PART : MATHEMATICS

1. The number of common terms of the progression 4, 9, 14 ....up to 25 terms and 3, 6, 9, ... Up to 37 terms, is

(1) 8 (2) 9 (3) 7 (4) 10

Ans. **(3)**

Sol. 4, 9, 14 .... and 3, 6, 9 , ...

$T_{25} = 124$ $\quad$ $T_{37} = 37 \times 3 = 111$

Common diff : 5 $\quad$ common diff = 3

Clearly first common term = 9

Common diff = L.C.M (5, 3) = 15

$\therefore$ T_n (of common terms of two given A.P)

$= 9 + (n-1) \times 15$

$= 15n-6$

$\therefore \quad 15n - 6 \le 111$

$15n \le 117$

$n \le 7.8$

$\therefore$ Number of common terms = 7

2. If $\int_0^1 \frac{1}{\sqrt{3+x}+\sqrt{1+x}}dx = a + b\sqrt{2} + c\sqrt{3}$. Then (a – 3b –4c) is equal to.

(1) 7 (2) 9 (3) 8 (4) 10

Ans. **(2)**

Sol. $\int_0^1 \frac{\sqrt{3+x}-\sqrt{1+x}}{2}dx = \frac{1}{2}\left(\frac{2}{3}(3+x)^{\frac{3}{2}} - \frac{2}{3}(1+x)^{\frac{3}{2}}\right)_0^1$

$= \frac{1}{3}\left[\left(4^{\frac{3}{2}} - 2^{\frac{3}{2}}\right) - \left(3^{\frac{3}{2}} - 1\right)\right]$

$= \frac{1}{3}\left(2^3 - 2\sqrt{2} - 3\sqrt{3} + 1\right) = \frac{1}{3}\left(9 - 2\sqrt{2} - 3\sqrt{3}\right) = 3 - \frac{2}{3}\sqrt{2} - \sqrt{3}$

$a = 3, b = -\frac{2}{3}, c = -1$

$a - 3b - 4c = 3 - 3\frac{(-2)}{3} - 4(-1) = 9$

3. If $8 = 3 + \frac{3+p}{4} + \frac{3+2p}{4^2} + ...\infty$,then p is equal to.

(1) 9 (2) $\frac{5}{4}$ (3) 3 (4) 1

Ans. **(1)**

Sol. $8 = 3 + \frac{3+p}{4} + \frac{3+2p}{4^2} + ...$ $\quad$ (1)

$2 = \frac{3}{4} + \frac{3+p}{4^2} + ...$ (2)

(1) – (2)

$6 = 3 + \frac{p}{4} + \frac{p}{4^2} + ...$

$3 = \frac{p}{4} + \frac{p}{4^2} + ...$

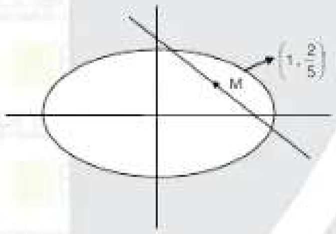
$\frac{\frac{p}{4}}{1-\frac{1}{4}} = 3 \quad \Rightarrow p = 9$

4. Chord length of ellipse $\frac{x^2}{25} + \frac{y^2}{16} = 1$ with mid point $\left(1, \frac{2}{5}\right)$ is equal to ______.

(1) $\frac{\sqrt{1681}}{5}$ (2) $\frac{\sqrt{1691}}{5}$ (3) $\frac{\sqrt{691}}{5}$ (4) $\frac{\sqrt{961}}{5}$

Ans. **(2)**

Sol.



$T = S_1 \Rightarrow \frac{x}{25} + \frac{y.\frac{2}{5}}{16} = \frac{1}{25} + \frac{4}{16 \times 25}$

$\frac{x}{25} + \frac{y}{8 \times 5} = \frac{5}{100}$

$4x + \frac{5}{2}y = 5 \Rightarrow 8x + 5y - 10 = 0$

$\frac{x^2}{25} + \frac{y^2}{16} = 1 \Rightarrow (4x)^2 + (5y)^2 = 400$

$\Rightarrow \quad (4x)^2 + (8x - 10)^2 = 400$

$\Rightarrow \quad 4x^2 + (4x - 5)^2 = 100 \quad \Rightarrow 20x^2 - 40x - 75 = 0$

