GATE 2022

Civil Engineering

Shift-2

Questions with Detailed Solution

1. The movie was funny and I…….

A. could help laughing

B. couldn't help laughed

C. couldn't help laughing

D. could helped laughed

[MCQ: 1 Mark]

Ans. (C)

Sol. The movie was funny and I **couldn't help laughing.**

2. $x : y : z = \frac{1}{2} : \frac{1}{3} : \frac{1}{4}$

What is the value of $\frac{x + z - y}{y}$?

A. 0.75 B. 1.25

C. 2.25 D. 3.25

[MCQ: 1 Mark]

Ans. (B)

Sol.

$$x : y : z = \frac{1}{2} : \frac{1}{3} : \frac{1}{4}$$

$$\Rightarrow \frac{x}{y} = \frac{1/2}{1/3} = \frac{3}{2}$$

$$\Rightarrow \frac{y}{z} = \frac{1/3}{1/4} = \frac{4}{3}$$

$$\frac{x + z - y}{y} = \frac{x}{y} + \frac{z}{y} - 1$$

$$= \frac{3}{2} + \frac{3}{4} - 1$$

$$= \frac{5}{4} = 1.25$$

3. Both the numerator and the denominator of $\frac{3}{4}$ are increased by a positive integer, x, and those of $\frac{15}{17}$ are decreased by the same integer. This operation results in the same value for both the fractions. What is the value of x?

A. 1 B. 2

C. 3 D. 4

[MCQ: 1 Mark]

Ans. (C)

Sol.

$$\frac{3 + x}{4 + x} = \frac{15 - x}{17 - x}$$

$$\Rightarrow (x + 3)(x - 17) = (x - 15)(x + 4)$$

$$\Rightarrow x^2 - 14x - 51 = x^2 - 11x - 60$$

$$\Rightarrow 3x - 9 = 0$$

$$x = 3$$

4. A survey of 450 students about their subjects of interest resulted in the following outcome.

- 150 students are interested in Mathematics.
- 200 students are interested in Physics.
- 175 students are interested in Chemistry.
- 50 students are interested in Mathematics and Physics.
- 60 students are interested in Physics and Chemistry.
- 40 students are interested in Mathematics and Chemistry.
- 30 students are interested in Mathematics, Physics and Chemistry.
- Remaining students are interested in Humanities.

Based on the above information, the number of students interested in Humanities is

A. 10 B. 30

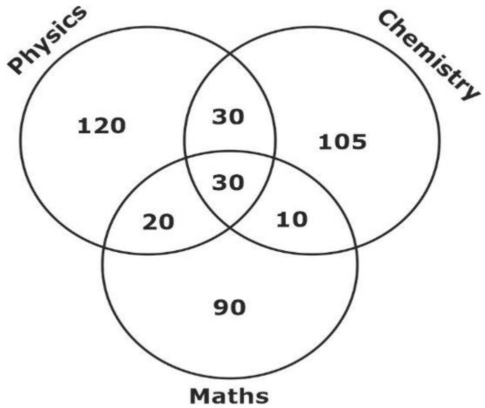
C. 40 D. 45

[MCQ: 1 Mark]

Ans. (D)

Sol.

The given data can be shown in the venn diagram as below:–



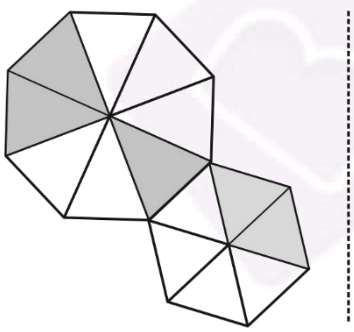
Total no. of students interested in physics, chemistry and maths all together

= 120 + 20 + 30 + 30 + 105 + 10 + 90 = 405.

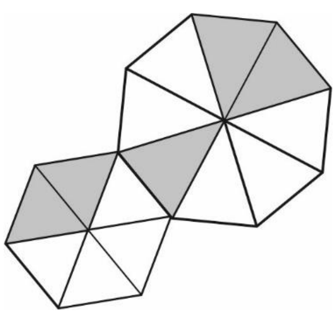
Total no. of students = 450

∴ No. of students interested in humanities = 450–405 = 45

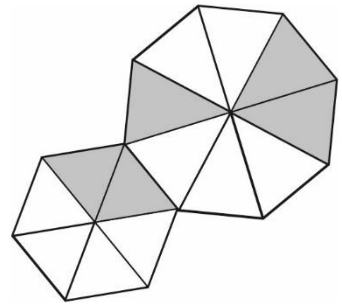
5. For the picture shown above, which one of the following is the correct picture representing reflection with respect to the mirror shown as the dotted line?



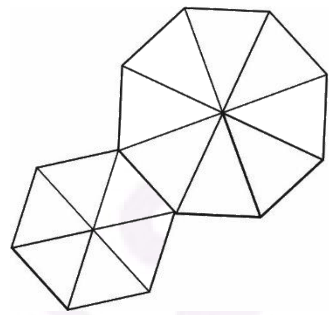
A.



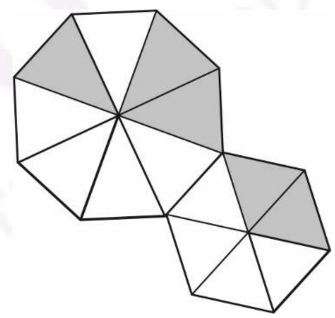
B.



C.



D.

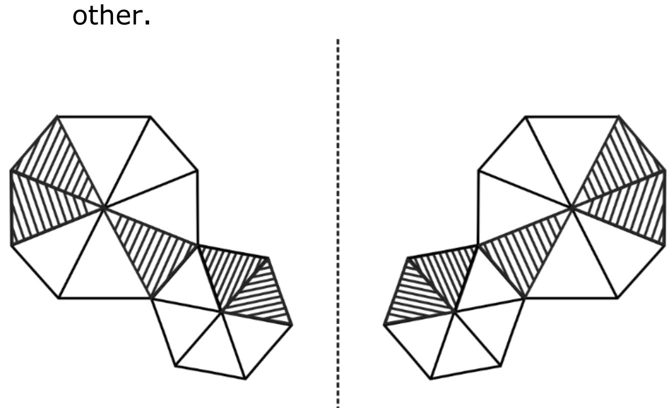


[MCQ: 1 Mark]

Ans. (A)

Sol.

The image will be symmetrical about the dotted line so that on folding about the dotted line, both pictures overlap each other.



6. In the last few years, several new shopping malls were opened in the city. The total number of visitors in the malls is impressive. However, the total revenue generated through sales in the shops in these malls is generally low.

Which one of the following is the CORRECT logical inference based on the information in the above passage?

A. Fewer people are visiting the malls but spending more

B. More people are visiting the malls but not spending enough

C. More people are visiting the malls and spending more

D. Fewer people are visiting the malls and not spending enough

Ans. (B)

[MCQ: 2 Mark]

Sol.

The inference drawn from the passage is that more people are visiting the malls but not spending enough.

7. In a partnership business the monthly investment by three friends for the first six months is in the ratio 3: 4: 5. After six months, they had to increase their monthly investments by 10%, 15% and 20%, respectively, of their initial monthly investment. The new investment ratio was kept constant for the next six months.

What is the ratio of their shares in the total profit (in the same order) at the end of the year such that the share is proportional to their individual total investment over the year?

A. 22 : 23 : 24 B. 22 : 33 : 50

C. 33 : 46 : 60 D. 63 : 86 : 110

[MCQ: 2 Mark]

Ans. (D)

Sol.

Let the initial investments be 3x, 4x and 5x, respectively.

Now, after increasing their monthly investment by 10%, 15% and 20%, respectively, the monthly instalment will be: 3.3x, 4.6x and 6x, respectively

Ratio of their profits at the end of year

= Ratio of their investments

= (3x + 3.3x) : (4x + 4.6x) : (5x + 6x)

= 6.3x : 8.6x : 11x

= 63 : 86 : 110

8. Consider the following equations of straight lines:

Line L1: 2x – 3y = 5

Line L2: 3x + 2y = 8

Line L3: 4x – 6y = 5

Line L4: 6x – 9y = 6

Which one among the following is the correct statement?

A. L1 is parallel to L2 and L1 is perpendicular to L3

B. L2 is parallel to L4 and L2 is perpendicular to L1

C. L3 is perpendicular to L4 and L3 is parallel to L2

D. L4 is perpendicular to L2 and L4 is parallel to L3

[MCQ: 2 Mark]

Ans. (D)

Sol.

L_1 : $2x - 3y = 5 \Rightarrow$ slope = 2/3

L_2 : $3x + 2y = 8 \Rightarrow$ slope = – 3/2

L_3 : $4x - 6y = 5 \Rightarrow$ slope = 4/6 = 2/3

L_4 : $6x - 9y = 6 \Rightarrow$ slope = 6/9 = 2/3

Slope of L_1 = slope of L_3 = slope of L_4

$\Rightarrow$ L_1, L_3 and L_4 are parallel to each other

slope of L_4 × slope of L_2 = $\frac{2}{3} \times \left(\frac{-3}{2}\right) = -1$.

$\Rightarrow$ L_2 and L_4 are perpendicular to each other

9. Given below are two statements and four conclusions drawn based on the statements.

Statement 1: Some soaps are clean.

Statement 2: All clean objects are wet.

Conclusion I: Some clean objects are soaps.

Conclusion II: No clean object is a soap.

Conclusion III: Some wet objects are soaps.

Conclusion IV: All wet objects are soaps.

Which one of the following options can be logically inferred?

A. Only conclusion I is correct

B. Either conclusion I or conclusion II is correct

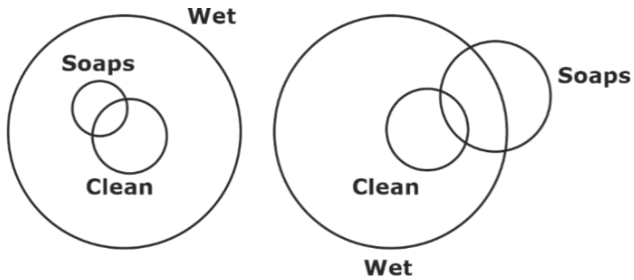
C. Either conclusion III or conclusion IV is correct

D. Only conclusion I and conclusion III are correct

[MCQ: 2 Mark]

Ans. (D)

Sol. The possible representations are:–



We can observe that:–

- Some clean objects are soaps
- Some wet objects are soaps

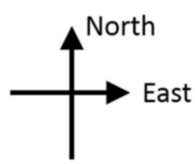
The above two criteria are fulfilled in both the diagrams, i.e. only conclusions I and III are correct.

10. An ant walks in a straight line on a plane leaving behind a trace of its movement. The initial position of the ant is at point P facing east.

The ant first turns 72° anticlockwise at P, and then does the following two steps in sequence exactly FIVE times before halting.

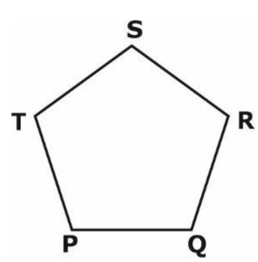
1. moves forward for 10 cm.

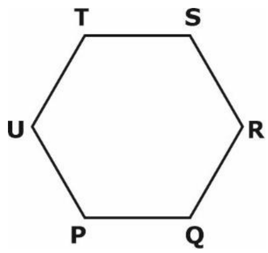
2. turns 144o clockwise.

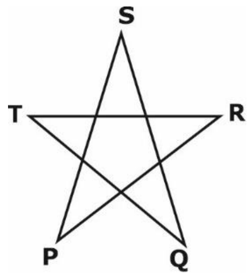


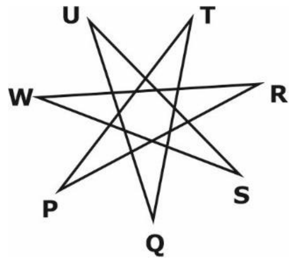
The pattern made by the trace left behind by the ant is

A.



