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JEE (Main)

PAPER-1 (B.E./B. TECH.)

2022

COMPUTER BASED TEST (CBT)
Memory Based Questions & Solutions

Date: 29 July, 2022 (SHIFT-2) | TIME : (3.00 p.m. to 6.00 p.m)

Duration: 3 Hours | Max. Marks: 300

SUBJECT: MATHEMATICS

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PART : MATHEMATICS

1. Domain of $f(x) = \sin^{-1}\left(\frac{x^2-3x+2}{x^2+2x+7}\right)$ is

(1) $[-1, \infty)$ (2) $[-1, 1]$ (3) $(-\infty, 1]$ (4) $[-2, 0)$

Ans. (1)

Sol. $-1 \le \frac{x^2-3x+2}{x^2+2x+7} \le 1$

$-x^2 - 2x - 7 \le x^2 - 3x + 2 \le x^2 + 2x + 7$
$0 \le 2x^2 - x + 9$ and $0 \le 5x + 5$
Domain = $[-1, \infty)$

2. If $f(x) = \begin{cases} \dfrac{\ln(1+5x) - \ln(1-\alpha x)}{x} & x \ne 0 \\ 10 & x = 0 \end{cases}$ is continuous at x = 0, then value of 'α' is equal to

(1) 5 (2) –5 (3) 45 (4) 0

Ans. (1)

Sol. f(x) is continuous at x = 0 so

$\lim_{x \to 0} \dfrac{\ln(1+5x) - \ln(1-\alpha x)}{x} = 10$

$\Rightarrow \lim_{x \to 0} \left(\dfrac{\ln(1+5x)}{x} - \dfrac{\ln(1-\alpha x)}{x} \right) = 10$

$\Rightarrow 5 + \alpha = 10$

$\Rightarrow \alpha = 5$

3. The value of $\sum_{r=1}^{20} (r^2 + 1)\, r!$ is equal to

(1) (22)! (2) 20.20! (3) 20. 21! (4) 21!

Ans. (3)

Sol. $\sum_{r=1}^{20} (r^2 + 1)\, r! = \sum_{r=1}^{20} \left((r+1)^2 r! - 2r.r!\right)$

$= \sum_{r=1}^{20} \left((r+1)(r+1)! - r.r!\right) - \sum_{r=1}^{20} r.r!$

$= (21 . 21! - 1) - \sum_{r=1}^{20} \left((r+1)! - r!\right)$

$= (21. 21! - 1) - [21! - 1]$

$= 20. (21)!$

4. If $|\vec{a} + \vec{b}|^2 = |\vec{a}|^2 + 2|\vec{b}|^2$, $\vec{a}.\vec{b} = 3$ and $|\vec{a} \times \vec{b}| = 75$ then the value of $|\vec{a}|^2$ is

Ans. (14)

Sol. $|\vec{a} + \vec{b}|^2 = |\vec{a}|^2 + |\vec{b}|^2 + 2\vec{a}.\vec{b} \Rightarrow |\vec{b}|^2 = 2\vec{a}.\vec{b} \Rightarrow |\vec{b}|^2 = 6$ ______(1)

$|\vec{a} \times \vec{b}|^2 = |\vec{a}|^2 |\vec{b}|^2 - (\vec{a}.\vec{b})^2 = 75 \Rightarrow 6|\vec{a}|^2 - 9 = 75$

$= 6|\vec{a}|^2 = 84 \Rightarrow |\vec{a}|^2 = 14$

5. The value of $\int \dfrac{\sec^2 x - 2022}{(\sin x)^{2022}} dx$

(1) $(\sin x)^{-2021} \tan x + C$ (2) $(\sin x)^{-2022} \cot x + C$
(3) $(\sin x)^{-2022} \tan x + C$ (4) $(\sin x)^{-2023} \tan x + C$

Ans. (3)

Sol. $I = \int (\sin x)^{-2022} \sec^2 x\, dx - 2022 \int (\sin x)^{-2022} dx$

$I = (\sin x)^{-2022} \tan x - \int (-2022)(\sin x)^{-2023} . \cos x . \tan x\, dx - 2022 \int (\sin x)^{-2022} dx$

$I = (\sin x)^{-2022} \tan x + 2022 \int (\sin x)^{-2022} dx - 2022 \int (\sin x)^{-2022} dx$

$\Rightarrow I = (\sin x)^{-2022} \tan x + C$

6. If system of equations

x + y + z = 6

2x + 8y + αz = β

and x + 2y + 3z = 14

has infinitely many solutions then value of $(\alpha + \beta)$ is

(1) 46 (2) – 74 (3) 74 (4) 84

Ans. (3)