$\Rightarrow \quad 4x^2 - 8x - 15 = 0$

$\ell = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} = \sqrt{(x_2 - x_1)^2 + \left\{\frac{10 - 8x_2}{5} - \frac{(10 - 8x_1)}{5}\right\}^2}$

$= \sqrt{(x_2 - x_1)^2 + \frac{64}{25}(x_2 - x_1)^2} = \sqrt{\frac{89}{4}}\sqrt{(x_2 - x_1)^2}$

$= \sqrt{\frac{89}{4}} + \sqrt{(x_1 - x_2)^2 - 4x_1x_2} = \sqrt{\frac{89}{5}}\sqrt{4 + 4 \times \frac{15}{4}} = \frac{\sqrt{89} \times \sqrt{19}}{5}$

5. If $\sum_{i=1}^{10} x_i = 50$, and $\sum_{\forall i \neq j} x_i x_j = 1100$, then standard deviation of $x_1, x_2, \ldots\ldots, x_{10}$ is

(1) 5 (2) 2 (3) $\sqrt{2}$ (4) $\sqrt{5}$

Ans. (4)

Sol. $\because \left(\sum_{i=1}^{10} x_i\right)^2 = \sum_{i=1}^{10} x_i^2 + 2\left(\sum_{i>j} x_i x_j\right)$

$\Rightarrow (50)^2 = \sum_{i=1}^{10} x_i^2 + 2(1100)$

$\Rightarrow \sum x_i^2 = 2500 - 2200 = 300$

$\because \sigma^2 = \frac{\sum x_i^2}{10} - \left(\frac{\sum x_i}{10}\right)^2$

$\Rightarrow \sigma^2 = \frac{300}{10} - \left(\frac{50}{10}\right)^2 = 30 - 25$

$\therefore$ standard deviation $= \sqrt{5}$

6. If the shortest distance of the parabola $y^2 = 4x$ from the centre of the circle $x^2 + y^2 - 4x - 16y + 64 = 0$ is 'd', then find d^2

(1) 10 (2) 20 (C) 30 (4) 15

Ans. (2)

Sol. $y^2 = 4x$; $x^2 + y^2 - 4x - 16y + 64 = 0$

circle (2,8) radius = 2

$\because$ shortest distance will lie along common normal

$\because$ normal at p(t) to parabola is

$\therefore y + tx = 2t + t^2$, now it will pass through (2, 8)

$\Rightarrow 8 + 2t = 2t + t^3$.

$t = 2 \therefore p(t) = \equiv p(t^2, 2t) \equiv P(4,4)$

$\therefore d = \sqrt{4+16} = \sqrt{20}$

$\therefore d^2 = 20$

7. If $f(x) = x^3 + x^2 f'(1) + xf''(2) + f'''(3)$. Then find $f'(10)$.

Ans. (202)

Sol. $f(x) = x^3 + ax^2 + bx + c$

$f'(x) = 3x^3 + 2xf'(1) + f''(2)$

$f''(x) = 6x + 2f'(1)$

$f'''(x) = 6$

$f'''(3) = 6$

$f''(2) = 12 + 2f'(1)$

$f'(1) = 3 + 2f'(1) + f''(2)$

adding

$0 = 15 + 3f'(x) \Rightarrow f'(1) = -5$

$f''(2) = 12 - 10 = 2$

$x = 10$

$f'(10) = 300 + 20(-5) + 2 = 300 - 100 + 2 = 202$

8. If ${}^{n-1}C_r = (k^2-8)\,{}^{n}C_{r+1}$, then an interval in which k lies is

(1) $(2\sqrt{2}, 3]$ (2) $\left(-2\sqrt{2}, 2\sqrt{2}\right)$ (3) $(3, \infty)$ (4) $(-\infty, -3)$