B. 

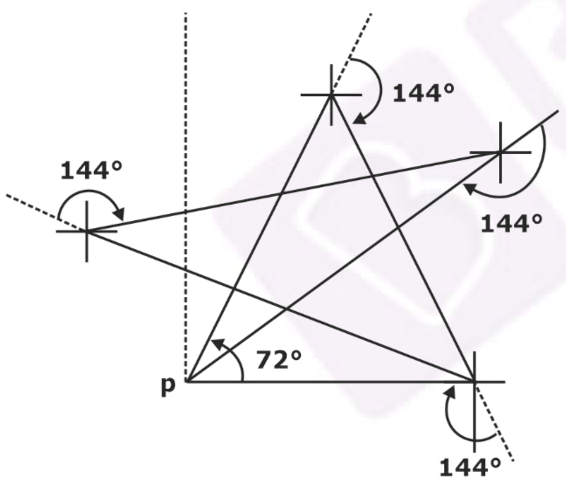
C. 

D. 

[MCQ: 2 Mark]

Ans. (C)

Sol. The path is shown below:



11. The function f(x, y) satisfies the Laplace equation

$\nabla^2 f(x, y) = 0$

on a circular domain of radius r = 1 with its center at point P with coordinates x = 0, y = 0. The value of this function on the circular boundary of this domain is equal to 3.

The numerical value of f(0, 0) is:

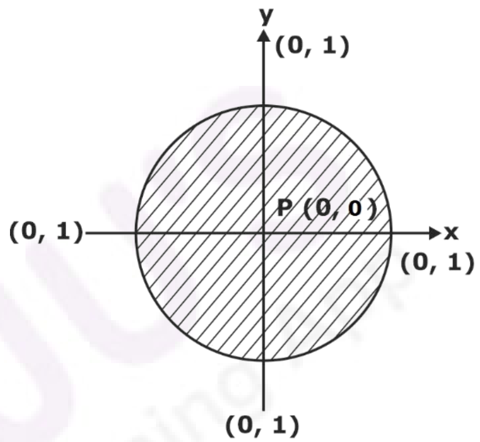
A. 0 B. 2

C. 3 D. 1

[MCQ: 1 Mark]

Ans. (C)

Sol.



f(x, y) = 3 within the circular region.

Given point P also lies in the circular domain

Hence, f(0, 0) = 3

12. $\int \left(x - \frac{x^2}{2} + \frac{x^3}{3} - \frac{x^4}{4} + \ldots \right) dx$ is equal to

A. $\frac{1}{1+x}$ + Constant

B. $\frac{1}{1+x^2}$ + Constant

C. $-\frac{1}{1+x}$ + Constant

D. $-\frac{1}{1+x^2}$ + Constant

[MCQ: 1 Mark]

Ans. (*)

Sol.

$$\int\left(x-\frac{x^2}{2}+\frac{x^3}{3}-\frac{x^4}{4}+\ldots\right)dx$$

$$=\frac{x^2}{2}-\frac{x^3}{6}+\frac{x^4}{12}-\frac{x^5}{20}+\ldots$$

$$\frac{1}{1+x}=1-x+x^2-x^3+\ldots$$

$$\frac{1}{1+x^2}=1-x^2+x^4-x^6+\ldots$$

$$-\frac{1}{1-x}=-(1+x+x^2+\ldots)$$

$$-\frac{1}{1-x^2}=-(1+x^2+x^4+\ldots)$$

None of the options are matching

Marks to all

13. For a linear elastic and isotropic material, the correct relationship among Young's modulus of elasticity (E), Poisson's ratio (v), and shear modulus (G) is

A. $G=\frac{E}{2(1+v)}$ B. $G=\frac{E}{(1+2v)}$

C. $E=\frac{E}{2(1+v)}$ D. $E=\frac{G}{(1+2v)}$

[MCQ: 1 Mark]

Ans. A

Sol. $E = 2G(1 + \mu) = 3K(1 - 2\mu)$

$$\Rightarrow \boxed{G=\frac{E}{2(1+\mu)}}$$

14. Read the following statements relating to flexure of reinforced concrete beams:

I. In over-reinforced sections, the failure strain in concrete reaches earlier than the yield strain in steel.

II. In under-reinforced sections, steel reaches yielding at a load lower than the load at which the concrete reaches failure strain.

III. Over-reinforced beams are recommended in practice as compared to the under-reinforced beams.

IV. In balanced sections, the concrete reaches failure strain earlier than the yield strain in tensile steel.

Each of the above statements is either True or False.

Which one of the following combinations is correct?

A. I (True), II (True), III (False), IV (False)

B. I (True), II (True), III (False), IV (True)

C. I (False), II (False), III (True), IV (False)

D. I (False), II (True), III (True), IV (False)

[MCQ: 1 Mark]

Ans. A

Sol. Under reinforced section ($x_u < x_{u,\,max}$)

- In this case, steel yields before the crushing of concrete and the failure is ductile.
- In construction, under-reinforced sections are preferred as it gives warning before the collapse.

Balanced section ($x_u = x_{u,max}$)

- In this case, the yielding of steel and crushing of concrete take place simultaneously.

Over-reinforced section($x_u > x_{u,max}$)

- In this case, crushing of concrete occurs before yielding steel, and sudden failure occurs.

It means statement I, II is true, and Statement III, IV is false.

15. Match all the possible combinations between Column X (Cement compounds) and Column Y (Cement properties):

Column X	**Column Y**
(i) C3S	(P) Early age strength
(ii) C2S	(Q) Later age strength
(iii) C3A	(R) Flash setting
	(S) Highest heat of hydration
	(T) Lowest heat of hydration

A. (i) - (P), (ii) - (Q) and (T), (iii) - (R) and (S)

B. (i) - (Q) and (T), (ii) - (P) and (S), (iii) - (R)

C. (i) - (P), (ii) - (Q) and (R), (iii) - (T)

D. (i) - (T), (ii) - (S), (iii) - (P) and (Q)

[MCQ: 1 Mark]

Ans. (A)

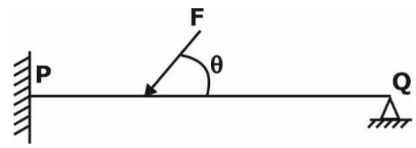
Sol.

C_3S →responsible for early age strength

C_2S → give lowest heat of hydration and provide later age strength

C_3A → Flash setting and highest heat of hydration

16. Consider a beam PQ fixed at P, hinged at Q, and subjected to a load F as shown in figure (not drawn to scale). The static and kinematic degrees of indeterminacy, respectively, are

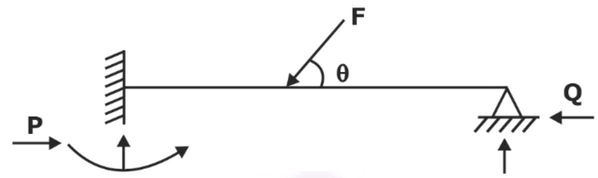


A. 2 and 1
B. 2 and 0
C. 1 and 2
D. 2 and 2

[MCQ: 1 Mark]

Ans. (A)

Sol.



This is the case of general loading

No. of support reactions = 3 + 2 = 5

No. of equilibrium equation available = 3.

Static indeterminacy = 5 – 3 = 2

Degrees of freedom at P = 0

Degrees of freedom at Q = 3 – 2

Kinematic indeterminacy = 0 + 1 = 1

17. Read the following statements:

(P) While designing a shallow footing in sandy soil, monsoon season is considered for critical design in terms of bearing capacity.

(Q) For slope stability of an earthen dam, sudden drawdown is never a critical condition.

(R) In a sandy sea beach, quicksand condition can arise only if the critical hydraulic gradient exceeds the existing hydraulic gradient.

(S) The active earth thrust on a rigid retaining wall supporting homogeneous cohesionless backfill will reduce with the lowering of water table in the backfill.

Which one of the following combinations is correct?

A. (P)-True, (Q)-False, (R)-False, (S)-False

B. (P)-False, (Q)-True, (R)-True, (S)-True

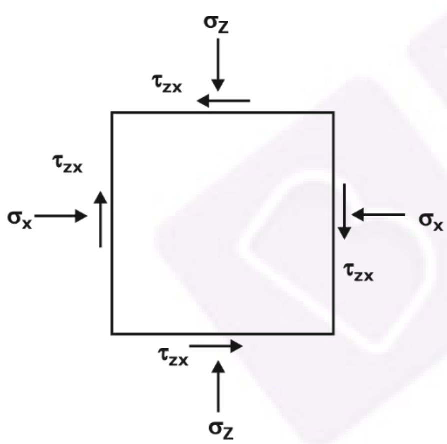
C. (P)-True, (Q)-False, (R)-True, (S)-True

D. (P)-False, (Q)-True, (R)-False, (S)-False

[MCQ: 1 Mark]

Ans. A

18. Stresses acting on an infinitesimal soil element are shown in the figure (with $\sigma_z > \sigma_x$). The major and minor principal stresses are σ_1 and σ_3, respectively. Considering the compressive stresses as positive, which one of the following expressions correctly represents the angle between the major principal stress plane and the horizontal plane?



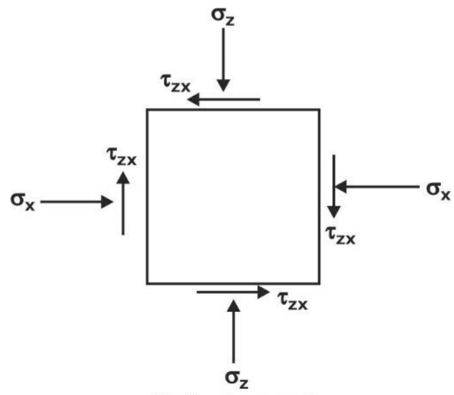
A. $\tan^{-1}\left(\frac{\tau_{zx}}{\sigma_1 - \sigma_x}\right)$ B. $\tan^{-1}\left(\frac{\tau_{zx}}{\sigma_3 - \sigma_x}\right)$

C. $\tan^{-1}\left(\frac{\tau_{zx}}{\sigma_1 - \sigma_x}\right)$ D. $\tan^{-1}\left(\frac{\tau_{zx}}{\sigma_1 - \sigma_3}\right)$

[MCQ: 1 Mark]

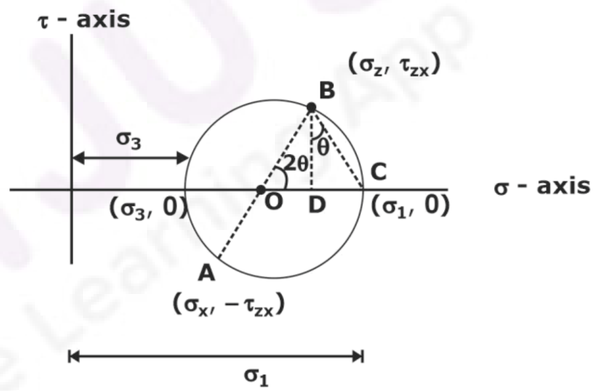
Ans. *

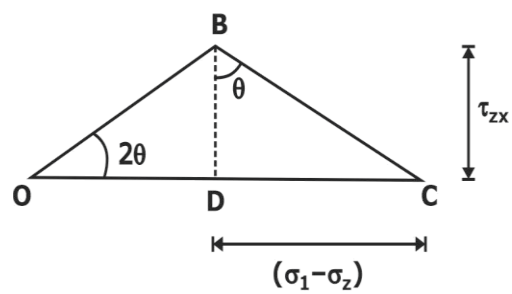
Sol.