Sol. For infinitely many solution's $D = D_1 = D_2 = D_3 = 0$

$$D = \begin{vmatrix} 1 & 1 & 1 \\ 2 & 8 & \alpha \\ 1 & 2 & 3 \end{vmatrix} = 0$$

$\Rightarrow$ $1(24 - 2\alpha) - 1(6 - \alpha) + 1(4 - 8) = 0$

$\Rightarrow$ $24 - 2\alpha - 6 + \alpha - 4 = 0$

$\Rightarrow$ $14 = \alpha$

$$D_1 = \begin{vmatrix} 6 & 1 & 1 \\ \beta & 8 & 14 \\ 14 & 2 & 3 \end{vmatrix} = 0$$

$\Rightarrow$ $6(24 - 28) - 1(3\beta - 196) + 1(2\beta - 112) = 0$

$\Rightarrow$ $-24 - 3\beta + 196 + 2\beta - 112 = 0$

$\Rightarrow$ $\beta = 60$

So $\alpha + \beta = 74$

7. If roots of $x^2 - x + 4 = 0$ are α and β such that $P_n = \alpha^n - \beta^n$, then which of the following is correct.

(1) $P_{15} = P_{14} + 4P_{13}$ (2) $P_{15} = P_{13} - 4P_{14}$

(3) $P_{13} = P_{15} - 4P_{14}$ (4) $P_{15} = P_{14} - 4P_{13}$

Ans. (4)

Sol. $\alpha^2 - \alpha + 4 = 0 \Rightarrow \alpha^n - \alpha^{n-1} + 4\alpha^{n-2} = 0$ ....(1)

Similarly $\beta^n - \beta^{n-1} + 4\beta^{n-2} = 0$ .... (2)

Subtract (2) from (1)

$\Rightarrow P_n - P_{n-1} + 4P_{n-2} = 0$

Put n = 15 $\Rightarrow P_{15} = P_{14} - 4P_{13}$

8. If $\left|z - \frac{1}{z}\right| = 2$ then maximum value of $|z|$ is

(1) $\sqrt{3-2\sqrt{2}}$ (2) $\sqrt{3+2\sqrt{2}}$ (3) $\sqrt{3-3\sqrt{2}}$ (4) $\sqrt{3+3\sqrt{2}}$

Ans. (2)

Sol. $\left||z_1| - |z_2|\right| \le |z_1 - z_2|$

$\left||z| - \frac{1}{|z|}\right| \le 2$ Let $|z| = t$

$\left|t - \frac{1}{t}\right| \le 2$

Squaring both sides

$t^4 - 6t^2 + 1 \le 0$

$(t^2 - 3)^2 - \left(2\sqrt{2}\right)^2 \le 0$

$t \in \left[\sqrt{3-2\sqrt{2}}, \sqrt{3+2\sqrt{2}}\right]$

Hence $|z|_{max} = \sqrt{3+2\sqrt{2}}$

9. If $\sum_{k=1}^{10} k^2\left({}^{10}C_k\right)^2 = 22000\,L$ then L is equal to

(1) 220 (2) 187 (3) 221 (4) 143

Ans. (3)

Sol. $\sum_{k=1}^{10} \left(k\ {}^{10}C_k\right)^2 = \sum_{k=1}^{10} (10)^2\left({}^{9}C_{k-1}\right)^2$

$= 100\left[\left({}^9C_0\right)^2 + \left({}^9C_1\right)^2 ....\left({}^9C_9\right)^2\right]$

$= 100\ {}^{18}C_9 = 22000\,L$

$= \frac{18!}{9!\,9!} = 220\,L \Rightarrow L = 17 \times 13 = 221$

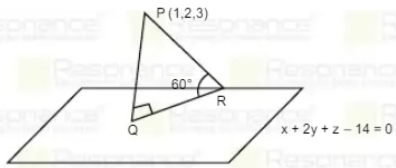
10. Foot of perpendicular drawn from a point P (1,2,3) to the plane x + 2y + z = 14 is Q. If a point R be on given plane such that $\angle PRQ = 60°$ then area of ΔPQR is

(1) $\sqrt{5}$ square units (2) $\sqrt{3}$ square units

(3) $\sqrt{6}$ square units (4) $\sqrt{2}$ square units

Ans. **(2)**

Sol.



