MATHEMATICS

SECTION - A

Multiple Choice Questions: This section contains 20 multiple choice questions. Each question has 4 choices (1), (2), (3) and (4), out of which **ONLY ONE** is correct.

Choose the correct answer :

1. If $\vec{a} = \hat{i} + 2\hat{j} + \hat{k}$

$\vec{b} = 3(\hat{i} - \hat{j} + \hat{k})$, $\vec{a} \cdot \vec{c} = 3$

$\vec{a} \times \vec{c} = \vec{b}$ then $\vec{a} \cdot [(\vec{c} \times \vec{b}) - \vec{b} - \vec{c}] =$

(1) 24 (2) 38

(3) 10 (4) None of these

Answer (1)

Sol. $\vec{a} \cdot [(\vec{c} \times \vec{b}) - \vec{b} - \vec{c}]$

$\Rightarrow \vec{a} \cdot (\vec{c} \times \vec{b}) - \vec{a} \cdot \vec{b} - \vec{a} \cdot \vec{c}$ ...(i)

Now $\vec{a} \times \vec{c} = \vec{b}$

$\Rightarrow (\vec{a} \times \vec{c}) \cdot \vec{b} = \vec{b} \cdot \vec{b} = |\vec{b}|^2 = 27$

$\Rightarrow [\vec{a}\,\vec{c}\,\vec{b}] = 27$

From (i)

27 – 0 – 3 = 24

$\therefore$ Option (1) is correct

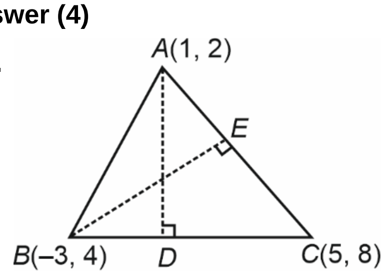
2. The vertices of a triangle *ABC* are *A*(1, 2), *B*(–3, 4), *C*(5, 8), then orthocentre of ΔABC is

(1) $\left(\frac{2}{3}, 1\right)$ (2) $\left(-\frac{7}{3}, 2\right)$

(3) (2, 3) (4) $\left(\frac{3}{2}, 1\right)$

Answer (4)

Sol.



$AD : (y - 2) = -2(x - 1)$

$BE : (y - 4) = -\frac{2}{3}(x + 3)$

Intersection of *AD* and *BE* : $H\left(\frac{3}{2}, 1\right)$

3. $S_1 = 3, 9, 15, \ldots$ 25 terms

$S_2 = 3, 8, 13, \ldots$ 37 terms

Number of common terms in S_1, S_2 is equal to

(1) 3 (2) 4

(3) 5 (4) 6

Answer (3)

Sol. $S_1 = 3, 9, 15 \ldots. 147$

$d_1 = 6, a_1 = 3$

$S_2 = 3, 8, 13, \ldots 183$

$d_2 = 5, a_2 = 3$

LCM $(d_1, d_2) = 30$

$\therefore$ 3, 33, ... 123

$\therefore$ $123 = 3 + (n - 1)30$

$\Rightarrow$ $n = 5$

$\therefore$ Number of common terms = 5

4. $\int_0^1 \frac{1}{\sqrt{3+x} + \sqrt{1+x}} dx = a + b\sqrt{2} + c\sqrt{3}$, then

$2a - 3b - 4c$ is equal to

(1) 10 (2) 0

(3) 12 (4) 20

Answer (3)

Sol. $I = \int_0^1 \frac{1}{\sqrt{x+3} + \sqrt{x+1}} dx$

On rationalization

$I = \int_0^1 \frac{\sqrt{x+3} - \sqrt{x+1}}{2} dx$

$I = \int_0^1 \frac{(x+3)^{1/2}}{2} dx - \int_0^1 \frac{(x+1)^{1/2}}{2} dx$

$I = \frac{(x+3)^{3/2}}{3}\Big|_0^1 - \frac{(x+1)^{3/2}}{3}\Big|_0^1$

$= \frac{1}{3}[4^{3/2} - 3^{3/2}] - \frac{1}{3}[2^{3/2} - 1^{3/2}]$

$= \frac{8 - 3\sqrt{3} - 2\sqrt{2} + 1}{3} = 3 - \sqrt{3} - \frac{2}{3}\sqrt{2}$

$\Rightarrow$ $2a = 6$

$3b = -2$

$4c = -4$

$2a - 3b - 4c = 6 + 2 + 4 = 12$

5. If ${}^{n-1}C_r = (k^2-8)\,{}^{n}C_{r+1}$ then

(1) $k \in \left[-3, -2\sqrt{2}\right) \cup \left(2\sqrt{2}, 3\right]$

(2) $k \in \left[-4, -2\sqrt{3}\right) \cup \left(2\sqrt{3}, 4\right]$

(3) $k \in \left[-2\sqrt{3}, 4\right]$

(4) $k \in \left[3, 2\sqrt{3}\right]$

Answer (1)

