

MATHEMATICS

SECTION - A

Multiple Choice Questions: This section contains 20 multiple choice questions. Each question has 4 choices (1), (2), (3) and (4), out of which **ONLY ONE** is correct.

Choose the correct answer :

1. Let a die rolled till 2 is obtained. The probability that 2 obtained on even numbered toss is equal to

- | | |
|--------------------|--------------------|
| (1) $\frac{5}{11}$ | (2) $\frac{5}{6}$ |
| (3) $\frac{1}{11}$ | (4) $\frac{6}{11}$ |

Answer (1)

Sol. $P(2 \text{ obtained on even numbered toss}) = k(\text{let})$

$$P(2) = \frac{1}{6}$$

$$P(\bar{2}) = \frac{5}{6}$$

$$k = \frac{5}{6} \times \frac{1}{6} + \left(\frac{5}{6}\right)^2 \times \frac{1}{6} + \left(\frac{5}{6}\right)^3 \times \frac{1}{6} + \dots$$

$$= \frac{\frac{5}{6} \times \frac{1}{6}}{1 - \left(\frac{5}{6}\right)^2}$$

$$= \frac{5}{11}$$

2.
$$\lim_{x \rightarrow \frac{\pi}{2}^-} \frac{\int_{x^3}^{\left(\frac{\pi}{2}\right)^2} \cos t^{1/3} dt}{\left(x - \frac{\pi}{2}\right)^2}$$

- | | |
|------------------------|----------------------|
| (1) $\frac{3\pi^2}{4}$ | (2) $\frac{3\pi}{4}$ |
| (3) $\frac{3\pi^2}{8}$ | (4) $\frac{3\pi}{8}$ |

Answer (3)

Sol.
$$\lim_{h \rightarrow 0} \frac{\int_{\left(\frac{\pi}{2}-h\right)^3}^{\left(\frac{\pi}{2}\right)^3} \cos(t^{1/3}) dt}{h^2}$$

$$= \lim_{h \rightarrow 0} \frac{0 + 3\left(\frac{\pi}{2}-h\right)^2 \cos\left(\frac{\pi}{2}-h\right)}{2h}$$

$$= \lim_{h \rightarrow 0} \frac{3\left(\frac{\pi}{2}-h\right)^2 \sin h}{2h}$$

$$= \frac{3\pi^2}{8}$$

3. Consider the equation $4\sqrt{2}x^3 - 3\sqrt{2}x - 1 = 0$.

Statement 1: Solution of this equation is $\cos \frac{\pi}{12}$.

Statement 2: This equation has only one real solution.

- (1) Both statement 1 and statement 2 are true
- (2) Statement 1 is true but statement 2 is false
- (3) Statement 1 is false but statement 2 is true
- (4) Both statement 1 and statement 2 are false

Answer (2)

Sol. $12x = \pi$

$$\Rightarrow 3x = \frac{\pi}{4}$$

$$\cos 3x = \frac{1}{\sqrt{2}}$$

$$\Rightarrow 4 \cos^3 x - 3 \cos x = \frac{1}{\sqrt{2}}$$

$$\Rightarrow 4\sqrt{2} \cos^3 x - 3\sqrt{2} \cos x - 1 = 0$$

$x = \frac{\pi}{12}$ is the solution of above equation.

\therefore Statement 1 is true

$$f(x) = 4\sqrt{2}x^3 - 3\sqrt{2}x - 1$$

$$f'(x) = 12\sqrt{2}x^2 - 3\sqrt{2} = 0$$

$$\Rightarrow x = \pm \frac{1}{2}$$

$$f\left(-\frac{1}{2}\right) = -\frac{1}{\sqrt{2}} + \frac{3}{\sqrt{2}} - 1 = \sqrt{2} - 1 > 0$$

$$f(0) = -1 < 0$$

\therefore one root lies in $\left(-\frac{1}{2}, 0\right)$, one root is $\cos \frac{\pi}{12}$ which

is positive. As the coefficients are real, therefore all the roots must be real.

\therefore Statement 2 is false.