Ans. **(1)**

Sol. ${}^{n-1}C_r = (k^2-8)\dfrac{n}{r+1}\,{}^{n-1}C_r$

$k^2 - 8 = \dfrac{r+1}{n}$

$0 < k^2 - 8 \le 1$

$8 < k^2 \le 9$

$2\sqrt{2} < k \le 3$ or $-3 \le k < -2\sqrt{2}$

9. Let $\vec{a} = \hat{i} + 2\hat{j} + \hat{k}$, $\vec{b} = 3(\hat{i} - \hat{j} + \hat{k})$ and a vector $\vec{c}$ is defined such that $\vec{a}\times\vec{c} = \vec{b}$ and $\vec{a}.\vec{c} = 3$, then $\vec{a}.\left((\vec{c}\times\vec{b}) - \vec{b} - \vec{c}\right)$ is

(1) 24 (2) 25 (3) 52 (4) 42

Ans. **(1)**

Sol. $(\vec{a}\times\vec{c}).\vec{b} = \vec{b}.\vec{b}$

$[\vec{a}\,\vec{c}\,\vec{b}] = 9(1+1+1) = 27$

$\vec{a}.\vec{b} = 3\,(1-2+1) = 0$

$\vec{a}.\vec{c} = 3$

$\vec{a}.\left((\vec{c}\times\vec{b}) - \vec{b} - \vec{c}\right) = [\vec{a}\,\vec{c}\,\vec{b}] - \vec{a}.\vec{b} - \vec{a}.\vec{c}$

$= 27 - 0 - 3 = 24$

10. Let $a \in R$ such that equation $\cos 2x + a\sin x = 2a - 7$ has a solution. If range of a is [p, q] and $r = \tan 9^\circ + \tan 81^\circ - \dfrac{1}{\cot 63^\circ} - \tan 27^\circ$, then pqr is ____.

Ans. **(48)**

Sol. $1 - 2\sin^2 x + a\sin x = 2a - 7$

$2t^2 - at + 2a - 8 = 0$

$\Rightarrow (t-2)(2t+4-a) = 0$

$\Rightarrow a = 2\sin x + 4$

$a \in [2, 6] \Rightarrow p = 2, q = 6$

$\tan\theta + \cot\theta = 2\,\text{cosec}\,2\theta$

$r = 2\text{cosec}18^\circ - 2\text{cosec}54^\circ = 2\left[\dfrac{4}{\sqrt{5}-1} - \dfrac{4}{\sqrt{5}+1}\right]$

$= 8\times\dfrac{2}{5-1} = 4$

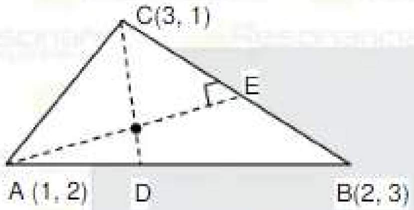
So $pqr = 2\times6\times4 = 48$

13. The vertices of a triangle are (1, 2), (2, 3), (3, 1) and its orthocentre is (a, b) if $I_1 = \int_a^b x\sin(4x - x^2)dx$

$I_2 = \int_a^b \sin(4x - x^2)dx$, then find $\frac{36\, I_1}{I_2}$

Ans. (72)

Sol.



$m|_{AB} = 1$ $\quad$ $m|_{BC} = -2$

$m|_{CD} = -1$ $\quad$ $m|_{AE} = \frac{1}{2}$

Equation of CD is

$y - 1 = -1\ (x - 3)$

$x + y = 4$ ______(1)

Equation of AE is

$y - 2 = \frac{1}{2}(x - 1)$

$2y - 4 = 4 - 1$

$x - 2y = -3$ ______ (2)

Now co-ordinates of orthocentre is $H\left(\frac{5}{3}, \frac{7}{3}\right)$

$$I_1 = \int_{5/3}^{7/3} x\sin(4x - x^2)dx = \int_{5/3}^{7/3} x\sin(x(4-x))dx$$

$$I_1 = \int_{5/3}^{7/3} (4-x)\sin((4-x)x)dx$$

Now $2I_1 = \int_{5/3}^{7/3} 4\sin(4x - x^2)dx$

$2I_1 = 4I_2$

$I_1 = 2I_2$

$\frac{I_1}{I_2} = 2$

$\frac{36 I_1}{I_2} = 72$

14. Let $S = \{1,2,3,......,10\}$, M is the set of all subsets of S, $R:M \to M$ be a relation such that

$(A, B) \in R \Rightarrow A \cap B = \phi$ then relation R is

(1) symmetric (2) Transitive (3) Reflexive (4) none

Ans. **(1)**

Sol. $A \in M$

$A \cap A = A \neq \phi$

So R is not reflexive

If $(A, B) \in R$

$A \cap B = \phi \Rightarrow B \cap A = \phi$

$\Rightarrow (B, A) \in R$

So R is symmetric

If (A, B) and $(B, C) \in R \Rightarrow A \cap B = \phi$ and $B \cap C = \phi$

$\Rightarrow A \cap C$ is not necessarily ϕ set. Hence $A \cap C$ not necessarily belongs to R.