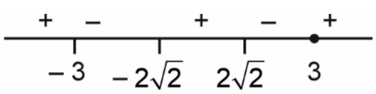
Sol. ${}^{n-1}C_r = (k^2-8) \times \frac{n}{r+1}\,{}^{n-1}C_r$

$\frac{1}{k^2-8} = \frac{n}{r+1} \qquad (n \ge r+1)$

$\Rightarrow \frac{1}{k^2-8} \ge 1$

$\frac{1-(k^2-8)}{k^2-8} \ge 0$

$\frac{k^2-9}{k^2-8} \le 0$



$k \in \left[-3, 2-\sqrt{2}\right) \cup \left(2\sqrt{2}, 3\right]$

6. The value of k for $(2k, 3k)$, $(0, 0)$, $(1, 0)$ and $(0, 1)$ to be on the circle is

(1) $\frac{2}{13}$ (2) $\frac{5}{13}$

(3) $\frac{1}{13}$ (4) $\frac{2}{13}$

Answer (2)

Sol. Circle passing through (0, 0), (1, 0) and (0, 1) will be a circle having (1, 0) and (0, 1) as the end points of diameter.

$C: (x-1)x + y(y-1) = 0$

$x^2 + y^2 - x - y = 0$

Now $(2k, 3k)$ lies on C.

$4k^2 + 9k^2 - 2k - 3k = 0$

$13k^2 - 5k = 0$

$k(13k - 5) = 0$

$\Rightarrow k = \frac{5}{13}$

7. Shortest distance between the parabola $y^2 = 4x$ and $x^2 + y^2 - 4x - 16y + 64 = 0$ is equal to

(1) $2\sqrt{3}-2$ (2) $3\sqrt{2}-3$

(3) $4\sqrt{5}-2$ (4) $2\sqrt{5}-2$

Answer (4)

Sol. $N : y = mx - 2am - am^3$

$N : y = mx - 2m - m^3$

It passes through (2, 8)

$8 = 2m - 2m - m^3$

$\Rightarrow m = -2$

$\therefore$ $N : y + 2x = 12$

Point of intersection of normal with $y^2 = 4x$ is (4, 4)

$\therefore$ Shortest distance : $\sqrt{(4-2)^2+(4-8)^2} - \sqrt{4+64-64}$

$= \sqrt{20} - 2$

$= 2\sqrt{5} - 2$

8. Let $f(x) = \begin{cases} 2^{\frac{\sin(x-3)}{x-[x]}} & x < 3 \\ \frac{a\left|-x^2-12+7\right|}{b\left(x^2+12x-7\right)} & x > 3 \\ b & x = 3 \end{cases}$

is continuous at $x = 3$, then (a, b) is

(1) (2, 3) (2) (1, 2)

(3) $(2^{\sin 2}, 3)$ (4) None of these

Answer (4)

Sol. $\lim_{x\to 3^-} f(x)$

$\lim_{x\to 3^-} 2^{\frac{\sin(x-3)}{\{x\}}} = 2^0 = 1 = b = f(3)$

$\therefore$ $b = 1$

$= \lim_{x\to 3^+} \frac{a}{b} \frac{\left|-x^2-12x+7\right|}{\left(x^2+12x-7\right)}$

$= \frac{a}{b}(1)$

$\lim_{x\to 3^+} f(x) = \lim_{x\to 3^-} f(x) = f(3)$

$\frac{a}{b} = 1 = b$

$\Rightarrow a = 1$

$\therefore$ $(a, b) \equiv (1, 1)$

$\therefore$ 4th option is correct

9. If $f(x)-f(y)=\ln\left(\frac{x}{y}\right)+x-y$, then find $\sum_{k=1}^{20} f'\left(\frac{1}{k^2}\right)$

(1) 2890 (2) 2390

(3) 1245 (4) None of these

Answer (1)

