, Oxides of nitrogen, acrolein & ozone gas.



**Sol9.** For 1 mole of gas

$$\left(P + \frac{a}{V^2}\right)(V - b) = RT$$

At low pressure (point A), volume of gas is more

$$V \gg b$$

$$(V - b) \rightarrow V$$

$$\left(P + \frac{a}{V^2}\right)V = RT$$

$$\Rightarrow PV = RT - \frac{a}{V}$$

$$\Rightarrow \frac{PV}{RT} = 1 - \frac{a}{VRT} \Rightarrow Z = 1 - \frac{a}{VRT}$$

**Sol10.** Mond's process given for Ni.

**Sol11.** Liberation of hydrogen gas from dilute acidic solution requires negative value of reduction potential.

**Sol12.** Cis -  $[\text{PtCl}_2(\text{en})_2]^{2+}$  is chiral.

**Sol13.** Reagents of different name reactions are given.

**Sol14.** AAA, BBB, ABA, BAB, are 4 possible cyclic structure.

**Sol15.** Less value of  $P_{\text{Ka}}$  means stronger acid. Electron withdrawing groups increases acidic strength and electron donating groups decreases acidic strength.

**Sol16.** Application of drugs are matched with the corresponding drugs.

**Sol17.** Last line will have shortest wave length and first line will have longest wavelength.

$$\frac{1}{\lambda} = R_{\text{H}} (1)^2 \left( \frac{1}{1^2} - \frac{1}{\infty^2} \right)$$

$$\frac{1}{\lambda_{\text{He}^+}} = R_{\text{H}} (2)^2 \left( \frac{1}{2^2} - \frac{1}{3^2} \right)$$

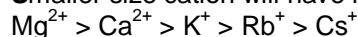
$$= R_{\text{H}} \times 4 \left( \frac{9-4}{9 \times 4} \right)$$

$$= R_{\text{H}} \times \frac{5}{9}$$

$$= \frac{5}{9} \times \frac{1}{\lambda}$$

$$\lambda_{\text{He}^+} = \frac{9\lambda}{5}$$

**Sol18.** Smaller size cation will have more hydration energy.



Sol19.



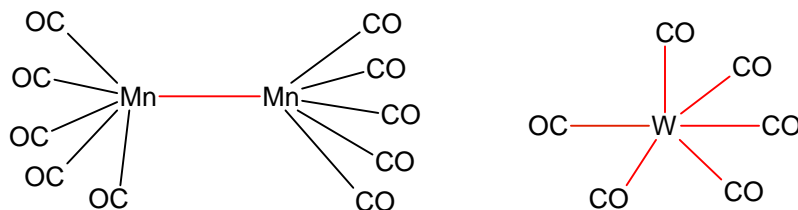
Sol20.  $\text{O}^{2-}$  &  $\text{O}_2^{2-}$  are diamagnetic.  
 $\text{O}_2^-$  is paramagnetic.

## SECTION – B

Sol1. Statement (B) &amp; (C) are correct.

Sol2.  $\text{ICl}_4^-, \text{BrF}_3$  and  $\text{NO}_2^+ \Rightarrow$  Not having odd number of electrons  
 $\text{NO}_2, \text{ClO}_2$  and  $\text{NO} \Rightarrow$  have odd number of electrons

Sol3.

Sol4.  $[\bar{\text{O}}\text{H}] = 10^{-2}\text{M}$ Number of milimole of  $\bar{\text{O}}\text{H} = 10^{-2} \times 100 = 1$ Number of milimole of  $\text{Ca}(\text{OH})_2 = \frac{1}{2} = 5 \times 10^{-1}$ Sol5.  $\text{Pb}(\text{NO}_3)_2 \longrightarrow \text{Pb}^{2+} + 2\text{NO}_3^-$  (i = 3)

$$\Delta T_b = i \times k_b \times M \Rightarrow M = 0.1$$

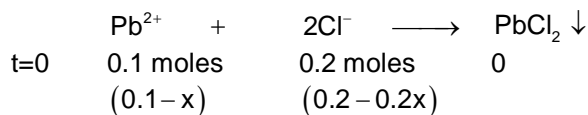
Number of moles of  $\text{Pb}^{2+} = 0.1$  $\text{NaCl} \rightarrow \text{Na}^+ + \text{Cl}^-$ , number of moles of  $\text{Cl}^- = 0.2$  moles