$PQ = \left|\frac{1+4+3-14}{\sqrt{6}}\right| = \sqrt{6}$

In Δ PRQ $\tan 60° = \frac{PQ}{QR} = \frac{\sqrt{6}}{QR} \Rightarrow QR = \sqrt{2}$

Area of $\Delta PQR = \frac{1}{2}$ (PQ) (QR) $= \frac{1}{2}\left(\sqrt{6}\right)\left(\sqrt{2}\right)$

$= \sqrt{3}$ square units

11. Find number of solutions of equation $2\cos\left(\frac{x^2+x}{6}\right) \geq 4^x + 4^{-x}$ is

(1) 0 (2) 1 (3) 3 (4) infinite

Ans. **(2)**

Sol. $\cos\left(\frac{x^2+x}{6}\right) \geq \frac{4^x+4^{-x}}{2}$ ________(1)

Here $\cos\left(\frac{x^2+x}{6}\right) \in [-1,1]$

$\frac{4^x+4^{-x}}{2} \in [1, \infty)$

It is possible only x = 0.

So number of solutions is 1.

12. If sum and product of mean & variance of binomial probability distribution are 82.5 and 1350 respectively then number of trial is

(1) 90 (2) 92 (3) 96 (4)95

Ans. **(3)**

Sol. Mean = np, Variance = npq

np + npq = 82.5 $\Rightarrow$ np (1 +.q) = 82.5 ....(1)

np. npq = 1350 $\Rightarrow$ $n^2p^2.q = 1350$ ....(2)

from (1) & (2)

$\frac{q}{(1+q)^2} = \frac{24}{121}$

$24q^2 - 73q + 24 = 0$

$(3q - 8)(8q - 3) = 0$

$q = \frac{8}{3}$ or $\frac{3}{8} \Rightarrow q = \frac{3}{8}$ $(\because q \leq 1)$

$\Rightarrow p = 1 - q = \frac{5}{8}$

n = 96 Ans.

13. If y = f(x) is satisfying differential equation $\frac{dy}{dx} = \frac{x+y-2}{x-y}$ such that y = f(x) is passing through (1,0) and (k + 1, 2) then which of the following is correct.

(1) $\ln(2 + k^2) = 2\tan^{-1}\left(\frac{1}{k}\right) + \pi$ (2) $\ln(1 + k^2) = 2\tan^{-1}\left(\frac{1}{k}\right) + \pi$

(3) $\ln(1 + k^2) = 2\tan^{-1}\left(\frac{1}{k}\right) - \pi$ (4) $\ln(2 + k^2) = 2\tan^{-1}\left(\frac{1}{k}\right) - \pi$

Ans. (2)

Sol. $\frac{dy}{dx} = \frac{x+y-2}{x-y}$ ______(1)

put x = X + h, y = Y + k

$\frac{dY}{dX} = \frac{X+Y}{X-Y}$ ______(2) where h + k – 2 = 0 and h – k = 0

$\Rightarrow$ h = k = 1

equation (2) is homogenous differential equation so

put Y = vX in equation (2)

$v + X\frac{dv}{dX} = \frac{1+v}{1-v} \Rightarrow \frac{v-1}{v^2+1}dv = \frac{-1}{X}dX$ ______(3)

Integrate

$\ln(v^2+1) - 2\tan^{-1}v = -2\ln X + c$

$\ln(v^2X^2 + X^2) = 2\tan^{-1}v + c$

$\ln(Y^2 + X^2) = 2\tan^{-1}\left(\frac{Y}{X}\right) + c$

$\ln\left[(x-1)^2 + (y-1)^2\right] = 2\tan^{-1}\left(\frac{y-1}{x-1}\right) + c$ ______(4)

It is passing through(1, 0) so c = π

$\Rightarrow \ln((x-1)^2 + (y-1)^2) = 2\tan^{-1}\left(\frac{y-1}{x-1}\right) + \pi$ _____(5)

Also it is passing through (k + 1, 2) so

$\ln(1 + k^2) = 2\tan^{-1}\left(\frac{1}{k}\right) + \pi$

14. $\vec{a}, \vec{b}, \vec{c}$ are three equally inclined , co-planer and con-current vectors such that $|\vec{a}||\vec{b}||\vec{c}| = 14$ and $(\vec{a}\times\vec{b}).(\vec{b}\times\vec{c}) + (\vec{b}\times\vec{c}).(\vec{c}\times\vec{a}) + (\vec{c}\times\vec{a}).(\vec{a}\times\vec{b}) = 168$ then value of $|\vec{a}| + |\vec{b}| + |\vec{c}|$ is

Ans. (16)

Sol. Since $\vec{a}, \vec{b}, \vec{c}$ are con current and co-planer and equally inclined.