Soil element

As given in the question, compressive stress is considered positive, the Mohr circle will be as shown below





$$\tan\theta = \frac{(\sigma_1 - \sigma_2)}{\tau_{zx}}$$

$$\Rightarrow \theta = \tan^{-1} = \left(\frac{\sigma_1 - \sigma_2}{\tau_{zx}}\right)$$

19. Match Column X with Column Y:

	Column X		Column Y
(P)	Horton equation	(I)	Design of alluvial channel
(Q)	Penman method	(II)	Maximum flood discharge
(R)	Chezy's formula	(III)	Evapotranspiration
(S)	Lacey's theory	(IV)	Infiltration
(T)	Dicken's formula	(V)	Flow velocity

Which one of the following combinations is correct?

A. (P)-(IV), (Q)-(III), (R)-(V), (S)-(I), (T)-(II)

B. (P)-(III), (Q)-(IV), (R)-(V), (S)-(I), (T)-(II)

C. (P)-(IV), (Q)-(III), (R)-(II), (S)-(I), (T)-(V)

D. (P)-(III), (Q)-(IV), (R)-(I), (S)-(V), (T)-(II)

[MCQ: 1 Mark]

Ans. (A)

Sol. Horton equation- Infiltration

Penman method- Evapotranspiration

Chezy's formula- Flow velocity

Lacey's theory- Design of alluvial channel

Dicken's formula- Maximum flood discharge

20. In a certain month, the reference crop evapotranspiration at a location is 6 mm/day. If the crop coefficient and soil coefficient are 1.2 and 0.8, respectively, the actual evapotranspiration in mm/day is

A. 5.76 B. 7.20

C. 6.80 D. 8.00

[MCQ: 1 Mark]

Ans. (A)

Sol. Actual evapotranspiration

$= k_c \times k_s \times$ reference evapotranspiration

$\Rightarrow k_s = 0.8, k_c = 1.2$

$\Rightarrow$ Actual evapotranspiration

$= 0.8 \times 1.2 \times 6$

$= \boxed{5.76\,\text{mm}}$

21. The dimension of dynamic viscosity is:

A. $M L^{-1} T^{-1}$

B. $M L^{-1} T^{-2}$

C. $M L^{-2} T^{-2}$

D. $M L^{0} T^{-1}$

[MCQ: 1 Mark]

Ans. A

Sol. $\tau = \mu = \frac{dv}{dy}$

$$F/A = \mu \frac{V}{Y}$$

$$\frac{[MLT^{-2}]}{[L^2]} = \mu \frac{[LT^{-1}]}{[L]}$$

$$\boxed{\mu = ML^{-1}T^{-1}}$$

22. A process equipment emits 5 kg/h of volatile organic compounds (VOCs). If a hood placed over the process equipment captures 95% of the VOCs, then the fugitive emission in kg/h is

A. 0.25 B. 4.75

C. 2.50 D. 0.48

[MCQ: 1 Mark]

Ans. (A)

Sol. Total VOCs = 5 kg/h

VOCs captured = 95% of total VOCs

$= \frac{95}{100} \times 5 = 4.75$ kg/h

Fugitive emission = VOCs escaped

= Total VOCs – VOCs captured

= 5 – 4.75

= 0.25 kg/h

23. Match the following attributes of a city with the appropriate scale of measurements.

Attribute		Scale of Measurement	
(P)	Average temperature (^{0}C) of a city	(I)	Interval
(Q)	Name of a city	(II)	Ordinal
(R)	Population density of a city	(III)	Nominal
(S)	Ranking of a city based on ease of Business	(IV)	Ratio

Which one of the following combinations is correct?

A. (P)-(I), (Q)-(III), (R)-(IV), (S)-(II)
B. (P)-(II), (Q)-(I), (R)-(IV), (S)-(III)
C. (P)-(II), (Q)-(III), (R)-(IV), (S)-(I)
D. (P)-(I), (Q)-(II), (R)-(III), (S)-(IV)

[MCQ: 1 Mark]

Ans. (A)

Sol. Attribute Scale of measurement

Average temperature (^{0}C) of a city
Interval
Name of a cityNominal
Population density of a city Ratio
Ranking of a city based on ease of business
Ordinal

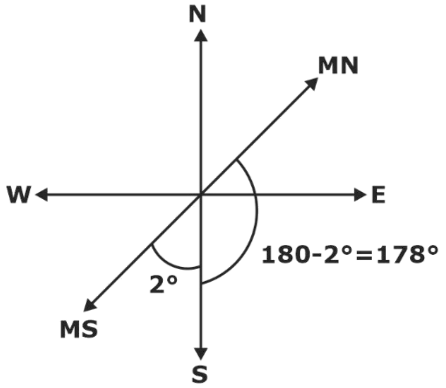
24. If the magnetic bearing of the Sun at a place at noon is S 2° E, the magnetic declination (in degrees) at that place is

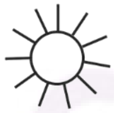
A. 2°E B. 2°W
C. 4°E D. 4°W

[MCQ: 1 Mark]

Ans. (A)

Sol.





TB at Noon = 180°
MB at noon = 180°–2° = 178°
Magnetic decimation
= TB – MB = 180°– 178° = + 2°
+ 2° means 2°E

25. P and Q are two square matrices of the same order. Which of the following statement(s) is/are correct?

A. If P and Q are invertible, then $[PQ]^{-1} = Q^{-1} P^{-1}$
B. If P and Q are invertible, then $[QP]^{-1} = P^{-1} Q^{-1}$
C. If P and Q are invertible, then $[PQ]^{-1} = P^{-1} Q^{-1}$
D. If P and Q are not invertible, then $[PQ]^{-1} = Q^{-1} P^{-1}$

[MCQ: 1 Mark]

Ans. (A, B)

Sol. $[PQ]^{-1} = Q^{-1} P^{-1}$ when both P and Q are invertible

$[QP]^{-1} = P^{-1} Q^{-1}$ when both P and Q are invertible

26. In a solid waste handling facility, the moisture contents (MC) of food waste, paper waste, and glass waste were found to be MCf, MCp, and MCg, respectively. Similarly, the energy contents (EC) of plastic waste, food waste, and glass waste were found to be ECpp, ECf, and ECg, respectively. Which of the following statement(s) is/are correct?

A. MCf > MCp > MCg
B. ECpp > ECf > ECg
C. MCf < MCp < MCg
D. ECpp < ECf < ECg

[MCQ: 1 Mark]

Ans. (A, B)

Sol. Typical specific weight and moisture content data for residential, commercial, industrial, and agricultural wastes.

	Specific weight, lb/yd^3		**Moisture content, % by weight**	
	Range	**Typical**	**Range**	**Typical**
Residential (uncompacted)				
*Food wastes (mixed)	220–810	490	50–80	70
*Paper	70–220	150	4–10	6
Cardboard	70–135	85	4–8	5
Plastics	70–220	110	1–4	2
Textiles	70–170	110	6–15	10
Rubber	170–340	220	1–4	2
Leather	170–440	270	8–12	10
Yard wastes	100–380	170	30–80	60
Wood	220–540	400	15–40	20
*Glass	270–810	330	1–4	2
Tin cans	85–270	150	2–4	3
Aluminium	110–405	270	2–4	2
Other metals	220–1940	540	2–4	3
Dirt, ashes, etc.	540–1685	810	6–12	6
Ashes	1095–1400	1255	6–12	6
Rubbish	150–305	220	5–20	15
Residential yard wastes				
Leaves (loose and dry)	50–250	100	20–40	30
Green grass (loose and moist)	350–500	400	40–80	60
Green grass (wet and compacted)	1000–1400	1000	50–90	80
Yard waste (shredded)	450–600	500	20–70	50
Yard waste (composted)	450–650	550	40–60	50

Municipal				
In compactor truck	300–760	500	15–40	20
In Landfill Normally compacted	610–840	760	15–40	25
In landfill Well compacted	995–1250	1010	15–40	25
Commercial				
Food wastes (wet)	800–1600	910	50–80	70
Appliances	250–340	305	0–2	1

Moisture content:

Mcf > Mcp > Mcg

Typical values for short residue and energy content of residential of residential MSW.

	Inert residue[a] percent		**Energy[b] (Btu/lb)**	
Component	**Range**	**Typical**	**Range**	**Typical**
Organic				
*Food wastes	2.8	5.0	1,500–3,000	2,000
*Paper	4–8	6.0	5,000–8,000	7,200
Cardboard	3.6	5.0	6,000–7,500	7,000
Plastics	6–20	10.0	12,000–10,000	14,000
Textiles	2–4	2.5	6,500–8,000	7,500
Rubber	8–20	10.0	9,000–12,000	10,000
Leather	8–20	10.0	6,500–8,500	7,500
Yard wastes	2–6	4.5	1,000–8,000	2.000
Wood	0.6.2	1.5	7,500–8.500	8.000
Misc. organics	–	–	–	–
Inorganic				
*Class	96.99	980	50–100[c]	60
Tin cans	96.99	98.0	100–500[c]	300
Aluminium	90.99	96.0	–	–
Other metal	94.99	98.0	100–500[c]	300
Dirt, ashes	60.80	70.0	1,000–5,000	3.000
Municipal solid wastes etc.			4,000–6,000	5,000

[a] After complete combustion
[b] As discarded basis
[c] Energy content is from coatings, labels and attached minerals

Energy content:

ECpp > ECf > ECg

27. To design an optimum municipal solid waste collection route, which of the following is/are NOT desired:

A. Collection vehicle should not travel twice down the same street in a day.

B. Waste collection on congested roads should not occur during rush hours in morning or evening.

C. Collection should occur in the uphill direction.

D. The last collection point on a route should be as close as possible to the waste disposal facility.

[MCQ: 1 Mark]

Ans. (C)

Sol. Some guidelines that should be taken into consideration when laying out routes are as follows:

(i) Existing policies and regulations related to such items as the point of collection and frequency of collection must be identified.

(ii) Existing system characteristics such as crew size and vehicle types must be coordinated.

(iii) Wherever possible, routes should be laid out so that they begin and end near arterial streets, using topographical and physical barriers as route boundaries.

(iv) In hilly areas, routes should start at the top of the grade and proceed downhill as the vehicle becomes loaded. Proceeding uphill will require large tractive force (vehicles gets loaded) and more fuel.

(v) Routes should be laid out so that the last container to be collected on the route is located nearest to the disposal site.

(vi) Waste generated at traffic congested locations should be collected as early in the day as possible as it will cause the generation of foul smell and the breeding of insects for a long time during congestion.

(vii) Sources at which extremely large quantities of waste are generated should be serviced during the start of the day as the residual of large wastes cause the generation of foul smell and the breeding of insects.

(viii) Scattered pickup points (where small quantities of solid wastes are generated) that receive the same collection frequency should, if possible, be serviced during one trip or on the same day.

28. For a traffic stream, v is the space mean speed, k is the density, q is the flow, v_f is the free-flow speed, and k_j is the jam density. Assume that the speed decreases linearly with density.

Which of the following relation(s) is/are correct?