$$n_{\text{Pb}^{2+}} = (0.1 - x) \text{ moles}$$

$$n_{\text{Cl}^-} = (0.2 - 0.2x) \text{ moles}$$

$$n_{\text{NO}_3^-} = 0.2 \text{ moles}$$

$$n_{\text{Na}^+} = 0.2 \text{ moles}$$



$$\Delta T_f = i \times K_f \times M$$

$$\Rightarrow 0.8 = 1.8 \times \left[ \frac{(0.1 - x) + (0.2 - 0.2x) + 0.2 + 0.2}{1} \right] \Rightarrow x = 0.085$$

$$K_{\text{sp}}(\text{PbCl}_2) = [\text{Pb}^{2+}] \times [\text{Cl}^-]^2 = 13.5 \times 10^{-6}$$

**Sol6.**  $K_{eq} = \frac{K_f}{K_b} = 10$   
 $\Delta G^0 = -RT \ln K_{eq} = -5.72 \text{ KJ/mole} \sim -6 \text{ KJ/mole}$

**Sol7.**  $\text{H}_2\text{O(g)} \rightleftharpoons \text{H}_2\text{(g)} + \frac{1}{2}\text{O}_2\text{(g)}$   
 Eq.  $(1-\alpha) \quad \alpha \quad \alpha/2$   
 $K_p = \frac{\alpha \times \sqrt{\alpha/2}}{(1-\alpha)} = 2 \times 10^{-3} \Rightarrow \alpha = 2 \times 10^{-2}$   
 $\% \alpha = 2 \times 10^{-2} \times 10^2 = 2\%$

**Sol8.** 1 mole of hydrocarbon reacts with 3 moles of  $\text{H}_2$  gas.

**Sol9.**  $R_f = \frac{\text{Distance moved by substance from base line}}{\text{Distance moved by solvent from base line}} = \frac{3\text{cm}}{5\text{cm}}$

**Sol10.** Statement (A) & (C) are incorrect.

A is weak electrolyte. So its equivalent conductance at infinite dilution can't be formed by extrapolation. B is strong electrolyte. So its degree of dissociation is unity at all concentrations.

## PART – C (MATHEMATICS)

### SECTION – A

**Sol1.** Given  $A^2 = 3A + \alpha I$  .....(i)  
 $\Rightarrow (A^2 - \alpha I)^2 = 9A^2$   
 $\Rightarrow A^4 = (9 + 2\alpha)A^2 - \alpha^2 I$   
 $= (9 + 2\alpha)(3A + \alpha I) - \alpha^2 I$   
 $= (27 + 6\alpha)A + (9\alpha + \alpha^2)I$   
 Compare with equation (i) to get  $27 + 6\alpha = 21$   
 $\Rightarrow \alpha = -1$   
 $\therefore \beta = 9\alpha + \alpha^2 = -9 + 1 = -8$

**Sol2.** Radius of the circle,

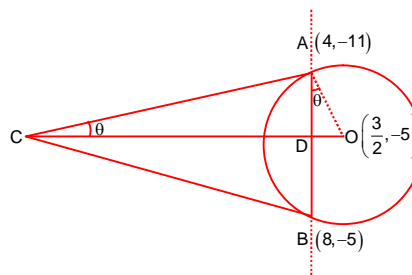
$$OA = \sqrt{\frac{9}{4} + 25 + 15} = \frac{13}{2}$$

$$AD = \frac{AB}{2} = \frac{1}{2} \sqrt{(8-4)^2 + (-5+11)^2} = \sqrt{13}$$

$$\therefore \cos \theta = \frac{AD}{AO} = \frac{\sqrt{13}}{(13/2)} = \frac{2}{\sqrt{13}}$$