$\vec{a} \wedge \vec{b} = \vec{b} \wedge \vec{c} = \vec{a} \wedge \vec{c} = 2\pi/3$

$\vec{a}\times\vec{b} \,||\, \vec{b}\times\vec{c} \,||\, \vec{c}\times\vec{a}$

$(\vec{a}\times\vec{b}).(\vec{a}\times\vec{c}) = |\vec{a}\times\vec{b}||\vec{b}\times\vec{c}|\cos(0^\circ)$

$= \left(|\vec{a}||\vec{b}|\sin\frac{2\pi}{3}\right)\left(|\vec{b}||\vec{c}|\sin(\frac{2\pi}{3})\right)$

$= |\vec{a}||\vec{b}|^2|\vec{c}| \times 3/4$

Similarly $(\vec{b}\times\vec{c})(\vec{c}\times\vec{a}) = \frac{3}{4}|\vec{a}||\vec{b}||\vec{c}|^2$

$(\vec{c}\times\vec{a}).(\vec{a}\times\vec{b}) = \frac{3}{4}|\vec{a}|^2|\vec{b}||\vec{c}|$

Now

$(\vec{a}\times\vec{b}).(\vec{b}\times\vec{c}) + (\vec{b}\times\vec{c})(\vec{c}\times\vec{a}) + (\vec{c}\times\vec{a}).(\vec{c}\times\vec{a}).(\vec{a}\times\vec{b}) = 168$

$\frac{3}{4}|\vec{a}||\vec{b}||\vec{c}|(|\vec{a}| + |\vec{b}| + |\vec{c}|) = 168$

$|\vec{a}| + |\vec{b}| + |\vec{c}| = 168 \times \frac{3}{4} \times \frac{1}{14} = 16$

15. Number of integers lying between 1024 and 23416 which are divisible by 55 and formed by digits 2,3, 4, 5, 6 without repetition of digits, is

Ans. **(6)**

Sol. Case I when number is four digit number

a	b	c	5

Divisibly by 11

a + c = b + 5

When b = 2   a = 3, c = 4
              a = 4, c = 3

b = 3   a = 2, c = 6
         a = 6, c = 2

b = 4   a = 3, c = 6
         a = 6, c = 3

So possible number of integers is 6.

Case II No five digit integer is possible since least 5 digit integers is 23456 which is grater than 23416

16. If the curve y = y (x) satisfies $\frac{dy}{dx}+\left(\frac{2x^2+11x+13}{x^3+6x^2+11x+6}\right)y = \frac{x+3}{x+1}$ and passes through point (0,1) then the value of y(1) is

(1) $\frac{1}{2}$ (B) $\frac{3}{2}$ (C) $\frac{5}{2}$ (D) $\frac{7}{2}$

Ans. **(2)**

Sol. $\frac{dy}{dx}+\frac{(2x^2+11x+13)}{(x+1)(x+2)(x+3)}\,y = \frac{x+3}{x+1}$

I.F. = $e^{\int\frac{2x^2+11x+13}{(x+1)(x+2)(x+3)}dx} = e^{\int\left(\frac{2}{x+1}+\frac{1}{x+2}-\frac{1}{x+3}\right)dx}$

$e^{2\ln(x+1)+\ln(x+2)-\ln(x+3)} = e^{\ln\frac{(x+1)^2(x+2)}{(x+3)}}$

I.F = $\frac{(x+1)^2(x+2)}{(x+3)}$

Curve $y.\frac{(x+1)^2\,(x+2)}{(x+3)} = \int\frac{(x+1)^2\,(x+2)}{(x+3)}\cdot\frac{(x+3)}{(x+1)}\,dx + c$

$= \int\left(x^2+3x+2\right)dx+c$

$\Rightarrow \quad y\,\frac{(x+1)^2(x+2)}{(x+3)} = \frac{x^3}{3}+\frac{3x^2}{2}+2x+c$

$\because$ Passes through (0, 1) $\Rightarrow$ c = $\frac{2}{3}$

Curve $\to y\,\frac{(x+1)^2\,(x+2)}{(x+3)} = \frac{x^3}{3} + \frac{3x^3}{2} + 2x + \frac{2}{3}$

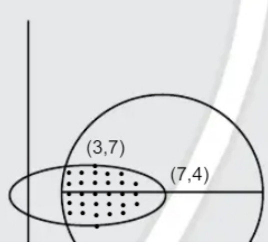
for x = 1 $\Rightarrow$ y = $\frac{3}{2}$

17. Let $A=\left\{(x,y):(x-7)^2+(y-4)^2\le\frac{169}{4}\ \&\ x,y\in R\right\}$ and $B=\left\{(x,y):\frac{(x-3)^2}{16}+\frac{(y-4)^2}{9}\le 1\ \&\ x,y\in N\right\}$ be two sets then number of elements in $A\cap B$ is

(1) 30   (2) 31   (3) 32   (4) 33

Ans. **(2)**

Sol.



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