A. $q = k_j k - \left(\frac{k_j}{v_f}\right)k^2$

B. $q = v_f k - \left(\frac{v_f}{k_j}\right)k^2$

C. $q = v_f v - \left(\frac{v_f}{kj}\right)v^2$

D. $q = k_j v - \left(\frac{k_j}{v_f}\right)v^2$

[MCQ: 1 Mark]

Ans. (B,D)

Sol. v = Space mean speed

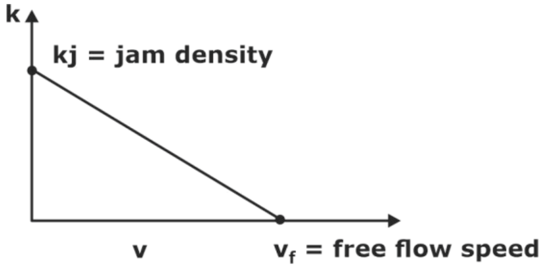
k = density

q = flow

v_f = free flow speed

k_j = jam density

Since it is given speed decreasing linearly with density



Let v = a k + b.......(1)

At k = 0

v = v_f

v_f = a...............(2)

at k = kj,

v = 0

$b = -\frac{v_f}{k_j}$ ....(3)

by (1), (2), (3)

$v = v_f - \frac{v_f}{k_j} \cdot k$ ...(4)

∴ know q = kv ...(5)

By (4) and (5)

$$q = v_f \cdot k - \frac{v_f}{k_j} k^2$$

$$q = v_f \cdot \left(\frac{q}{v}\right) - \frac{v_f}{k_j}\left(\frac{q}{v}\right)^2$$

$$q = q\left(\frac{v_f}{v}\right) - \frac{v_f}{k_j}\left(\frac{q}{v}\right)^2$$

$$1 = \left(\frac{v_f}{v}\right) - \frac{q}{v^2}\left(\frac{v_f}{k_j}\right)$$

$$v^2 = v_f v - q\left(\frac{v_f}{k_j}\right)$$

$$q = k_j v - \left(\frac{k_j}{v_f}\right) v^2$$

Hence B and D are correct

29. The error in measuring the radius of a 5 cm circular rod was 0.2%. If the cross-sectional area of the rod was calculated using this measurement, then the resulting absolute percentage error in the computed area is_______.

(round off to two decimal places)

[NAT: 1 Mark]

Ans. (0.39 to 0.41)

Sol. R = radius,

A = area

R = 5 cm

e_r = 0.2% of r

$e_r = \frac{0.2}{100} \times 5 = 0.01\,\text{cm}$

$A = \pi r^2$

e_A = error in area

$e_A = 2\pi r\,(e_r)$

% Error in area $= \frac{e_A}{A} \times 100 = \frac{2\pi r(e_r)}{\pi r^2} \times 100$

% Error in area $= \frac{2e_r}{r} \times 100$

% Error in area $= \frac{2 \times 0.01}{5} \times 100 = 0.40$

30. The components of pure shear strain in a sheared material are given in the matrix form:

$\varepsilon = \begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix}$

Here, Trace$(\varepsilon) = 0$. Given, P = Trace (ε^8) and Q = Trace(ε^{11}).

The numerical value of (P + Q) is ________. (in integer)

[NAT: 1 Mark]

Ans. (32-32)

Sol. $\varepsilon = \begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix}$

$\varepsilon^2 = \begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix} \begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix}$

$= \begin{bmatrix} 2 & 0 \\ 0 & 2 \end{bmatrix} = 2I$

$\varepsilon^8 = \left(\varepsilon^2\right)^4 = (2I)^4 = 16I = \begin{bmatrix} 16 & 0 \\ 0 & 16 \end{bmatrix}$

P = Trace (ε^8) = 32

$\varepsilon^{11} = \varepsilon^{10}\ \varepsilon$

$= (\varepsilon^2)^5\ \varepsilon$

$= (2I)^5\ \varepsilon$

$= 32\ \varepsilon$

$= \begin{bmatrix} 32 & 32 \\ 32 & -32 \end{bmatrix}$

Q = Trace (ε^{11}) = 0

$\boxed{P + Q = 32}$

31. The inside diameter of a sampler tube is 50 mm. The inside diameter of the cutting edge is kept such that the Inside Clearance Ratio (ICR) is 1.0 % to minimize the friction on the sample as the sampler tube enters into the soil.

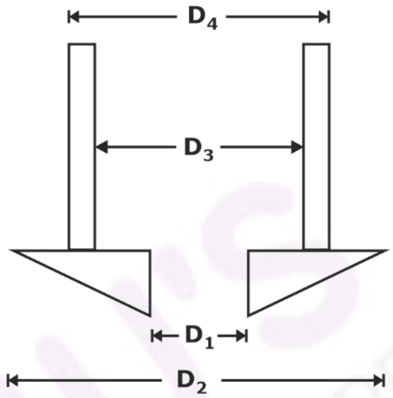
The inside diameter (in mm) of the cutting edge is ________________.

(round off to two decimal places)

[NAT: 1 Mark]

Ans. (49.49 - 49.52)

Sol. D_3 = 50 mm , inside clearance ratio=1%



Inside clearances ratio = $\dfrac{D_3 - D_1}{D_1}$

$\Rightarrow 0.01 = \dfrac{50 - D_1}{D_1}$

$\boxed{D_1 = 49.50\text{mm}}$

32. A concentrically loaded isolated square footing of size 2 m × 2 m carries a concentrated vertical load of 1000 kN. Considering Boussinesq's theory of stress distribution, the maximum depth (in m) of the pressure bulb corresponding to 10 % of the vertical load intensity will be ___________. (round off to two decimal places)

[NAT: 1 Mark]

Ans. (4.35-4.39)

Sol. vertically load intensity (Q) at top

$Q = \dfrac{\text{Load}}{\text{area}} = \dfrac{1000}{2^2}$

$\boxed{Q = 250\,\text{kN/m}^2}$

At depth (Z), σ_z = 10% (250)

σ_2 = 25 kN/m^2

By Boussinesq's theory

$$\sigma_z = \frac{3}{2\pi}\frac{Q}{z^2}\left[\frac{1}{1+\frac{r^2}{z^2}}\right]^{5/2}$$

For, r = 0, $\Rightarrow \sigma_z = \frac{3}{2\pi}\frac{Q}{z^2}$

$$\Rightarrow 25 = \frac{3}{2\pi}\times\frac{1000}{z^2}$$

$$\boxed{z = 4.37\,m}$$

33. In a triaxial unconsolidated undrained (UU) test on a saturated clay sample, the cell pressure was 100 kPa. If the deviatoric stress at failure was 150 kPa, then the undrained shear strength of the soil is _________ kPa. (in integer)

[NAT: 1 Mark]

Ans. (75-75)

Sol. Cell pressure = σ_c = 100 kPa

Deviatoric stress at Failure $\sigma_d = \Delta\sigma$ = 150 kPa

σ_1 = 150 + 100 = 250 kPa

undrained shear strength

$$= C_{uu} = \tau_f = \frac{\sigma_1 - \sigma_2}{2}$$

$$= \frac{(100+150)-100}{2}$$

$$\boxed{C_{uu} = 75\,kPa}$$

34. A flood control structure having an expected life of n years is designed by considering a flood of return period T years. When T = n, and n $\to\infty$, the structure's hydrologic risk of failure in percentage is.
(round off to one decimal place)

[NAT: 1 Mark]

Ans. (63.0 to 63.5)

Sol. Risk = 1 – q^n

q = 1 – p

$P = \frac{1}{T}$

$\Rightarrow q = 1 - \frac{1}{T}$

For T = n $\to \infty$

$$\Rightarrow \text{Risk (R)} = \left[1 - \left(1 - \frac{1}{n}\right)^n\right]$$

$$\lim_{x\to\infty}\left(1+\frac{1}{x}\right)^x = e$$

$$\Rightarrow \lim_{n\to\infty}\left(1+\left(-\frac{1}{n}\right)^{-n}\right)^{-1} = e^{-1}$$

$\Rightarrow$ R = 1 – e^{-1}

$$R = 1 - \frac{1}{e} = 0.6321$$

$$\Rightarrow \boxed{R = 63.2\%}$$

35. The base length of the runway at the mean sea level (MSL) is 1500 m. If the runway is located at an altitude of 300 m above the MSL, the actual length (in m) of the runway to be provided is ____________. (round off to the nearest integer)

[NAT: 1 Mark]

Ans. (1602–1606)

Sol. As per the provision, elevation must be increased by 7% per 300-meter rise in MSL.

L = Basic runway length = 1500 m

Altitude = 300 m

L' = actual length

$$L' = L + \frac{7}{100}\times\frac{300}{300}\times L$$

$$L' = 1500 + \frac{7}{100}\times\frac{300}{300}\times 1500$$

L' = 1605 m

36. Consider the polynomial $f(x) = x^3 - 6x^2 + 11x - 6$ on the domain S given by $1 \le x \le 3$. The first and second derivatives are $f'(x)$ and $f''(x)$.

Consider the following statements:

I. The given polynomial is zero at the boundary points x = 1 and x = 3.

II. There exists one local maxima of f(x) within the domain S.

III. The second derivative $f''(x) > 0$ throughout the domain S.

IV. There exists one local minima of f(x) within the domain S.

The correct option is:

A. Only statements I, II and III are correct.

B. Only statements I, II and IV are correct.

C. Only statements I and IV are correct.

D. Only statements II and IV are correct.

[MCQ: 2 Marks]

Ans. (B)

Sol. $f(x) = x^3 - 6x^2 + 11x - 6, \quad S : 1 \le x \le 3$

$f'(x) = 3x^2 - 12x + 11$

$f''(x) = 6x - 12$

$f(x) = 0$

$\Rightarrow x^3 - 6x^2 + 11x - 6 = 0$

$\Rightarrow x^3 - x^2 - 5x^2 + 5x + 6x - 6 = 0$

$\Rightarrow x^2 (x-1) - 5x (x-1) + 6 (x-1) = 0$

$\Rightarrow (x-1) (x^2 - 5x + 6) = 0$

$\Rightarrow (x - 1) (x - 2) (x - 3) = 0$

x= 1, 2, 3

Hence, Statement 1 is correct

$f'(x) = 0$

$\Rightarrow 3x^2 - 12x + 11 = 0$

$$x = \frac{12 \pm \sqrt{12^2 - 4 \times 3 \times 11}}{6} = \frac{12 \pm 2\sqrt{3}}{6} = 2 \pm \frac{1}{\sqrt{3}}$$

$$= 2 + \frac{1}{\sqrt{3}}, 2 - \frac{1}{\sqrt{3}}$$

= 2.577, 1.423

$f''(x) = 6x - 12$

$f'(2.577)$ = +ve ⇒ point of local minima

$f'(1.423)$ = − ve ⇒ point of local maxima

There exists one local minima and one local maxima within the domain S.

Hence statements II and IV are correct.

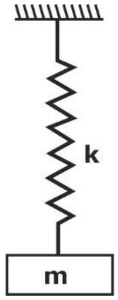
$f''(x) > 0$

$\Rightarrow 6x - 12 > 0$

$x > 2$

$f''(x) > 0$ for $2 < x \le 3$. Hence, Statement III is incorrect.

37. An undamped spring-mass system with mass m and spring stiffness k is shown in the figure. The natural frequency and natural period of this system are ω rad/s and T s, respectively. If the stiffness of the spring is doubled and the mass is halved, then the natural frequency and the natural period of the modified system, respectively, are



A. 2 ω rad/s and T/2 s

B. ω /2 rad/s and 2T s

C. 4 ω rad/s and T/4 s

D. ω rad/s and T s

[MCQ: 2 Marks]

Ans. (A)

Sol. $\omega = \sqrt{\frac{k}{m}}$

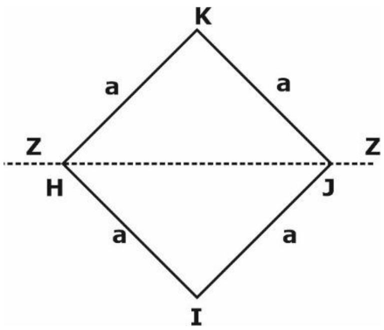
$T = \frac{2\pi}{\omega}$

When stiffness is doubled, and mass is halved

$$\omega' = \sqrt{\frac{2k}{m/2}} = 2\sqrt{\frac{k}{m}} = 2\omega$$

$$T' = \frac{2\pi}{\omega'} = \frac{2\pi}{2\omega} = \frac{T}{2}$$

38. For the square steel beam cross-section shown in the figure, the shape factor about z − z axis is S and the plastic moment capacity is M_p. Consider yield stress



fy = 250 MPa and a = 100 mm.