$$\therefore \text{Required radius} = CD = AD \cot \theta$$

$$= \sqrt{13} \times \frac{2}{3} = \frac{2\sqrt{13}}{3}$$



**Sol3.** By separating the variables and integrating, we get

$$\int \frac{x+1}{x^2} dx = \int \frac{dy}{y}$$

$$\Rightarrow \ln|x| - \frac{1}{x} = \ln|y| + c$$

$$y(1) = e \Rightarrow 0 - 1 = \ln e + c \Rightarrow c = -2$$

$$\therefore \text{ we get } \ln|x| - \frac{1}{x} + 2 = \ln|y|$$

$$\Rightarrow |y| = e^{\ln|x| - \frac{1}{x} + 2} = |x| \cdot e^{2 - \frac{1}{x}}$$

$$\therefore \lim_{x \rightarrow 0^+} f(x) = \lim_{x \rightarrow 0^+} x \cdot e^{2 - \frac{1}{x}} = 0.0 = 0$$

**Sol4.** Condition of coplanarity :  $\begin{vmatrix} \lambda & \mu & 4 \\ -2 & 4 & -2 \\ 2 & 3 & 1 \end{vmatrix} = 0$

$$\Rightarrow 5\lambda - \mu = 28 \dots\dots\dots(i)$$

Again,  $\vec{a} \cdot \hat{b} = \pm\sqrt{54}$

$$\Rightarrow (\lambda\hat{i} + \mu\hat{j} + 4\hat{k}) \cdot \left( \frac{-\hat{i} + 2\hat{j} - \hat{k}}{\sqrt{6}} \right) = \pm\sqrt{54}$$

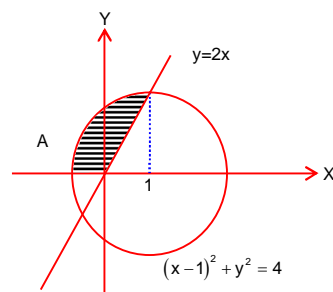
$$\Rightarrow \begin{cases} -\lambda + 2\mu - 4 = 18 \\ \text{or, } -\lambda + 2\mu - 4 = -18 \end{cases} \dots\dots\dots(ii)$$

Solve equation (i) and (ii) to get

$$(\lambda, \mu) = \left( \frac{26}{3}, \frac{46}{3} \right) \text{ or } \left( \frac{14}{3}, \frac{-14}{3} \right)$$

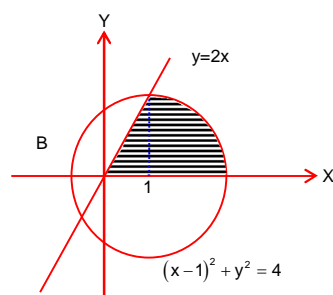
$$\therefore \sum(\lambda + \mu) = \frac{72}{3} + 0 = 24$$

**Sol5.**  $\text{Area}_A = \pi \cdot \frac{2^2}{4} - \frac{1}{2} \times 1 \times 2 = \pi - 1$



$$\text{Area}_B = \pi \cdot \frac{2^2}{4} + \frac{1}{2} \times 1 \times 2 = \pi + 1$$

$$\therefore \frac{\text{area}(A)}{\text{area}(B)} = \frac{\pi - 1}{\pi + 1}$$



**Sol6.**  $\Delta = \begin{vmatrix} \alpha & 2 & 1 \\ 2\alpha & 3 & 1 \\ 3 & \alpha & 2 \end{vmatrix} = (\alpha - 3)(\alpha + 1)$

$\Delta = 0 \Rightarrow \alpha = 3$  or  $-1$  (case of infinite or no solution)

For  $\alpha = 3$ , system becomes

$$3x + 2y + z = 1$$

$$6x + 3y + z = 1$$

$$3x + 3y + 2z = \beta$$

$$\text{Let } (3, 3, 2) = \ell(3, 2, 1) + m(6, 3, 1)$$

$$\Rightarrow m = -1, \ell = 3$$

$$\text{We get } \beta = 3 \times 1 - 1 \times 1 = 2$$

$\therefore$  For  $\alpha = 3$  and  $\beta = 2 \rightarrow$  infinite solution

For  $\alpha = 3$  and  $\beta \neq 2 \rightarrow$  No solution

Again, for  $\alpha = -1$ , system becomes

$$-x + 2y + z = 1$$

$$-2x + 3y + z = 1$$

$$3x - y + 2z = \beta$$

$$\text{Let } (3, -1, 2) = \ell(-1, 2, 1) + m(-2, 3, 1)$$

$$\Rightarrow m = -5, \ell = 7$$

$$\beta = 7(1) + (-5) = 2$$

So,  $\alpha = -1, \beta = 2 \rightarrow$  infinite solution

$\alpha = -1, \beta \neq 2 \rightarrow$  No solution

Hence the result.