Sol. Rearranging,

$f(x) - \ln(x) - x = f(y) - \ln y - y$

$\Rightarrow f(x) - \ln(x) - x = c$ (some constant)

$\Rightarrow f(x) = c + x + \ln x$

$$f'(x) = 0+1+\frac{1}{x}$$

$$f'\left(\frac{1}{k^2}\right) = 1+\frac{1}{1/k^2} = (1+k^2)$$

$$\sum_{k=1}^{20}(1+k^2) = \sum_{k=1}^{20}1+\sum_{k=1}^{20}k^2$$

$$= 20+\frac{20\times21\times41}{6}$$

$= 20 + 2870 = 2890$

10. If $f(x)=\begin{bmatrix}\cos x & -\sin x & 0\\ \sin x & \cos x & 0\\ 0 & 0 & 1\end{bmatrix}$

Then

S-I : $f(x).f(y) = f(x + y)$.

S-II : $f(-x) = 0$ is invertible.

(1) S-I True, S-II False (2) S-I True, S-II True

(3) S-I False, S-II True (4) S-I False, S-II False

Answer (2)

Sol. $f(x)=\begin{bmatrix}\cos x & -\sin x & 0\\ \sin x & \cos x & 0\\ 0 & 0 & 1\end{bmatrix}$

S-I:

$$f(x).f(y)=\begin{bmatrix}\cos x & -\sin x & 0\\ \sin x & \cos x & 0\\ 0 & 0 & 1\end{bmatrix}\begin{bmatrix}\cos y & -\sin y & 0\\ \sin y & \cos y & 0\\ 0 & 0 & 1\end{bmatrix}$$

$$=\begin{bmatrix}\cos(x+y) & -\sin(x+y) & 0\\ \sin(x+y) & \cos(x+y) & 0\\ 0 & 0 & 1\end{bmatrix}=f(x+y)$$

$\therefore$ S-I is true

Now,

$$f(-x)=\begin{bmatrix}\cos x & \sin x & 0\\ -\sin x & \cos x & 0\\ 0 & 0 & 1\end{bmatrix}$$

$\det(f(-x)) = \cos^2 x + \sin^2 x = 1$

$\Rightarrow |f(-x)| \neq 0$

$\therefore$ Non-singular

$\therefore$ S-II is true

11. If $\lim_{x\to0}\frac{\sqrt{1+\sqrt{1+x^4}}-\sqrt{2}}{x^4}=A$ and $\lim_{x\to0}\frac{\sin^2 x}{\sqrt{2}-\sqrt{1+\cos x}}=B$ then $AB^3 =$

(1) 8 (2) 32

(3) 6 (4) None of these

Answer (2)

Sol. $A=\lim_{x\to0}\frac{\sqrt{1+\sqrt{1+x^4}}-\sqrt{2}}{x^4}$

$$=\lim_{x\to0}\frac{1+\sqrt{1+x^4}-2}{x^4\times\left(\sqrt{1+\sqrt{1+x^4}}+\sqrt{2}\right)}$$

$$=\lim_{x\to0}\frac{1+x^4-1}{x^4\times\left(\sqrt{1+\sqrt{1+x^4}}+\sqrt{2}\right)\left(\sqrt{1+x^4}+1\right)}$$

$$A=\frac{1}{2\sqrt{2}\times2}=\frac{1}{4\sqrt{2}}$$

$$B=\lim_{x\to0}\frac{\sin^2 x}{\sqrt{2}-\sqrt{1+\cos x}}$$

$$=\lim_{x\to0}\frac{\sin^2 x\left(\sqrt{2}+\sqrt{1+\cos x}\right)}{2-(1+\cos x)}$$

$$=\lim_{x\to0}\frac{\sin^2 x}{1-\cos x}\left(\sqrt{2}+\sqrt{1+\cos x}\right)$$

$$=\lim_{x\to0}\frac{\sin^2 x}{2\sin^2\frac{x}{2}}\left(\sqrt{2}+\sqrt{1+\cos x}\right)$$

$$=\frac{1}{2\times\frac{1}{4}}\times2\sqrt{2}$$

$$B=4\sqrt{2}$$

$$AB^3=\frac{1}{4\sqrt{2}}\times\left(4\sqrt{2}\right)^3=\left(4\sqrt{2}\right)^2=32$$