A. S = 2.0, M_p = 58.9 kN-m

B. S = 2.0, M_p = 100.2 kN-m

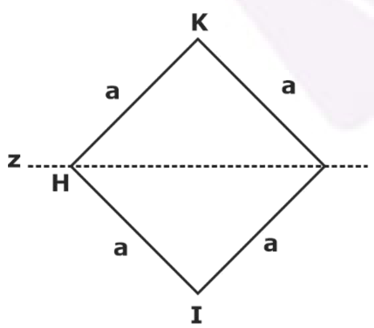
C. S = 1.5, M_p = 58.9 kN-m

D. S = 1.5, M_p = 100.2 kN-m

[MCQ: 2 Marks]

Ans. A

Sol. For diamond section, shape factor (S) is 2.



$$\frac{\sqrt{2}a}{2} = \frac{a}{\sqrt{2}}$$

Plastic moment capacity (M_P)

M_P = (shape factor) × M_g

$M_P = Sf_yZ_e$

Z_e = elastic section modulus

$$z_e = \frac{I_{zz}}{y_{max}}$$

$$z_e = \frac{\frac{a^4}{12}}{a/\sqrt{2}} = \frac{\sqrt{2}a^3}{12}\,mm^3$$

Now, $M_P = Sf_yZ_e$

$$= 2 \times 250 \times \frac{\sqrt{2} \times (100)^3}{3}$$

$$\boxed{M_P = 58.93\,kNm}$$

39. A post-tensioned concrete member of span 15 m and cross-section of 450 mm ☐ 450 mm is prestressed with three steel tendons, each of cross-sectional area 200 mm2. The tendons are tensioned one after another to a stress of 1500 MPa. All the tendons are straight and located at 125 mm from the bottom of the member. Assume the prestress to be the same in all tendons and the modular ratio to be 6. The average loss of prestress, due to elastic deformation of concrete, considering all three tendons is

A. 14.16 MPa B. 7.08 MPa

C. 28.32 MPa D. 42.48 MPa

[MCQ: 2 Marks]

Ans. (A)

Sol. L = 15 m

B = 450 mm

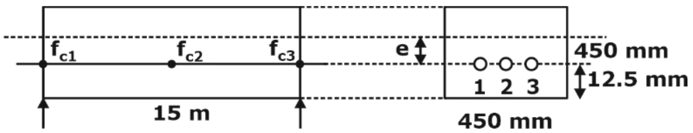
D = 450 mm

N = 3

As = 200 mm^2

e = 1500 MPa

m = 6



e = eccentricity

$$e = \frac{450}{2} - 125 = 100\,mm$$

A_s = Area of steel of each bar = 200 mm^2

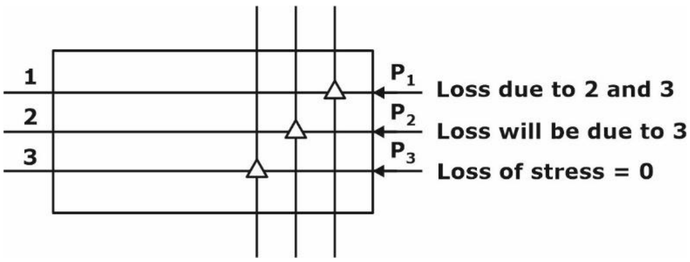
p_0 = 1500 MPa

P – force in each tandon

P = p_0 × As

$$p = 1500 \times \frac{200}{100} = 300\ kN$$

This force will be in each wire



$$f_{c1} = f_{c2} = \frac{P_3}{A} + \frac{P_3 e^2}{I}$$

$$f_{c1} = f_{c2} = \frac{300 \times 1000}{450 \times 450} + \frac{300 \times 1000 \times 100^2}{450 \times \frac{450^3}{12}}$$

= 2.36 MPa

Loss due to elastic shortening = $m \times f_{c,avg}$

Loss due to elastic shortening

$$= 6 \times \left(\frac{2.36 + 2.36}{2}\right)$$

Loss due to elastic shortening

= 14.16 N/mm^2

Loss of stress due to elastic shortening in wire number 1 (due to P_2 and P_3)

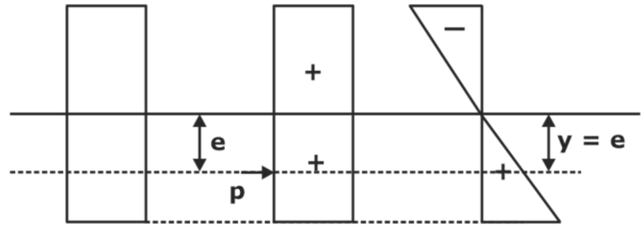
$P_2 + P_3 = 2P = 600$ kN

In wire – 1

$$f_{c1} = f_{c2} = \frac{P_2 + P_3}{A} + \frac{P_2 + P_3}{I} e^2 = 4.72\ N/mm^2$$

Δ = fixing wire at different location

We are neglecting self-weight because self-weight will create gain of stress



$$f_{c1} = f_{c2} = \frac{P}{A} + \frac{Pe}{I} \times y$$

$$= \frac{P}{A} + \frac{Pe^2}{I}$$

Loss of stress due to elastic shorting in wire number 3 = 0

Loss of stress due to elastic sharing in wire number 2 (due to P_3 force)

Loss of stress = m × $f_{c,\,avg}$

Loss of stress = 6 × 4.72 = 28.32 N/mm^2

Average loss in 3 wires $= \frac{0 + 14.16 + 28.32}{3}$

Average loss in 3 wires = 14.16 N/mn^2

40. Match the following in Column x with Column Y:

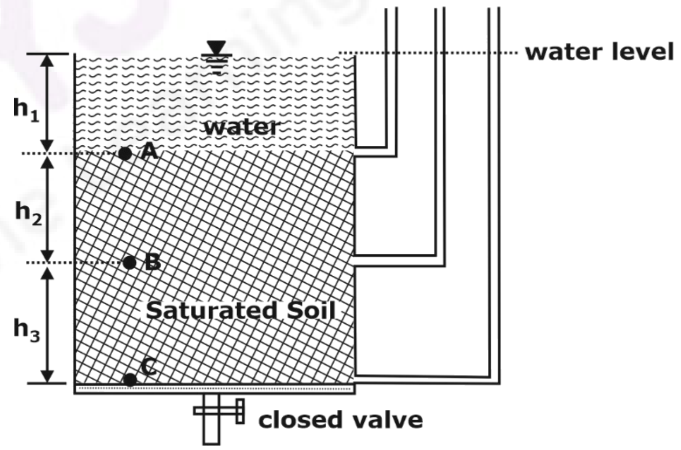
Column X		**Column Y**	
(P)	In a triaxial compression test, with increase of axial strain in loose sand under drained shear condition, the volumetric strain	(I)	decreases.

(Q)	In a triaxial compression test, with increase of axial strain in loose sand under undrained shear condition, the excess pore water pressure	(II)	increases.
(R)	In a triaxial compression test, the pore pressure parameter 'B' for a saturated soil	(III)	remains same.
(S)	For shallow strip footing in pure saturated clay, Terzaghi's bearing capacity factor (Nq) due to surcharge	(IV)	is always 0.0.
		(V)	is always 1.0.
		(VI)	is always 0.5.

Sol.

- In case pf loose sand as axial strain is increased volume of sand decreases for drained shear condition.
- As volume of voids decrease in loose sand on the axial strain, the excess pore water pressure increases for undrained condition
- Terzaghi's bearing capacity factor (N_γ) is one for pure clay.
- The pore pressure parameter B is one for saturate

41. A soil sample is underlying a water column of height h1, as shown in the figure. The vertical effective stresses at points A, B, and C are σ'_A, σ'_B, σ'_C respectively. Let γ_{sat} and γ' be the saturated and submerged unit weights of the soil sample, respectively, and γ_w be the unit weight of water. Which one of the following expressions correctly represents the sum $(\sigma'_A + \sigma'_B + \sigma'_C)$?



A. $(2h_2 + h_3)\gamma'$

B. $(2h_1 + h_2 + h_3)\gamma'$

C. $(h_2 + h_3)(\gamma_{sat} - \gamma_w)$

D. $(h_1 + h_2 + h_3)\gamma_{sat}$

[MCQ: 2 Marks]

Ans. A

Sol. effective stress at A, $\sigma'_A = 0$

Effective stress at B, $\sigma'_B = h_2 \times \gamma'$

Effective stress at C , $\sigma_C' = (h_2 + h_3)\gamma'$

$$\Rightarrow \sigma_A' + \sigma_B' + \sigma_C' = 0 + h_2\gamma' + (h_2 + h_3)\gamma'$$

$= (2h_2 + h_2)\ \gamma'$

42. A 100 mg of HNO3 (strong acid) is added to water, bringing the final volume to 1.0 litre. Consider the atomic weights of H, N, and O, as 1 g/mol, 14 g/mol, and 16 g/mol, respectively. The final pH of this water is (Ignore the dissociation of water.)

A. 2.8 B. 6.5

C. 3.8 D. 8.5

[MCQ: 2 Marks]

Ans. (A)

Sol. $HNO_3 \rightarrow H^+ + NO_3^-$

1 mol of HNO_3 gives 1 mol of H^+

No. of moles of HNO_3 in 100 mg of $HNO_3 = \frac{100}{1+14+3\times16}$ m mol

$= 1.587 \times 10^{-3}$ mol

Concentration of HNO_3 in mol/lit = 1.587×10^{-3} ml/lit

$[H^+] = [HNO_3]$

$= 1.587 \times 10^{-3}$ mol/lit

$pH = -\log_{10} [H^+]$

$= 3 - \log_{10} 1.587$

$= 2.8$

43. In a city, the chemical formula of biodegradable fraction of municipal solid waste (MSW) is $C_{100}H_{250}O_{80}N$. The waste has to be treated by forced-aeration composting process for which air requirement has to be estimated.

Assume oxygen in air (by weight) = 23 %, and density of air = 1.3 kg/m^3. Atomic mass:

C = 12

H = 1, O = 16, N = 14.