**Sol7.** Numerator :

$$x - 2 > 0, x + 1 \neq 1 \text{ and } x + 1 > 0$$

$$\Rightarrow x > 2$$

$$\text{Denominator } e^{2\ln x} - 2x - 3$$

$$= x^2 - 2x - 3, x > 0$$

$$\text{Now, } x^2 - 2x - 3 = 0 \Rightarrow x = -1, 3$$

$\therefore x = 3$  must be excluded from the interval  $(2, \infty)$

**Sol8.** 3R  
7G

X = Number of rotten apples out of the 4 apples drawn one by one without replacement.

|     |   |
|-----|---|
| $x$ | $P(x)$  |
| 0   | $\frac{{}^3C_0 \cdot {}^7C_4}{{}^{10}C_4} = \frac{1}{6}$  |
| 1   | $\frac{{}^3C_1 \cdot {}^7C_3}{{}^{10}C_4} = \frac{1}{2}$  |
| 2   | $\frac{{}^3C_2 \cdot {}^7C_2}{{}^{10}C_4} = \frac{3}{10}$ |

$$3 \frac{{}^3C_3 \cdot {}^7C_1}{{}^{10}C_4} = \frac{1}{30}$$

$$\text{We've : } \sigma^2 = \Sigma x^2 P(x) - (\bar{x})^2$$

$$\Rightarrow \sigma^2 + \mu^2 = \Sigma x^2 P(x) = 0 + \frac{1}{2} + \frac{12}{10} + \frac{9}{30} = 2$$

$$\therefore 10(\mu^2 + \sigma^2) = 20$$

$$\text{Sol9. } f(\theta) = 3(\cos^4 \theta + \sin^4 \theta) - 2(1 - \sin^2 2\theta)$$

$$= 3 \left[ 1 - \frac{\sin^2 2\theta}{2} \right] - 2(1 - \sin^2 2\theta)$$

$$= 1 + \frac{1}{2} \sin^2 2\theta$$

$$\therefore f'(\theta) = \sin 4\theta = \frac{-\sqrt{3}}{2}, 0 \leq 4\theta \leq 4\pi$$

$$\Rightarrow 4\theta = \frac{4\pi}{3}, \frac{5\pi}{3}, \frac{10\pi}{3}, \frac{11\pi}{3}$$

$$\Rightarrow \theta = \frac{\pi}{3}, \frac{5\pi}{12}, \frac{5\pi}{6}, \frac{11\pi}{12}$$

$$\therefore 4\beta = \Sigma \theta = \frac{5\pi}{2}$$

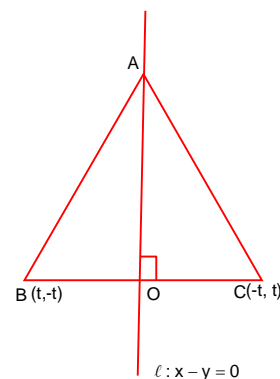
$$f\left(\frac{5\pi}{8}\right) = 1 + \frac{1}{4} \left(1 - \cos \frac{5\pi}{2}\right) = \frac{5}{4}$$