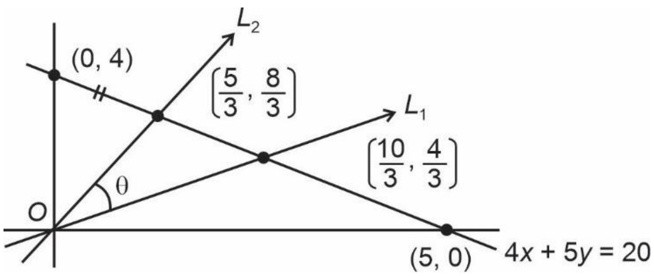
12. Two lines L_1 and L_2 passing through origin trisecting the line segment intercepted by the line $4x + 5y = 20$ between the coordinate axes. Then the tangent of angle between the lines L_1 and L_2 is

(1) $\sqrt{3}$ (2) $\frac{1}{\sqrt{3}}$

(3) 1 (4) $\frac{30}{41}$

Answer (4)

Sol.



$\Rightarrow \quad m_1 = \frac{4/3}{10/3} = \frac{2}{5}$ (Slope of line 1)

$m_2 = \frac{8/3}{5/3} = \frac{8}{5}$ (Slope of line 2)

Angle between L_1 and L_2 is

$$\tan\theta = \left|\frac{m_1 - m_2}{1 + m_1 m_2}\right| = \left|\frac{\frac{8}{5} - \frac{2}{5}}{1 + \frac{8}{5} \times \frac{2}{5}}\right|$$

$$= \left|\frac{\frac{6}{5}}{\frac{25 + 16}{25}}\right| = \frac{30}{41}$$

13. If $S = \{z : |z + i| = |z - i| = |z - 1|, \ z \in c\}$, then number of elements in set S is equal to

(1) 01 (2) 02

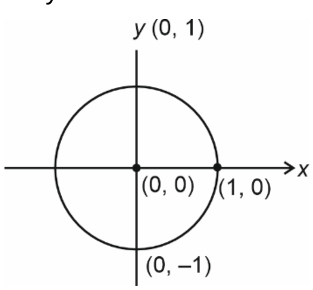
(3) 03 (4) 04

Answer (01)

Sol. z will be circumcentre of the triangle with vertices i, $-i$ and 1. (which is unique)

$\Rightarrow \quad z = 0 + 0i$

Only one element exists in S



14. If $\cos 2x - a\sin x = 2a - 7$, then range of a is

(1) $-2 \le a \le 0$ (2) $2 \le a \le 6$

(3) $a \ge 6$ (4) $6 \le a \le 8$

Answer (2)

Sol. $1 - 2\sin^2 x - a\sin x = 2a - 7$

$2\sin^2 x + a\sin x + 2a - 8 = 0$

$\sin x = \frac{-a \pm \sqrt{a^2 - 8(2a - 8)}}{4}$

$= \frac{-a \pm |a - 8|}{4}$

$= -2$ or $\frac{8 - 2a}{4}$

(-2) Not possible

$\therefore \ \sin x = \frac{8 - 2a}{4}$

$-1 \le \frac{8 - 2a}{4} \le 1$

$-4 \le 8 - 2a \le 4$

$2 \le a \le 6$

15. $A = \{1, 2, 3, \ldots 10\}$, S be the set of subset of A and $R = \{(a, b) : a, b \in S$ and $a \cap b \ne \phi\}$, then R is

(1) Reflexive only

(2) Symmetric only

(3) Symmetric and transitive

(4) Transitive only

Answer (2)

Sol. When $a = \phi$, then $a \cap a = \phi$,

hence $(a, a) \notin R \ \therefore \ R$ is not reflexive

as $(a, a) \ne \phi \ \forall \ a \in S$ is not true

for $(a, b) \in R \Rightarrow (b, a) \in R \ \forall a, b \in S$

as $a \cap b \ne \phi \Rightarrow b \cap a \ne \phi$

$\therefore \quad R$ is symmetric

For transitive,

Let $b = \{1, 2\}$, $c = \{1, 3, 4\}$, $d = \{4, 5, 6\}$

$b \cap c \ne \phi$, $c \cap d \ne \phi$, $b \cap d = \phi$

$\therefore \quad (b, c) \in R$, $(c, d) \in R \not\Rightarrow (b, d) \in R \ \forall b, c, d \in R$

$\therefore \quad R$ is not transitive

Option (2) is correct.