So R is not a transitive relation

eg. $A = \{1, 2\}$, $B = \{3, 4\}$, $C = \{2, 5\}$

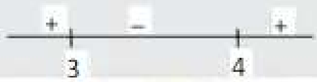
$A \cap B = \phi$, $B \cap C = \phi$ but $A \cap C = \{2\} \neq \phi$

15. If $f(x) = \begin{cases} \dfrac{a(7x - x^2 - 12)}{b|x^2 - 7x + 12|} ; & x < 3 \\ b ; & x = 3 \\ 2^{\frac{\sin(x-3)}{x-[x]}} ; & x > 3 \end{cases}$, is continuous at x = 3, then number of ordered pairs (a, b) are

(1) 1 (2) 2 (3) 3 (4) 4

Ans. **(1)**

Sol. $x^2 - 7x + 12 = (x - 3)(x - 4)$



$$\begin{cases} -\dfrac{a}{b} ; & x < 3 \\ b ; & x = 3 \\ 2^{\frac{\sin(x-3)}{x-3}} ; & 3 < x < 4 \end{cases}$$

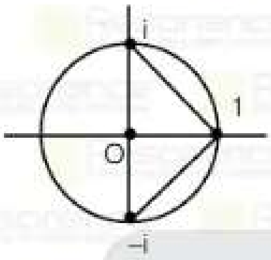
$-\dfrac{a}{b} = b = 2^1 \quad \Rightarrow \quad b = 2, \quad -a = 4$

$(a, b) = (-4, 2)$

16. $S = \{z : |z+i| = |z-i| = |z-1|\}$, then n(S) is equal to

Ans. (1)

Sol.



17. If α satisfies the equations $1 + x + x^2 = 0$ and $(1+\alpha)^7 = a + b\alpha + c\alpha^2$, where $a, b, c \in R$ then $5(3a - 2b - c)$ is______

Ans. (5)

Sol. $1 + x + x^2 = 0 \quad \Rightarrow \quad \alpha = \omega$ or ω^2

Let $\alpha = \omega$

so $(1+\omega)^7 = a + b\omega + c\omega^2 \qquad \omega = \frac{-1}{2} + \frac{\sqrt{3}}{2}i$

$-\omega^2 = a + b\omega + c\omega^2 \qquad \omega^2 = -\frac{1}{2} - \frac{\sqrt{3}}{2}i$

$a + b\omega + (c+1)\omega^2 = 0$

$\Rightarrow \quad a + b\left(-\frac{1}{2} + \frac{\sqrt{3}}{2}i\right) + (c+1)\left(-\frac{1}{2} - \frac{\sqrt{3}}{2}i\right) = 0$

$\Rightarrow \quad a - \frac{b}{2} - \frac{c+1}{2} = 0 \quad \& \quad \frac{\sqrt{3}}{2}b - \frac{\sqrt{3}}{2}(c+1) = 0$

$2a - b - c - 1 = 0 \qquad \Rightarrow b - c = 1$

$2a - b - b = 0 \qquad b = c + 1$

$a = b$

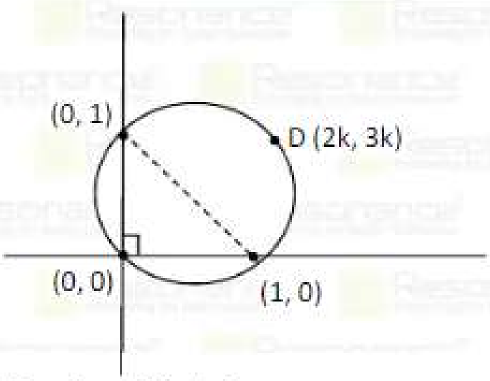
so $5(3a - 2b - c) = 5(3b - 2b - c) = 5(b - c) = 5 \times 1 = 5$

18. If the points (2k, 3k), (1, 0), (0, 0) and (0, 1) lie on a circle then the value of k is

(1) $\frac{5}{13}$ (2) $\frac{10}{13}$ (3) $\frac{13}{5}$ (4) $\frac{13}{10}$

Ans. (1)

Sol.