**Sol10.** A lies on the intersection of the lines  $x - y = 0$  and  $y - 2x = 2$

$$\therefore A = (-2, -2)$$

$$AO = \frac{|-2 - 2|}{\sqrt{2}} = 2\sqrt{2}$$

$$\therefore \text{Area} = \frac{8}{\sqrt{3}}$$



**Sol11.** Question dropped by NTA

$$\text{Sol12. } f(x) = x + \sin x \int_0^{\pi/2} \cos y f(y) dy + \cos x \int_0^{\pi/2} \sin y f(y) dy$$

$$= x + \ell \sin x + m \cos x (\text{say})$$

$$(i) \ell = \int_0^{\pi/2} \cos y f(y) dy = \int_0^{\pi/2} \cos y (y + \ell \sin y + m \cos y) dy$$

$$\Rightarrow \frac{\ell}{2} = \frac{\pi}{2} - 1 + \frac{m\pi}{4} \dots\dots\dots(i)$$



$$(ii) m = \int_0^{\pi/2} \sin y (y + \ell \sin y + m \cos y) dy$$

$$\Rightarrow \frac{m}{2} = 1 + \frac{\ell \pi}{4} \dots\dots\dots(ii)$$

$$\text{Solve equation (i) \& (ii) to get } \ell = \frac{8(\pi-1)}{4-\pi^2}, m = 2 + \frac{\pi(\pi-1)4}{4-\pi^2}$$

$$\therefore a = 8 - 8\pi, b = 4\pi - 2\pi^2 - 8$$

$$\therefore a + b = -2\pi(2 + \pi)$$

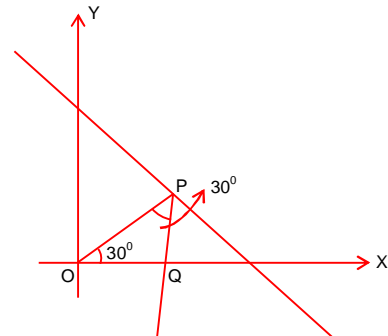
**Sol13.** Solve  $x + y = 1$  and  $y = \frac{1}{\sqrt{3}}x$  to get point of

$$\text{incidence } P = \left( \frac{\sqrt{3}}{2}(\sqrt{3}-1), \frac{\sqrt{3}-1}{2} \right)$$

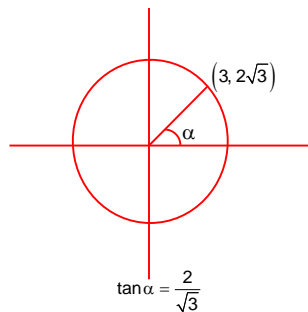
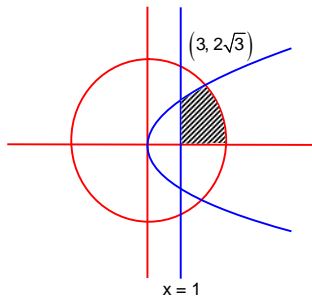
$$\Rightarrow OP = (\sqrt{3}-1)$$

$$\text{In } \triangle OPQ, \frac{OQ}{\sin 30^\circ} = \frac{OP}{\sin(120^\circ)}$$

$$\Rightarrow OQ = \frac{\sqrt{3}-1}{(\sqrt{3}/2)} \times \frac{1}{2} = \frac{2}{3+\sqrt{3}}$$



**Sol14.**



$$\Delta = 2 \times \text{Area of shaded region}$$

$$= 2 \left[ \int_1^3 2\sqrt{x} dx + \frac{21}{2}\alpha - \frac{1}{2} \times 3 \times 2\sqrt{3} \right]$$

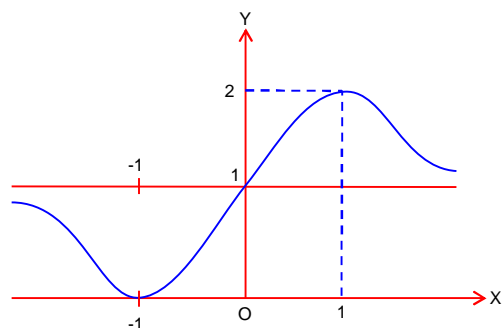
$$\Rightarrow \Delta = 2\sqrt{3} - \frac{8}{3} + 21 \sin^{-1} \left( \frac{2}{\sqrt{7}} \right)$$

$$\Rightarrow \frac{1}{2} \left( \Delta - 21 \sin^{-1} \left( \frac{2}{\sqrt{7}} \right) \right) = \sqrt{3} - \frac{4}{3}$$