16. The shortest distance between the line $\frac{x-1}{2}=\frac{y+1}{4}=\frac{z-1}{3}$ and $\frac{2x-1}{5}=\frac{y-2}{3}=\frac{z}{6}$ is equal to

(1) 10 unit (2) $\frac{7}{10}$ unit

(3) 0 unit (4) $\frac{34}{\sqrt{1045}}$ unit

Answer (4)

Sol. $\frac{x-1}{2}=\frac{y+1}{4}=\frac{z-1}{3}$ and

$$\frac{x-\frac{1}{2}}{\left(\frac{5}{2}\right)}=\frac{y-2}{3}=\frac{z}{6},$$

$$\vec{a}_1-\vec{a}_2=\frac{1}{2}\hat{i}-3\hat{j}+\hat{k}$$

$$\vec{n}_1\times\vec{n}_2=\begin{vmatrix}\hat{i}&\hat{j}&\hat{k}\\2&4&3\\\frac{5}{2}&3&6\end{vmatrix}$$

$$=15\hat{i}-\frac{9}{2}\hat{j}+\left(-4k\right)$$

$$d=\left|\frac{(\vec{a}_1-\vec{a}_2),(\vec{n}_1\times\vec{n}_2)}{|\vec{n}_1\times\vec{n}_2|}\right|$$

$$=\left|\frac{\frac{15}{2}+\frac{27}{2}-4}{\sqrt{225+\frac{81}{4}+16}}\right|$$

$$=\frac{\frac{34}{2}}{\sqrt{\frac{1045}{4}}}=\frac{34}{\sqrt{1045}}\text{ unit}$$

17. If α is a root of $x^2+x+1=0$ satisfying $(1+\alpha)^7=a+b\alpha+c\alpha^2$, then the ordered triplet (a, b, c) is

(1) (2, 3, 4) (2) (1, 3, 5)

(3) (3, 3, 2) (4) (–1, 5, 4)

Answer (3)

Sol. $x^2+x+1=0$ → ω, ω^2 } roots, $1+\omega+\omega^2=0$, $\omega^3=1$

$\Rightarrow$ $(1+\alpha)^7=(1+\omega)^7=(-\omega^2)^7=-\omega^{14}=-\omega^2$

$\Rightarrow$ Since $1+\omega+\omega^2=0$

$\Rightarrow$ $(1+\alpha)^7=\lambda(1+\omega+\omega^2)-\omega^2=a+b\omega+c\omega^2$

$\Rightarrow$ $\lambda+\lambda\omega+(\lambda-1)\omega^2=a+b\omega+c\omega^2$

$\Rightarrow$ $(\lambda, \lambda, \lambda-1)$, will be ordered triplets where $\lambda\in R$.

18.

19.

20.

SECTION - B

Numerical Value Type Questions: This section contains 10 questions. In Section B, attempt any five questions out of 10. The answer to each question is a **NUMERICAL VALUE.** For each question, enter the correct numerical value (in decimal notation, truncated/rounded-off to the second decimal place; e.g. 06.25, 07.00, –00.33, –00.30, 30.27, –27.30) using the mouse and the on-screen virtual numeric keypad in the place designated to enter the answer.

21. Find P if

$$3+\frac{1}{4}(3+P)+\frac{1}{4^2}(3+2P)+\ldots\infty=8$$

Answer (9)

Sol. $3+\frac{1}{4}(3+P)+\frac{1}{4^2}(3+2P)+\ldots\infty=8$

$$\left(3+\frac{3}{4}+\frac{3}{4^2}+\ldots\right)+\underbrace{\left(\frac{P}{4}+\frac{2P}{4^2}+\ldots\right)}_{k}=8$$

$$3\left[\frac{1}{1-\frac{1}{4}}\right]+k=8$$

$4+k=8$

Now,

$$k=\frac{P}{4}+\frac{2P}{4^2}+\frac{3P}{4^3}+\ldots$$

$$\frac{k}{4}=\qquad\frac{P}{4^2}+\frac{2P}{4^3}+\ldots$$

$$\frac{3k}{4}=\frac{P}{4}+\frac{P}{4^2}+\frac{P}{4^3}+\ldots$$

$$\frac{3k}{4}=\frac{\frac{P}{4}}{1-\frac{1}{4}}$$

$$k=\frac{4}{9}P$$

$$4+\frac{4}{9}P=8$$

$P=9$

22. If $f(x)=x^3+2x^2 f'(1)+xf''(2)+f'''(3)$. The value of $f'(10)$ is equal to ______.

Answer (218)