C and H are oxidized completely whereas N is converted only into NH_3 during oxidation. For oxidative degradation of 1 tonne of the waste, the required theoretical volume of air (in m^3/tonne) will be (round off to the nearest integer)

A. 4749 B. 8025

C. 1418 D. 1092

[MCQ: 2 Marks]

Ans. (A)

Sol. Balanced chemical reaction:

$4\ C_{100}H_{250}O_{80}N + 487\ O_2 \rightarrow 400\ CO_2 + 494\ H_2 + 4NH_3$

⇒ 4 moles of MSW requires 487 moles of oxygen

⇒ 1 mole of MSW requires 121.75 moles of oxygen

No. of moles of MSW in 1 tonne

$= \frac{1000\times100}{100\times12+250+80\times16+14}$

$= \frac{1000\times100}{2744}$ moles

Moles of O_2 required

$= \frac{1000\times1000}{2744}\times121.75$

Wt. of O_2 required

$= \frac{1000\times1000}{2744}\times121.75\times32\,\text{gm}$

$= \frac{121.75\times32\times1000}{2744}$ kg

= 1420 kg

Mass of air $= \frac{1420}{0.23} \approx 6174\,\text{kg}$

Density of air = 1.3 kg/m^3

Volume of air $= \frac{6174}{1.3}$

$\approx 4749 \text{ m}^3$

44. A single-lane highway has a traffic density of 40 vehicles/km. If the time-mean speed and space-mean speed are 40 kmph and 30 kmph, respectively, the average headway (in seconds) between the vehicles is

A. 3.00 B. 2.25

C. 8.33×10^{-4} D. 6.25×10^{-4}

[MCQ: 2 Marks]

Ans. (A)

Sol. k= 40 veh/km

v_t= 40 kmph

v_s= 30 kmph

q= k x v_s

= 40 x 30

= 1200 veh/hr

$q = \frac{3600}{H_t}$

$1200 = \frac{3600}{H_t}$

H_t = 3 sec

45. Let y be a non-zero vector of size 2022 × 1. Which of the following statement(s) is/are TRUE?

A. yy^T is a symmetric matrix.

B. y^Ty is an eigenvalue of yy^T.

C. yy^T has a rank of 2022.

D. yy^T is invertible.

[MSQ: 2 Marks]

Ans. (A, B)

Sol.

$$y = \begin{bmatrix} a_1 \\ a_2 \\ \vdots \\ : \\ a_{2022} \end{bmatrix}$$

$$y^T = \begin{bmatrix} a_1 & a_2 & a_3 & \dots\dots & a_{2022} \end{bmatrix}$$

$$yy^T = \begin{bmatrix} a_1 \\ a_2 \\ \vdots \\ : \\ : \\ a_{2022} \end{bmatrix} \begin{bmatrix} a_1 & a_2 & a_3 & \dots\dots & a_{2022} \end{bmatrix}$$

$$= \begin{bmatrix} a_1^2 & a_1a_2 & a_1a_3 & \dots\dots & a_1\ a_{2022} \\ a_2a_1 & a_2^2 & a_2a_3 & \dots\dots & a_2\ a_{2022} \\ a_3a_1 & a_3a_2 & a_3^2 & \dots\dots & a_3\ a_{2022} \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ : & : & : & : & : \\ a_{2022}\ a_1 & a_{2022}\ a_2 & \dots & \dots & a_{2022}^2 \end{bmatrix}$$

We can see that yy^T is a symmetric matrix

$$\det\left(yy^T\right) = \begin{vmatrix} a_1^2 & a_1a_2 & a_1a_3 & \dots\dots & a_1\ a_{2022} \\ a_2a_1 & a_2^2 & a_2a_3 & \dots\dots & a_2\ a_{2022} \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ a_{2022}a_1 & \dots\dots & \dots\dots & \dots\dots & a_{2022}^2 \end{vmatrix}$$

$$= (a_1a_2\dots\dots a_{2022}) \begin{vmatrix} a_1 & a_2 & a_3 & \dots\dots & a_{2022} \\ a_1 & a_2 & a_3 & \dots\dots & a_{2022} \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ a_1 & a_2 & a_3 & \dots\dots & a_{2022} \end{vmatrix}$$

Hence, $\det(yy^T) = 0$

i.e., rank of $(yy^T) < 2022$

yy^T is not invertible

2021 rows are identical after elementary transformation hence out of 2022 eigen values 0 eigen value will occur 2021 times.

Remaining eigen value = Trace (yy^T)

$= a_1^2a_2^2 + \dots a_{2022}^2$

$= y^T y$

46. Which of the following statement(s) is/are correct?

A. If a linearly elastic structure is subjected to a set of loads, the partial derivative of the total strain energy with respect to the deflection at any point is equal to the load applied at that point.

B. If a linearly elastic structure is subjected to a set of loads, the partial derivative of the total strain energy with respect to the load at any point is equal to the deflection at that point.

C. If a structure is acted upon by two force system P_a and P_b , in equilibrium separately, the external virtual work done by a system of forces P_b during the deformations caused by another system of forces P_a is equal to the external virtual work done by the P_a system during the deformation caused by the P_b system.

D. The shear force in a conjugate beam loaded by the M/EI diagram of the real beam is equal to the corresponding deflection of the real beam.

[MSQ: 2 Marks]

Ans. (A, B, C)

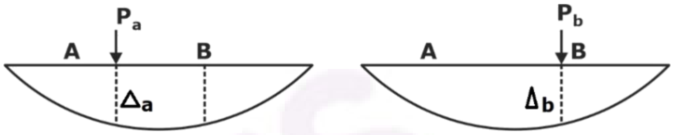
Sol.

Castigliano's first theorem states that the first partial derivative of the total internal energy (strain energy) in a structure with respect to any particular deflection component at a point is equal to the force applied at that point and in the direction corresponding to that deflection component.

$$\frac{\partial U}{\partial \delta} = P$$

Castigliano's second theorem states that the first partial derivative of the total internal energy in a structure with respect to the force applied at any point is equal to the deflection at the point of application of that force in the direction of its line of action.

$$\frac{\partial u}{\partial P} = \delta$$



From Betti's theorem, $P_a \Delta_b = P_b \ \Delta_a$

i.e. virtual work done by a system of forces P_b during the deformations caused by another system of forces P_a is equal to the virtual work done by a system of forces P_a during the deformations caused by P_b system.

Hence A, B and C are correct.

47. Water is flowing in a horizontal, frictionless, rectangular channel. A smooth hump is built on the channel floor at a section and its height is gradually increased to reach choked condition in the channel. The depth of water at this section is y_2 and that at its upstream section is y_1. The correct statement(s) for the choked and unchoked conditions in the channel is/are

A. In choked condition, y_1 decreases if the flow is supercritical and increases if the flow is subcritical. B. In choked condition, y_2 is equal to the critical depth if the flow is supercritical or subcritical.

C. In unchoked condition, y_1 remains unaffected when the flow is supercritical or subcritical.

D. In choked condition, y_1 increases if the flow is supercritical and decreases if the flow is subcritical.

[MSQ: 2 Marks]

Ans. A, B, C

48. The concentration (x, t) of pollutants in a one-dimensional reservoir at position x and time t satisfies the diffusion equation

$$\frac{\partial s(x,t)}{\partial t} = D\frac{\partial^2 s(x,t)}{\partial x^2}$$

on the domain $0 \le x \le L$, where D is the diffusion coefficient of the pollutants. The initial condition $s(x, 0)$ is defined by the step-function shown in the figure.

The boundary conditions of the problem are given by $\frac{\partial s(x,t)}{\partial t} = 0$ at the boundary point x = 0 and x = L at all times. Consider D = 0.1 m^2/s. S_0 = 5 μmol/m, and L = 10 ,

The steady concentration $\tilde{S}\left(\frac{L}{2}\right) = S\left(\frac{L}{2}, \infty\right)$ at the center $x = \frac{L}{2}$ of the reservoir (inμ/mol/m) ______. (in integer)

[NAT: 2 Marks]

Ans. (2–2)

Sol. $\frac{\partial^2 S}{\partial x^2}(x,t) = \frac{1}{D}\frac{\partial S}{\partial t}(x,t)$

Under steady state condition

$\Rightarrow \frac{\partial S}{\partial t} = 0$

$\Rightarrow \frac{\partial^2 S}{\partial x^2} = 0$

$\Rightarrow \frac{d^2 s}{dx^2} = 0$

$\Rightarrow \frac{ds}{dx} = A$

$\Rightarrow \boxed{S = A_x + B}$ ......(1)

At x = 0, S = S_0

i) $\Rightarrow S_0 = O + B$

$\boxed{B = S_o}$

And at x = L S = O

i) $\Rightarrow O = AL + S_0$

$\boxed{A = -\frac{S_o}{L}}$ ......(2)

Put A and B in (1)

i) $\boxed{S = -\frac{S_o}{L} \cdot x + S_o}$

$\boxed{S = (x, t) = -\frac{S_o}{L} \cdot x + S_o}$ .....(3)

At $x = \frac{L}{2}$

$S\left(\frac{L}{2}, \infty\right) = \frac{-S_o}{L} \cdot \frac{L}{2} + S_o = \frac{-S_o}{L} + S_o$

$S\left(\frac{L}{2}, \infty\right) = \frac{S_o}{2}$

Given

$$S\left(\frac{L}{2},\infty\right)=\frac{5}{2}=2.5$$

$$S\left(\frac{L}{2},\infty\right)=2 \text{ (only integer part)}$$

49. A pair of six-faced dice is rolled thrice. The probability that the sum of the outcomes in each roll equals 4 in exactly two of the three attempts is _______. (round off to three decimal places)

[NAT: 2 Marks]

Ans. (0.018-0.020)

Sol. Required outcomes

$\equiv$ (1, 3) (2, 2) and (3, 1)

Probability of success, $p=\frac{3}{36}=\frac{1}{12}$

Probability of failure, $q=\frac{11}{12}$

Probability of 2 success in 3 attempts

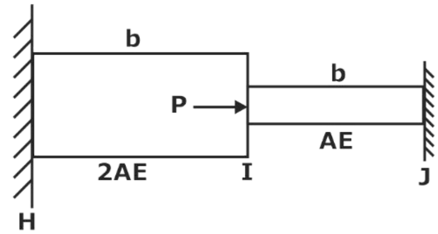
$=3c_2p^2q^1$

$=3c_2\times\left(\frac{1}{12}\right)^2\left(\frac{11}{12}\right)$

= 0.01909

≈ 0.019

50. Consider two linearly elastic rods HI and IJ, each of length b, as shown in the figure. The rods are co-linear, and confined between two fixed supports at H and J. Both the rods are initially stress free. The coefficient of linear thermal expansion is α for both the rods. The temperature of the rod IJ is raised by ΔT, whereas the temperature of rod HI remains unchanged. An external horizontal force P is now applied at node I. It is given that $\alpha = 10^{-6}$°C^{-1}, ΔT = 50°C, b= 2m AE= 10^6N. The axial rigidities of the rods HI and IJ are 2AE and AE, respectively.



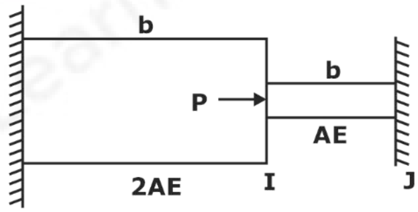
To make the axial force in rod HI equal to zero, the value of the external force P (in N) is __________. (round off to the nearest integer)

[NAT: 2 Marks]

Ans. (47-53)

Sol. To have zero axial force in rod HI, axial deformation of HI rod shall be zero.

$\Rightarrow \boxed{\Delta_{\text{axial load}}=\Delta_{\text{temp}}}$ .....(1)



By temperature raise of rod IJ

$\Delta_{\text{temp}} = \ell\alpha\Delta t$

$\Delta_{\text{temp}} = b\alpha\Delta t$

By force 'P', deflection is

$$\Delta_{\text{axial load}}=\frac{Pb}{AE}$$

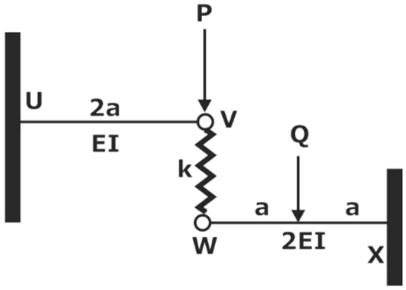
$$\Rightarrow \frac{Pb}{AE}=b\alpha\Delta t$$

P = AE × αΔt

$= 10^6 \times (10^{-6} \times 50) = 50N$

51. The linearly elastic planar structure shown in the figure is acted upon by two vertical concentrated forces. The horizontal beams UV and WX are connected with the help of the vertical linear spring with spring constant k = 20 kN/m. The fixed supports are provided at U and X. It is given that flexural rigidity

$EI = 10^5$ kN-m^2, P = 100 kN, and a = 5 m. Force Q is applied at the center of beam WX such that the force in the spring VW becomes zero.