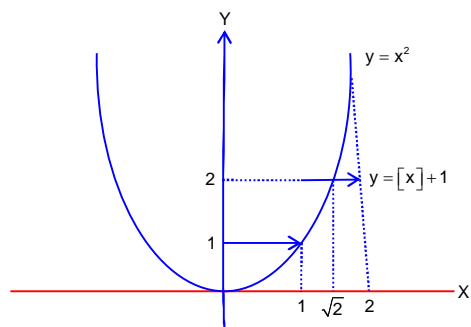
**Sol15.**

| p | q | r | $\sim p$ | $\sim q$ | $p \vee q$ | $\sim p \vee r$ | $(p \vee q) \wedge (\sim p \vee r)$ | $\sim q \vee r$ | S        |
|---|---|---|----------|----------|------------|-----------------|-------------------------------------|-----------------|----------|
| F | F | F | T        | T        | F          | T               | F                                   | T               | T        |
| F | T | F | T        | F        | T          | T               | T                                   | F               | <b>F</b> |
| T | F | F | F        | T        | T          | F               | F                                   | T               | T        |
| T | T | F | F        | F        | T          | F               | F                                   | F               | T        |
| F | F | T | T        | T        | F          | T               | F                                   | T               | T        |
| F | T | T | T        | F        | T          | T               | T                                   | T               | T        |
| T | F | T | F        | T        | T          | T               | T                                   | T               | T        |
| T | T | T | F        | F        | T          | T               | T                                   | T               | T        |

**Sol16.**  $f(x) = 1 + \frac{2x}{x^2 + 1}$   
 $\therefore f(x)$  is one-one in  $[1, \infty)$  but not in  $(-\infty, \infty)$



**Sol17.**  $I = \int_0^1 f(x) dx + \int_1^{\sqrt{2}} f(x) dx + \int_{\sqrt{2}}^2 f(x) dx$   
 $= \int_0^1 1 dx + \int_1^{\sqrt{2}} 2 dx + \int_{\sqrt{2}}^2 x^2 dx$   
 $= 1 + 2(\sqrt{2} - 1) + \frac{8 - 2\sqrt{2}}{3} = \frac{5}{3} + \frac{4\sqrt{2}}{3}$



**Sol18.**  $14x^2 - 31x + 3\lambda = 0$  .....(i)  
 $35x^2 - 53x + 4\lambda = 0$  .....(ii)  
Equation (i)  $\times 5$  - equation (ii)  $\times 2$ , we get  
 $x = \frac{\lambda}{7}$

Substitute in equation (i),  $14 \cdot \left(\frac{\lambda}{7}\right)^2 - 31\left(\frac{\lambda}{7}\right) + 3\lambda = 0$

$\Rightarrow \lambda = 0, 5.$

Given,  $\lambda \neq 0$

$\Rightarrow \lambda = 5.$

$\therefore$  we get  $\alpha = \frac{5}{7}, \beta = \frac{3}{2}$  and  $\gamma = \frac{4}{5}$

$\therefore \frac{3\alpha}{\beta} = \frac{10}{7}$  and  $\frac{4\alpha}{\gamma} = \frac{25}{7}$

$\therefore$  required quadratic equation is  $49x^2 - 245x + 250 = 0$

**Sol19.** We've,  $2p + q = -4$

$$\text{Let } (x - 2p)^2 = \theta$$

$$\therefore f(x) = \frac{1 - \cos \theta}{\theta^2}$$

$$\therefore \lim_{x \rightarrow 2p^+} [f(x)] = \lim_{\theta \rightarrow 0^+} \left[ \frac{1 - \cos \theta}{\theta^2} \right] = \lim_{\theta \rightarrow 0^+} \left[ \frac{2 \sin^2 \theta / 2}{\theta^2} \right] = \left[ \frac{1}{2} \right] = 0$$

**Sol20.** Let  $z_1 = x_1 + iy_1$  and  $z_2 = x_2 + iy_2$

$$\therefore \operatorname{Re}(z_1 z_2) = 0 \Rightarrow x_1 x_2 = y_1 y_2 \dots\dots\dots(i)$$

$$\operatorname{Re}(z_1 + z_2) = 0 \Rightarrow x_1 + x_2 = 0 \dots\dots\dots(ii)$$

$$\text{From (i) and (ii), } y_2 = \frac{-x_1^2}{y_1}$$

Statement (ii) & (iii) are possible.