Equation of circle is

$(x-1)(x-0)+(y-0)(y-1)=0$

In satisfy D(2k, 3k)

$(2k-1)(2k)+(3k)(3k-1)=0$

$4k^2-2k+9k^2-3k=0$

$13k^2-5k=0$

$\Rightarrow k=0,\ \frac{5}{13}$

When k = 0 Point D(0, 0)

So value of $k=\frac{5}{13}$.

19. If $f(x)-f(y)=\ell n\left(\frac{x}{y}\right)+x-y$, then find $\sum_{k=1}^{20} f'\left(\frac{1}{k^2}\right)$

Ans. **(2890)**

Sol. $f'(x)=\lim_{h\to 0}\frac{f(x+h)-f(x)}{h}=\lim_{h\to 0}\frac{\ell n\frac{x+h}{x}+h}{h}=\lim_{h\to 0}\frac{\ell n\left(1+\frac{h}{x}\right)}{x\left(\frac{h}{x}\right)}+1=\frac{1}{x}+1$

$\sum_{k=1}^{20} f'\left(\frac{1}{k^2}\right)=\sum_{k=1}^{20}(k^2+1)=\sum_{k=1}^{20}k^2+\sum_{k=1}^{20}1$

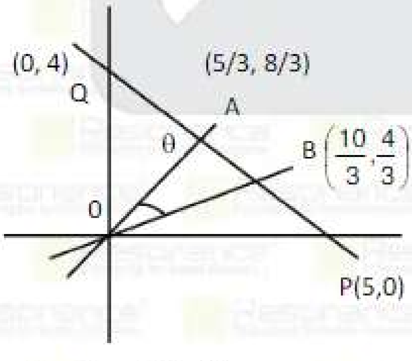
$=\frac{20\times 21\times 41}{6}+20=2870+20=2890$

20. Let L_1 and L_2 be two lines passing through the origin and trisect the portion of the line $4x+5y=20$ which is intercepted between the co-ordinate axes, then the tangent of angle between the lines L_1 and L_2 is equal to

(1) $\frac{8}{5}$ (2) $\frac{41}{15}$ (3) $\frac{15}{41}$ (4) $\frac{30}{41}$

Ans. **(4)**

Sol.



$A\left(\frac{5}{3},\frac{8}{3}\right)$, $B\left(\frac{10}{3},\frac{4}{3}\right)$

$m_{0A}=\ m_2=\frac{8}{5}$

$m_{0B} = m_1 = \frac{2}{5}$

$\tan\theta = \left|\frac{m_2 - m_1}{1 + m_1 m_2}\right| = \left|\frac{\frac{8}{5} - \frac{2}{5}}{1 + \frac{8}{5} \times \frac{2}{5}}\right|$

$= \frac{\frac{6}{5}}{1 + \frac{16}{25}} = \frac{6 \times 5}{25 + 16} = \frac{30}{41}$

21. If one root of equation $x^2 - px + 5 = 0$ is 4 and equation $x^2 - px + q = 0$ has equal roots, then value of q is

(1) $\frac{441}{16}$ (2) $\frac{21}{64}$ (3) $\frac{441}{64}$ (4) $\frac{21}{16}$

Ans. **(3)**

Sol. $16 - 4p + 5 = 0$

$\Rightarrow p = \frac{21}{4}$

$x^2 - \frac{21}{4}x + q = 0$

$4x^2 - 21x + 4q = 0$

Equation has equal roots, so

D = 0

441 – 64q = 0

$q = \frac{441}{64}$

22. Shortest distance between the lines

$\frac{x-1}{2} = \frac{y+1}{4} = \frac{z-1}{3}$ and $\frac{2x-1}{5} = \frac{y-2}{3} = \frac{z}{6}$ is equal to