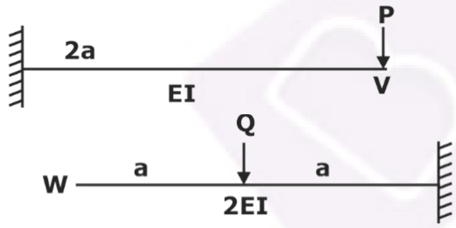


The magnitude of force Q (in kN) is ________. (round off to the nearest integer)

[NAT: 2 Marks]

Ans. (620-660)

Sol. For force in spring to be zero, deflection of point V and deflection of point w shall be equal



$\Rightarrow \delta_v = \delta_w$

$$\frac{P(2a)^3}{3EI} = \frac{Qa^3}{3(2EI)} + \frac{Qa^2 \times a}{2(2EI)}$$

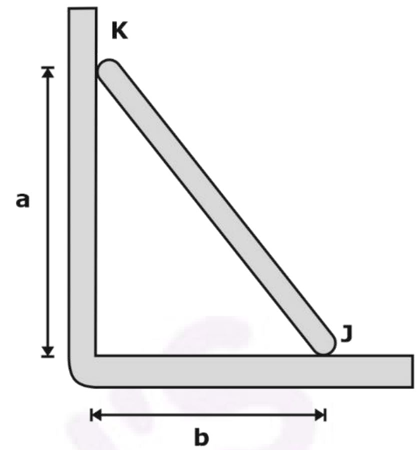
$$\frac{8}{3}P = \frac{25}{6}Q$$

$$\Rightarrow Q = \frac{32}{5}P$$

$$Q = \frac{32 \times 100}{5}P$$

$$\boxed{Q = 640\,\text{kN}}$$

52. A uniform rod KJ of weight w shown in the figure rests against a frictionless vertical wall at the point K and a rough horizontal surface at point J. It is given that w = 10 kN, a = 4 m and b = 3 m.

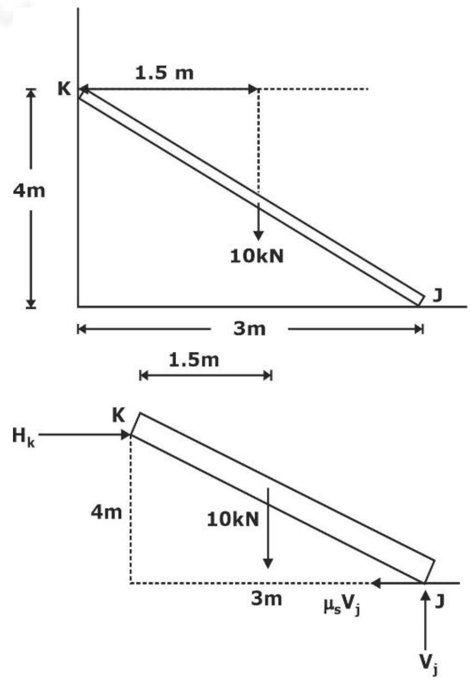


The minimum coefficient of static friction that is required at the point J to hold the rod in equilibrium is ___________. (round off to three decimal places)

[NAT: 2 Marks]

Ans. (0.350-0.40)

Sol.



Free body diagram

$\Sigma v = 0 \Rightarrow \boxed{V_j = 10\,kN}$

$\Sigma M_k = 0$

$\Rightarrow V_j \times 3 - 10 \times 1.5 - \mu_s V_j \times 4 = 0$

$\Rightarrow 10 \times 3 - 15 - \mu_s \times 10 \times 4 = 0$

$\boxed{\mu_s = 0.375}$

53. The activities of a project are given in the following table along with their durations and dependency.

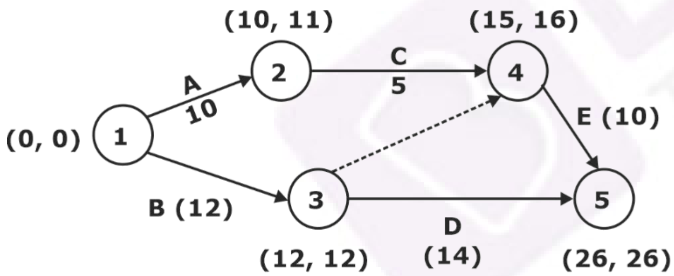
Activities	Duration (days)	Depends on
A	10	-
B	12	-
C	5	A
D	14	A
E	10	B, C

The total float of the activity E (in days) is ________. (in integer)

[NAT: 2 Marks]

Ans. (1 to 1)

Sol.



Total float of the activity E

$F_T = T_L^5 - T_E^4 - t_E$

$= 26 - 15 - 10 = 1$ day

$\{F_T^{ij} = T_T^j - T_E^i - t_{ij}\}$

54. A group of total 16 piles are arranged in a square grid format. The center-to center spacing (s) between adjacent piles is 3 m. The diameter (d) and length of embedment of each pile are 1 m and 20 m, respectively. The design capacity of each pile is 1000 kN in the vertical downward direction. The pile group efficiency (η_g) is given by

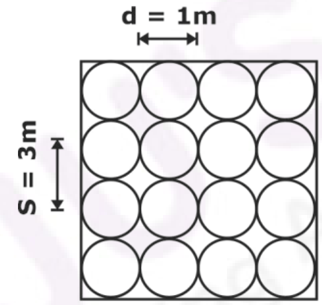
$$\eta_g = 1 - \frac{\theta}{90}\left[\frac{(n-1)m + (m-1n)}{mm}\right]$$

where m and n are number of rows and columns in the plan grid of pile arrangement, and $\theta = \tan^{-1}\left(\frac{d}{s}\right)$.

[NAT: 2 Marks]

Ans. (11000-11200)

Sol.



$\theta = \tan^{-1}\left(\frac{d}{s}\right)$

$\theta = \tan^{-1}\left(\frac{1}{3}\right)$

$\boxed{\theta = 18.435°}$

m = 4, n = 4

Pile group efficiency (ng)

$$\eta_g = 1 - \frac{\theta}{90}\left[\frac{(n-1)^{n-1} n(n-1)}{nn}\right]$$

$$\eta_g = 1 - \frac{18.345}{90}\left[\frac{(4-1) \times 4 + 4(4-1)}{4 \times 4}\right]$$

$\boxed{\eta_g = 0.693}$

design value of pile group

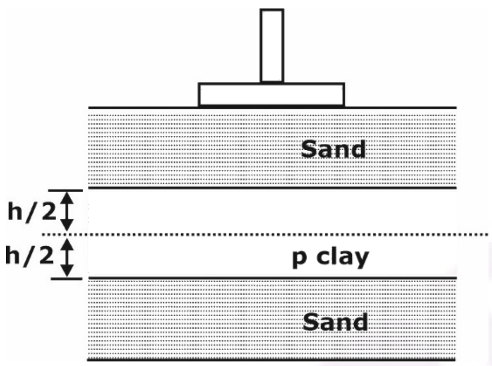
Capacity = $\eta_g \times 16 \times 1000$

= 0.693 × 16 × 1000

= $\boxed{11088\,kN}$

55. A saturated compressible clay layer of thickness h is sandwiched between two sand layers, as shown in the figure. Initially, the total vertical stress and pore water pressure at point P, which is located at the mid-depth of the clay layer, were 150 kPa and 25 kPa, respectively. Construction of a building caused an additional total vertical stress of 100 kPa at P. When the vertical effective stress at P is 175 kPa, the percentage of consolidation in the clay layer at P is_______. (in integer)

[NAT: 2 Marks]



[NAT: 2 Marks]

Ans. (50-50)

Sol. At point 'p' initial effective stress

$\sigma_P' = 150 - 25 = 125$ kPa

After construction of building, effective stress at P.

$\sigma_{P2}' = 125 + 100 = 225$ kPa

Percentage of consolidation when effective stress at P is 175 kPa

$$= \frac{175-125}{225-125} \times 100$$

= 50%

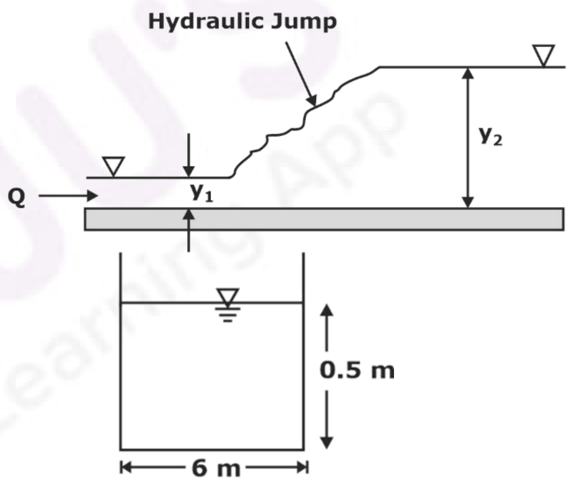
56. A hydraulic jump takes place in a 6 m wide rectangular channel at a point where the upstream depth is 0.5 m (just before the jump). If the discharge in the channel is 30 m3/s and the energy loss in the jump is 1.6 m, then the Froude number computed at the end of the jump is ___________. (round off to two decimal places)

(Consider the acceleration due to gravity as 10 m/s^2.)

[NAT: 2 Marks]

Ans. (0.30-0.41)

Sol.



discharge Q = 30m^3/sec

Width (B) = 6m

Energy loss (E_L) = 1. 6 m

Pre-jump depth = y_1 = 0.5 m

$$E_L = \frac{(y_2 - y_1)^3}{4y_1y_2}$$

$$1.6 = \frac{(y_2 - 0.5)^3}{4 \times 0.5 \times y_2}$$

$3.2y_2 = y_2^3 - 3y_2^2 \times 0.5 + 3y_2(0.5)^2 - 0.5^3$

$\Rightarrow y_2^3 - 1.5y_2^2 - 2.45y_2 - 0.125 = 0$

Y_2 = 2.5 m, – 0.0527 m – 0.947m

$\Rightarrow \boxed{y_2 = 2.5\text{m}}$

Froude number after jump (Fr_2)

$$Fr_2 = \frac{V_2}{\sqrt{gy_2}}$$

$$v_2 = \frac{Q}{By_2} = \frac{30}{6 \times 2.5}$$

V_2 = 2m/sec.

$$\Rightarrow Fr_2 = \frac{2}{\sqrt{9.81 \times 2.5}}$$

Fr_2 = 0.4038

$$\boxed{Fr_2 = 0.40}$$

57. A pump with an efficiency of 80% is used to draw groundwater from a well for irrigating a flat field of area 108 hectares. The base period and delta for paddy crop on this field are 120 days and 144 cm, respectively. Water application efficiency in the field is 80%. The lowest level of water in the well is 10 m below the ground. The minimum required horse power (h.p.) of the pump is________. (round off to two decimal places)

(Consider 1 h.p. = 746 W; unit weight of water = 9810 N/m^3)

[NAT: 2 Marks]

Ans. (30-32)

Sol. Base period (B) = 120 days

Delta (Δ) = 144 cm

$$\text{Duty (D)} = \frac{864\,B}{\Delta} = \frac{864 \times 120}{144}$$

= 720 ha/cumecs

$$\text{Discharge, } Q = \frac{\text{Area}}{\text{Duty}} = \frac{108}{720} = 0.15\,m^3/s$$

$$Q_{applied} = \frac{Q}{\eta_a} = \frac{0.15}{0.8} = \frac{3}{16} m^3/s$$

Power required = $\gamma_w Q_{applied} H$

$$= 9810 \times \frac{3}{16} \times 10$$

= 18400 W

= 24.66 HP

$$\text{Horse power of pump} = \frac{\text{Power required}}{\eta_{pump}}$$

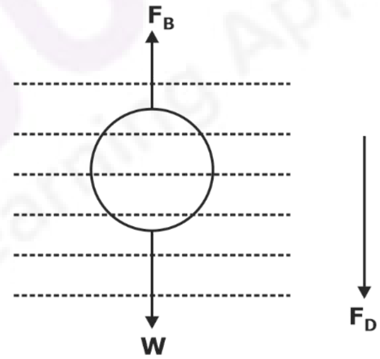
$$= \frac{24.66}{0.8} = 30.83 \text{ HP}$$

58. Two discrete spherical particles (P and Q) of equal mass density are independently released in water. Particle P and particle Q have diameters of 0.5 mm and 1.0 mm, respectively. Assume Stokes' law is valid. The drag force on particle Q will be________ times the drag force on particle P. (round off to the nearest integer)

[NAT: 2 Marks]

Ans. (8-8)

Sol.