## SECTION - B

**Sol1.**

| <u>Form</u>           | <u>Total number</u>              |
|-----------------------|----------------------------------|
| 7                     | $5^4 = 625$                      |
| 5                     | $5^4 = 625$                      |
| 3    7                | $5^3 = 125$                      |
| 3    5                | $2 \times 5 \times 5 = 50$       |
| 3    5    3           | $2 \times 5 = 10$                |
| 3    5    3    3    7 | 1                                |
|                       | 1436 $\rightarrow$ rank of 35337 |

**Sol2.**  $f(x) = cx$

$$f(1) = \frac{1}{5} \Rightarrow c = \frac{1}{5} \Rightarrow f(x) = \frac{x}{5}$$

$$\sum_{n=1}^m \frac{f(n)}{n(n+1)(n+2)} = \frac{1}{12}$$

$$\Rightarrow \sum_{n=1}^m \frac{1}{n+1} - \frac{1}{n+2} = \frac{5}{12}$$

$$\Rightarrow \frac{1}{2} - \frac{1}{m+2} = \frac{5}{12} \Rightarrow m = 10$$

**Sol3.**  $f(x+y) = f(x) + f(y) - 1$

Partially differentiate wrt x to get

$$f'(x+y) = f'(x)$$

$$\text{Put } x = 0 : f'(y) = f'(0) = 2$$

$$\Rightarrow f(x) = 2x + c$$

$$f(0) = 2f(0) - 1 \Rightarrow f(0) = 1 \Rightarrow c = 1$$

$$\therefore f(x) = 2x + 1 \Rightarrow |f(-2)| = 3$$

**Sol4.** Given plane :  $ax + by + 3z = 2(a + b)$

$$\therefore \text{We've } -10a + 8b = 2(a + b)$$

$$\Rightarrow b = 2a \dots\dots\dots(i)$$

$$\text{Also, } a - 2b + 3 = 0$$

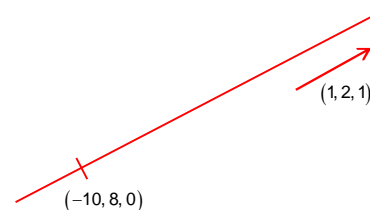
$$\Rightarrow 2b = a + 3 \dots\dots\dots(ii)$$

$$\text{From (i) \& (ii), } a = 1, b = 2$$

$$\therefore \text{ plane is } x + 2y + 3z = 6$$

$$c = \frac{|1 + 54 + 21 - 6|}{\sqrt{1 + 4 + 9}} = 5\sqrt{14}$$

$$\therefore a^2 + b^2 + c^2 = 355$$



**Sol5.** In the first expression,

$$T_{r+1} = {}^{11}C_r (\alpha x^3)^{11-r} \cdot \left(\frac{1}{\beta x}\right)^r$$

$$= {}^{11}C_r \cdot \alpha^{11-r} \cdot \beta^{-r} \cdot x^{33-4r}$$

$$\text{for } x^9, 33 - 4r = 9 \Rightarrow r = 6$$

$$\therefore T_7 = {}^{11}C_6 \cdot \alpha^5 \cdot \beta^{-6} \cdot x^9$$

Similarly, in the second expression,

$$T_{r+1} = (-1)^r \cdot {}^{11}C_r \alpha^{11-r} \cdot \beta^{-r} \cdot x^{11-4r}$$

$$\text{for } x^{-9}, 11 - 4r = -9 \Rightarrow r = 5$$

$$\therefore T_6 = -{}^{11}C_5 \alpha^6 \beta^{-5} \cdot x^{-9}$$

$$\therefore \text{ we get } {}^{11}C_6 \cdot \alpha^5 \beta^{-6} = -{}^{11}C_5 \cdot \alpha^6 \beta^{-5}$$