(1) $\frac{17}{\sqrt{1045}}$ (2) $\frac{34}{\sqrt{1045}}$ (3) $\frac{21}{\sqrt{964}}$ (4) $\frac{17}{\sqrt{964}}$

Ans. **(2)**

Sol. $SD = \left|\frac{[\vec{a}_2 - \vec{a}_1 \ \vec{b}_1 \ \vec{b}_2]}{|\vec{b}_1 \times \vec{b}_2|}\right|$

$$\Rightarrow S.D. = \frac{\begin{vmatrix} \frac{1}{2} - 1 & 2+1 & 0-1 \\ \frac{5}{2} & 3 & 6 \\ 2 & 4 & 3 \end{vmatrix}}{\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \frac{5}{2} & 3 & 6 \\ 2 & 4 & 3 \end{vmatrix}}$$

$$\Rightarrow \text{S.D.} = \left|\frac{-\frac{1}{2}(9-24)-3\left(\frac{15}{2}-12\right)-1(10-6)}{\hat{i}(9-24)-\hat{j}\left(\frac{15}{2}-12\right)+\hat{k}(10-6)}\right|$$

$$\Rightarrow \text{S.D.} = \left|\frac{\frac{15}{2}+\frac{27}{2}-4}{\sqrt{225+\frac{81}{4}+16}}\right| = \left|\frac{21-4}{\sqrt{241+\frac{81}{4}}}\right| = \frac{34}{\sqrt{964+81}} = \frac{34}{\sqrt{1045}}$$

23. $A = \begin{bmatrix} 2 & 0 & 1 \\ 1 & 0 & 0 \\ 3 & 2 & 0 \end{bmatrix}$; $B = [B_1\ B_{11}\ B_{111}]$, where B_1, B_{11}, B_{111} are column vectors such that

$AB_1 = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$, $AB_{11} = \begin{bmatrix} 2 \\ 0 \\ 1 \end{bmatrix}$, $AB_{111} = \begin{bmatrix} 3 \\ 2 \\ 1 \end{bmatrix}$.

If α = sum of the diagonal elements of B,
β = Det(B), then value of $8|\alpha^3 + \beta^3|$ =

Ans. **(9)**

Sol. Let $B_1 = \begin{bmatrix} p \\ q \\ r \end{bmatrix}$

$AB_1 = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$.

$\Rightarrow AB_1 = \begin{bmatrix} 2 & 0 & 1 \\ 1 & 0 & 0 \\ 3 & 2 & 0 \end{bmatrix} \begin{bmatrix} p \\ q \\ r \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} \Rightarrow$ $2p + r = 1 \quad \Rightarrow r = 1$

$p = 0$

$3p + 2q = 0 \Rightarrow q = 0$

$B_1 = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$

Let $B_{11} = \begin{bmatrix} x \\ y \\ z \end{bmatrix}$

$AB_{11} = \begin{bmatrix} 2 \\ 0 \\ 1 \end{bmatrix}$

$$\Rightarrow \begin{bmatrix} 2 & 0 & 1 \\ 1 & 0 & 0 \\ 3 & 2 & 0 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 2 \\ 0 \\ 1 \end{bmatrix} \Rightarrow \quad 2x + z = 2 \Rightarrow z = 2$$

$$x = 0$$

$$3x + 2y = 1 \Rightarrow y = \frac{1}{2}$$

$$B_{11} = \begin{bmatrix} 0 \\ 1/2 \\ 2 \end{bmatrix}$$

$$\text{Let } B_{111} = \begin{bmatrix} \lambda \\ \mu \\ \phi \end{bmatrix}$$

$$AB_{111} = \begin{bmatrix} 3 \\ 2 \\ 1 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 2 & 0 & 1 \\ 1 & 0 & 0 \\ 3 & 2 & 0 \end{bmatrix} \begin{bmatrix} \lambda \\ \mu \\ \phi \end{bmatrix} = \begin{bmatrix} 3 \\ 2 \\ 1 \end{bmatrix} \Rightarrow \quad 2\lambda + \phi = 3 \Rightarrow \phi = -1$$

$$\lambda = 2$$

$$3\lambda + 2\mu = 1 \Rightarrow \mu = -\frac{5}{2}$$

$$B_{111} = \begin{bmatrix} 2 \\ -5/2 \\ -1 \end{bmatrix} \Rightarrow B = \begin{bmatrix} 0 & 0 & 2 \\ 0 & 1/2 & -5/2 \\ 1 & 2 & -1 \end{bmatrix}$$