Drag force (F_D) = wt. of the sphere (W) – Buoyant force (F_B)

$$F_D = \rho_{sphere} V g - \rho_{water} V g$$

$$= (\rho_{sphere} - \rho_{water}) g \times \frac{4}{3}\pi r^3$$

$$F_D \propto r^3$$

$$\Rightarrow F_D \propto D^3$$

$$\frac{F_{D,Q}}{F_{D,P}} = \left(\frac{D_Q}{D_P}\right)^3 = \left(\frac{1}{0.5}\right)^3 = 8$$

$$\boxed{F_{D,Q} = 8\,F_{D,P}}$$

59. At a municipal waste handling facility, 30 metric ton mixture of food waste, yard waste, and paper waste was available. The moisture content of this mixture was found to be 10%. The ideal moisture content for composting this mixture is 50%. The amount of water to be added to this mixture to bring its moisture content to the ideal condition is _______metric ton. (in integer)

[NAT: 2 Marks]

Ans. (24-24)

Sol. MSW = 30 metric ton

Moisture content = 10%

Amount of water present

$= \frac{10}{100} \times 30 = 3 \text{ metric ton}$

Amount of solids present

= 30-3

= 27 metric ton

Ideal moisture content = 50%

Amount of solids will not change.

Let the amount of water present in the ideal sample be W_w.

Then, $\frac{W_w}{27 + W_w} \times 100 = 50$

W_w = 27 metric ton

Amount of water required to be added to bring to ideal condition

= 27 – 3

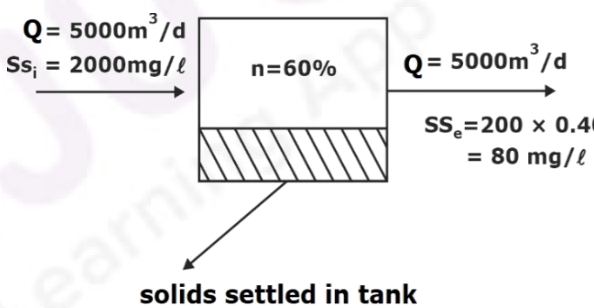
= 24 metric ton

60. A sewage treatment plant receives sewage at a flow rate of 5000 m^3/day. The total suspended solids (TSS) concentration in the sewage at the inlet of primary clarifier is 200 mg/L. After the primary treatment, the TSS concentration in sewage is reduced by 60%. The sludge from the primary clarifier contains 2 % solids concentration. Subsequently, the sludge is subjected to gravity thickening process to achieve a solids concentration of 6%. Assume that the density of sludge, before and after thickening, is 1000 kg/m^3. The daily volume of the thickened sludge (in m^3/day) will be_________.(round off to the nearest integer)

[NAT: 2 Marks]

Ans. (10-10)

Sol.



Inflow rate of sewage = 5000 m^3/day

Concentration of sewage at inlet = 200 mg/L

Amount of total suspended solids at inlet

= 5000×10^3 lit/day $\times 200 \times 10^{-6}$ kg/l

= 1000 kg/day

Amount of solids settled in tank

$= \frac{60}{100} \times 1000 \text{ kg/day}$ = 600 kg/day

Given: Solids concentration is 2 %

If the total weight of the sludge is W kg/day

Then, $\frac{2}{100} \times W = 600$

W= 30000 kg/day

Moisture content before thickening = 98 %

Moisture content after thickening= 94 %

Amount of solids before and after thickening remains the same.

Hence, 600 = W' (1-0.94)

where, W': weight of sludge after thickening

W' = 10000 kg/day

Daily volume of thickened sludge

$$= \frac{10000}{1000} = 10\ m^3$$

61. A sample of air analyzed at 25°C and 1 atm pressure is reported to contain 0.04 ppm of SO_2. Atomic mass of S = 32, O = 16.

The equivalent SO_2 concentration (in μg/m^3) will be__________. (round off to the nearest integer)

[NAT: 2 Marks]

Ans. (102-108)

Sol. From combined gas law equation, we have:

$$\frac{P_1V_1}{T_1} = \frac{P_2V_2}{T_2}$$

Since, 1 mol of SO_2 at STP (0ºC, 1 atm) occupies vol. of 22.4 lit

P_1 = 1 atm

V_1 = 22.4 lit

T_1 = 0°C = 273 k

P_2 = 1 atm

T_2 = 25°C = 298 k

$$\Rightarrow \frac{1 \times 22.4}{273} = \frac{1 \times V_2}{298}$$

V_2 = 24.45 lit

Given that air contains 0.04 ppm of SO_2

i.e. in 10^6 m^3 of air amount of SO_2 present is 0.04 m^3 (or 40 lit)

Moles of SO_2 in 10^6 m^3 of air = 40/24.45

Weight of SO_2 in 10^6 m^3 of air

$$= \frac{40}{24.45} \times 64\,gm$$

= 104.7 g

Equivalent SO_2 concentration

$$= \frac{104.7}{10^6}\,gm/m^3$$

$$\approx 105\,\mu g/m^3$$

62. A parabolic vertical crest curve connects two road segments with grades +1.0% and –2.0%. If a 200 m stopping sight distance is needed for a driver at a height of 1.2 m to avoid an obstacle of height 0.15 m, then the minimum curve length should be ______ m. (round off to the nearest integer)

[NAT: 2 Marks]

Ans. (270–275)

Sol.

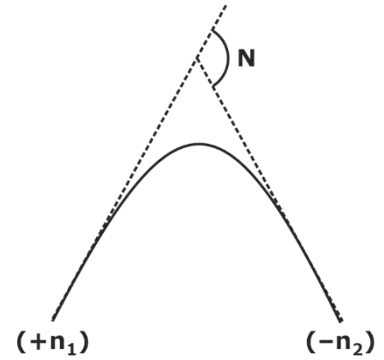
n_1 =+ 1

n_2 = –2%

N= n_1 – n_2

N= 1 – (–2) = + 3%

SSD = 200 m



So, it's a submit curve

L > SSD or OSD

The general equation for length L of the parabolic curve is given by:

$$L = \frac{NS^2}{(\sqrt{2H} + \sqrt{2h})^2}$$

where, L = length of summit curve, m

S = sight distance

N =Deviation angle, equal to the algebraic difference in grades, radians or tangent of deviation angle.

H = height of eye level of the driver above roadway surface, m

h = height of an object above the road surface, m

For SSD (H = 1.2 and h = 0.15 m)

$$L = \frac{NS^2}{4.4}$$

$$L = \frac{3}{100} \times \frac{(200)^2}{4.4}$$

L= 272.72 m > 200 m Hence OK

$L \simeq 273\,m$

63. Assuming that traffic on a highway obeys the Greenshields model, the speed of a shockwave between two traffic streams (P) and (Q) as shown in the schematic is _______ kmph. (in integer)

Direction of Traffic

(P) Flow = 1200 vehicles

Speed = 60 kmph

(Q) Flow = 1800 vehicles/hour

Speed = 30 kmph

[NAT: 2 Marks]

Ans. (15–15)

Sol.

Direction of Traffic

(P) Flow = 1200 vehicles

Speed = 60 kmph

(Q) Flow = 1800 vehicles/hour

Speed = 30 kmph

$$\text{Speed} = \frac{\partial q}{\partial k} = \frac{q_Q - k_p}{k_Q - k_p}$$

∂q = change in flow

∂k = change in density

q_Q = 1800 veh/h

q_P = 1800 veh/h

q_p = 1200 veh/h

$$k_Q = \frac{q_Q}{v_Q} = \frac{1800}{30} = 60 \text{ veh/km}$$

$$k_p = \frac{1200}{60} = 20 \text{ veh/cm}$$

$$\text{Speed} = \frac{1800 - 1200}{60 - 20} = 15\,\text{kmph}$$

64. It is given that an aggregate mix has 260 grams of coarse aggregates and 240 grams of fine aggregates. The specific gravities of the coarse and fine aggregates are 2.6 and 2.4, respectively. The bulk specific gravity of the mix is 2.3.

The percentage air voids in the mix is ____________. (round off to the nearest integer)

[NAT: 2 Marks]

Ans. (8–8)

Sol. G_m = 2.3

G_{CA} = 2.6

G_{FA} = 2.4

W_{CA} = 260 gm

W_{FA} = 240 gm

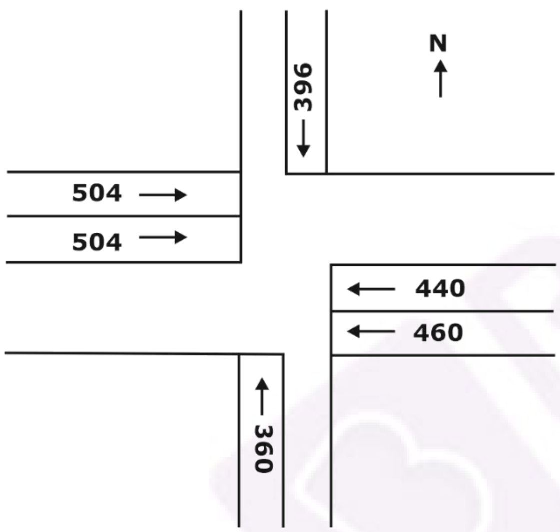
$$G_t = \frac{W_{FA} + W_{CA}}{\frac{W_{FA}}{G_{FA}} + \frac{W_{CA}}{G_{CA}}} = \frac{260 + 240}{\left(\frac{260}{2.6}\right) + \left(\frac{240}{2.4}\right)} = 2.5$$

% air void

$$= \frac{G_t - G_m}{G_t} \times 100 = \frac{2.5 - 2.3}{2.5} \times 100 = 8\%$$

65. The lane configuration with lane volumes in vehicles per hour of a four-arm signalized intersection is shown in the figure. There are only two phases: the first phase is for the East-West and the West-East through movements, and the second phase is for the North-South and the South-North through movements. There are no turning movements. Assume that the saturation flow is 1800 vehicles per hour per lane for each lane and the total lost time for the first and the second phases together is 9 seconds.

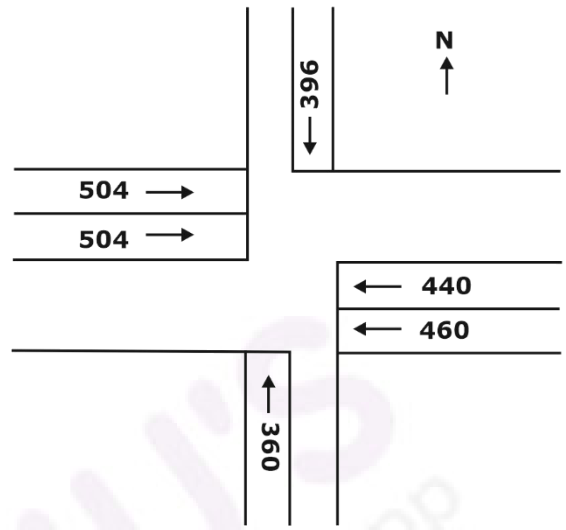
[NAT: 2 Marks]



The optimum cycle length (in seconds), as per the Webster's method, is ________. (round off to the nearest integer)

Ans. (37–37)

Sol.



q_S = 1800 veh/h

For N–S

$$y_{N-S} = \frac{396}{180} = 0.22$$

For E – W

$$y_{EW} = \frac{504}{1800} = 0.28$$

$$Co = \frac{1.5L+5}{1-y} = \frac{1.5\times 9+5}{1-(0.22+0.20)} = 37\,\text{sec}$$

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