$$\Rightarrow \alpha\beta = -1 \Rightarrow (\alpha\beta)^2 = 1$$

**Sol6.** B, C lies on  $\frac{x+\alpha}{5} = \frac{y-1}{2} = \frac{z+4}{3} = t$

$$\text{Let D be } (-\alpha + 5t, 1 + 2t, -4 + 3t)$$

$$\overrightarrow{AD} = (-\alpha + 5t, -1 + 2t, -4 + 3t - \alpha)$$

$$\overrightarrow{AD} \cdot \overrightarrow{BC} = 0 \Rightarrow \overrightarrow{AD} \cdot (5, 2, 3) = 0$$

$$\Rightarrow -5\alpha + 25t - 2 + 4t - 12 + 9t - 3\alpha = 0$$

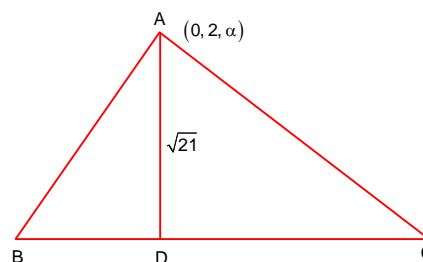
$$\Rightarrow t = \frac{4\alpha + 7}{19} \dots\dots\dots(i)$$

$$\text{We've, } \frac{1}{2} \times AD \times BC = \Delta \Rightarrow AD = \frac{2\Delta}{BC} = \sqrt{21}$$

$$|AD|^2 = 21 \Rightarrow (5t - \alpha)^2 + (2t - 1)^2 + (3t - 4 - \alpha)^2 = 21$$

$$\Rightarrow 3\alpha^2 + 20\alpha - 87 = 0 \text{ (using (i))}$$

$$\Rightarrow \alpha = 3, \frac{-58}{6}$$



But  $\alpha \in \mathbb{Z}$ ,

$$\Rightarrow \alpha = 3$$

$$\Rightarrow \alpha^2 = 9$$

**Sol7.**

Form

Total number (satisfying given constraint)

|   |  |  |  |  |  |
|---|--|--|--|--|--|
| 1 |  |  |  |  |  |
|---|--|--|--|--|--|

$${}^8C_5 = 56$$

|   |   |  |  |  |  |
|---|---|--|--|--|--|
| 2 | 3 |  |  |  |  |
|---|---|--|--|--|--|

$${}^6C_4 = 15$$

$\therefore$  71<sup>st</sup> number is 23 67 89

$\therefore$  72<sup>nd</sup> number is 24 56 78

$\Rightarrow$  sum of digits = 32

**Sol8.** Coplanarity  $\Rightarrow \begin{vmatrix} \lambda - 1 & -2 & 3 \\ -2 & 3 & -4 \\ 1 & -3 & 5 \end{vmatrix} = 0$

$$\Rightarrow \lambda = 2$$

**Sol9.**  $a > 0, r > 1$

$$ar^3 \cdot ar^5 = 9$$

$$\Rightarrow a^2 r^8 = 9$$

$$\Rightarrow ar^4 = 3$$

$$\text{Again } ar^4 + ar^6 = 24$$

$$\Rightarrow ar^4(1 + r^2) = 24$$

$$\Rightarrow 1 + r^2 = 8 \Rightarrow r^2 = 7$$

$$\Rightarrow a = \frac{3}{49}$$

$$\text{Now, } a_1 a_9 + a_2 a_4 a_9 + a_5 + a_7$$

$$= a^2 r^8 + a^3 \cdot r^{12} + ar^4 + ar^6 = 60$$

**Sol10.**  ${}^nC_{r-1} \cdot 2^{r-1} : {}^nC_r \cdot 2^r : {}^nC_{r+1} \cdot 2^{r+1} = 2 : 5 : 8$

$$\Rightarrow \frac{n-r+1}{r} = \frac{5}{4} \text{ and } \frac{n-r}{r+1} = \frac{4}{5}$$

Solve to get  $n = 8, r = 4$ .

$\therefore$  The middle of these terms

$$= {}^nC_r \cdot 2^r = {}^8C_4 \cdot 2^4 = 1120$$