$\alpha = 0 + 1/2 - 1 = -1/2$

$$\beta = \begin{vmatrix} 0 & 0 & 2 \\ 0 & 1/2 & -5/2 \\ 1 & 2 & -1 \end{vmatrix} = 2\left(-\frac{1}{2}\right) = -1$$

$$\left|\alpha^3 + \beta^3\right| = \left|-\frac{1}{8} - 1\right| = \frac{9}{8} \Rightarrow 8|\alpha^3 + \beta^3| = 9$$

24. If $f : N - \{1\} \to N - \{1\}$ is given by $f(n)$ = largest prime factor of n, then f is

(1) one-one and onto
(2) one-one and into
(3) Many one and onto
(4) Many one and into

Ans. **(4)**

Sol. Largest prime factors of two different natural numbers can be same so it is many-one.
example $f(6) = 3$, $f(12) = 3$
As range of $f(x)$ is set of prime numbers so range of $f(x)$ is proper subset of co-domain so f is into.

25. Matrix $f(x)=\begin{bmatrix}\cos x & -\sin x & 0\\ \sin x & \cos x & 0\\ 0 & 0 & 1\end{bmatrix}$

Statement 1: f(–x) is inverse of f(x)
Statement 2: f(x) f(y) = f(x +y)
(1) Statement- 1 is true & Statement -2 is true (2) Statement- 1 is true & Statement -2 is false
(3) Statement- 1 is false & Statement -2 is true (4) Statement- 1 is false & Statement -2 is false

Ans. **(1)**

Sol. $f(x)=\begin{bmatrix}\cos x & -\sin x & 0\\ \sin x & \cos x & 0\\ 0 & 0 & 1\end{bmatrix}$

$f(x).f(y)=\begin{bmatrix}\cos x & -\sin x & 0\\ \sin x & \cos x & 0\\ 0 & 0 & 1\end{bmatrix}\begin{bmatrix}\cos y & -\sin y & 0\\ \sin y & \cos y & 0\\ 0 & 0 & 1\end{bmatrix}$

$=\begin{bmatrix}\cos(x+y) & -\sin(x+y) & 0\\ \sin(x+y) & \cos(x+y) & 0\\ 0 & 0 & 1\end{bmatrix}=f(x+y)$

So statement 2 is true
f(x). f(–x) = I so, f(–x) is inverse of f(x).
so statement 1 is true

26. If $\frac{dx}{dt}+ax=0$, x(0) = 2,

$\frac{dy}{dt}+by=0$, y(0) = 1, and

3(x(1)) = 2(y(1)) and x(t) = y(t) then t is equal to
(1) $\log_3 2$ (2) $\log_2 3$ (3) $2\log_3 2$ (4) $3\log_2 3$

Ans. **(1)**

Sol. $\int\frac{1}{x}dx=-\int a\,dt \quad\Rightarrow\quad \ell n|x|=-at+C$

$\Rightarrow x=\pm e^{C}.e^{-at} \qquad x=\lambda.e^{-at}$

$x(0)=2\Rightarrow\lambda=2$ so $x=2e^{-at}$ ______(1)

$\frac{dy}{dt}=-by\Rightarrow\int\frac{1}{y}dy=\int-b\,dt$

$\Rightarrow \ell n|y|=-bt+c\Rightarrow y=\pm e^{c}.e^{-bt}$

$\Rightarrow y=\mu.\,e^{-bt}$

$y(0)=1\Rightarrow 1=\mu \;\Rightarrow y=e^{-bt}$ ______(2)

Now $3(x(1))=2\,(y(1))\Rightarrow 3.2.e^{-a}=2.e^{-b}$

$\Rightarrow e^{a-b}=3$

Now again x(t) = y(t)

$\Rightarrow 2\,e^{-at}=e^{-bt}\Rightarrow e^{(a-b)t}=2$

$\Rightarrow 3^{t}=2\Rightarrow t=\log_3 2$