Sol. $f'(x) = 3x^2 + 4x\,f'(1) + f''(2)$

$f'(1) = 3 + 4f'(1) + f''(2)$ ….(i)

$f''(x) = 6x + 4f'(1)$

$f''(2) = 12 + 4f'(1)$ ….(ii)

$f'''(x) = 6 \Rightarrow f'''(3) = 6$

using (i) and (ii)

$3f'(1) + f''(2) = -3$

$\Rightarrow 3f'(1) + 12 + 4f'(1) = -3$

$\Rightarrow 7f'(1) = -15$

$\Rightarrow \boxed{f'(1) = \frac{-15}{7}} \Rightarrow \boxed{f''(2) = 12 - \frac{60}{7} = \frac{24}{7}}$

$\Rightarrow f(x) = x^3 + 2x^2\left(-\frac{15}{7}\right) + x\cdot\frac{24}{7} + 6$

$f'(x) = 3x^2 - \frac{60}{7}x + \frac{24}{7}$

$f'(10) = 300 - \frac{600}{7} + \frac{24}{7}$

$f'(10) = 217.7142$

$f'(10) = 217.7$

23. Given $A = \begin{bmatrix} 2 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{bmatrix}, B = [B_1\ B_2\ B_3]$

Which satisfying the conditions

$A\cdot B_1 = \begin{bmatrix} 2 \\ 3 \\ 1 \end{bmatrix}, AB_2 = \begin{bmatrix} 2 \\ 0 \\ 0 \end{bmatrix}, AB_3 = \begin{bmatrix} 3 \\ 2 \\ 1 \end{bmatrix}$

and $\alpha = |B|$, β = Diagonal sum of matrix B

Then the value of $\alpha^3 + \beta^3$ equals to.

Answer (117)

Sol. $B = \begin{bmatrix} x_1 & x_2 & x_3 \\ y_1 & y_2 & y_3 \\ z_1 & z_2 & z_3 \end{bmatrix}, A = \begin{bmatrix} 2 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{bmatrix}$

$\Rightarrow B_1 = \begin{bmatrix} x_1 \\ y_1 \\ z_1 \end{bmatrix}, B_2 = \begin{bmatrix} x_2 \\ y_2 \\ z_2 \end{bmatrix}, B_3 = \begin{bmatrix} x_3 \\ y_3 \\ z_3 \end{bmatrix}$

$A\cdot B_1 = \begin{bmatrix} 2x_1 + y_1 \\ z_1 \\ x_1 \end{bmatrix} = \begin{bmatrix} 2 \\ 3 \\ 1 \end{bmatrix} \Rightarrow 2x_1 + y_1 = 2$

$z_1 = 3$

$x_1 = 1$

$\Rightarrow x_1 = 1, y_1 = 0, z_1 = 3$

$A\cdot B_2 = \begin{bmatrix} 2x_2 + y_2 \\ z_2 \\ x_2 \end{bmatrix} = \begin{bmatrix} 2 \\ 0 \\ 0 \end{bmatrix}$

$\Rightarrow 2x_2 + y_2 = 2, z_2 = 0, x_2 = 0$

$x_2 = 0$

$z_2 = 0$

$y_2 = 2$

$A\cdot B_3 = \begin{bmatrix} 2x_3 + y_3 \\ z_3 \\ x_3 \end{bmatrix} = \begin{bmatrix} 3 \\ 2 \\ 1 \end{bmatrix}$

$2x_3 + y_3 = 3, z_3 = 2, x_3 = 1$

$x_3 = 1$

$z_3 = 2$

$y_3 = 1$

$\Rightarrow B = \begin{bmatrix} 1 & 0 & 1 \\ 0 & 2 & 1 \\ 3 & 0 & 2 \end{bmatrix}$

$\alpha = |B| = 4 - 6 = -2$

$\beta = 5$

$\alpha^3 + \beta^3 = -8 + 125$

$= 117$

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