4. If $|2A|^3 = 2^{21}$

and $A = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \alpha & \beta \\ 0 & \beta & \alpha \end{bmatrix}$ then α is (if $\alpha, \beta \in I$)

- (1) 5 (2) 3
(3) 9 (4) 17

Answer (1)

Sol. $|2A| = 2^7$

$8|A| = 2^7$

$|A| = 2^4$

Now $|A| = \alpha^2 - \beta^2 = 2^4$

$\alpha^2 = 16 + \beta^2$

$\alpha^2 - \beta^2 = 16$

$(\alpha - \beta)(\alpha + \beta) = 16$

$\Rightarrow \alpha + \beta = 8$ and

$\alpha - \beta = 2$

$\Rightarrow \alpha = 5$, and $\beta = 3$

5. In a 64 terms GP if sum of total terms is seven times sum of odd terms, then common ratio is

- (1) 3 (2) 4
(3) 5 (4) 6

Answer (4)

Sol. $a, ar, ar^2, \dots, ar^{63}$

$a + ar + ar^2 + \dots + ar^{63} = 7[a + ar^2 + ar^4 + \dots + ar^{62}]$

$\frac{a(1-r^{64})}{(1-r)} = 7 \frac{a(1-r^{64})}{(1-r^2)}$

$1+r = 7$

$r = 6$

6. If $\frac{dy}{dx} - \left(\frac{\sin 2x}{1+\cos^2 x}\right)y = \frac{\sin x}{1+\cos^2 x}$ and $y(0) = 0$ then

$y\left(\frac{\pi}{2}\right)$ is

- (1) -1 (2) 1
(3) 0 (4) 2

Answer (2)

Sol. $\frac{dy}{dx} - \left(\frac{\sin 2x}{1+\cos^2 x}\right)y = \frac{\sin x}{1+\cos^2 x}$

IF = $e^{-\int \frac{\sin 2x dx}{1+\cos^2 x}}$

= $e^{\ln(1+\cos^2 x)} = (1+\cos^2 x)$

So, $y(1+\cos^2 x) = \int \frac{\sin x}{(1+\cos^2 x)} \cdot (1+\cos^2 x) dx$

$y(1+\cos^2 x) = -\cos x + c$

$\therefore y(0) = 0$

$0 = -1 + c$

$\Rightarrow c = 1$

$y = \frac{1 - \cos x}{1 + \cos^2 x}$

Now, $y\left(\frac{\pi}{2}\right) = 1$

7. $4\cos\theta + 5\sin\theta = 1$

Then find $\tan\theta$, where $\theta \in \left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$.

- (1) $\frac{10 - \sqrt{10}}{6}$ (2) $\frac{10 - \sqrt{10}}{12}$
(3) $\frac{\sqrt{10} - 10}{6}$ (4) $\frac{\sqrt{10} - 10}{12}$

Answer (4)

Sol. $16\cos^2\theta + 25\sin^2\theta + 40\sin\theta\cos\theta = 1$

$16 + 9\sin^2\theta + 20\sin 2\theta = 1$

$16 + 9\left(\frac{1 - \cos 2\theta}{2}\right) + 20\sin 2\theta = 1$

$\frac{-9}{2}\cos 2\theta + 20\sin 2\theta = \frac{-39}{2}$

$-9\cos 2\theta + 40\sin 2\theta = -39$

$-9\left(\frac{1 - \tan^2\theta}{1 + \tan^2\theta}\right) + 40\left(\frac{2\tan\theta}{1 + \tan^2\theta}\right) = -39$

$48\tan^2\theta + 80\tan\theta + 30 = 0$

$24\tan^2\theta + 40\tan\theta + 15 = 0$

$\tan\theta = \frac{-40 \pm \sqrt{(40)^2 - 15 \times 24 \times 4}}{2 \times 24}$

$\tan\theta = \frac{-40 \pm \sqrt{160}}{2 \times 24}$

= $\frac{-10 \pm \sqrt{10}}{12}$

$\Rightarrow \tan\theta = \frac{\sqrt{10} - 10}{12}$, $\tan\theta = \frac{-\sqrt{10} - 10}{12}$

So $\tan\theta = \frac{-\sqrt{10} - 10}{12}$ will be rejected as

$\theta \in \left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$

Option (4) is correct.

Sol. Given

$$\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \left(\frac{x^2 \cos x}{1 + \pi^x} + \frac{1 + \sin^2 x}{1 + e^{(\sin x)^{2023}}} \right) dx = \frac{\pi}{4}(\pi + \alpha) - 2$$

$$\int_0^{\frac{\pi}{2}} \left\{ \left(\frac{x^2 \cos x}{1 + \pi^x} + \frac{1 + \sin^2 x}{1 + e^{(\sin x)^{2023}}} \right) + \left(\frac{x^2 \cos x}{1 + \pi^{-x}} + \frac{1 + \sin^2 x}{1 + e^{-(\sin x)^{2023}}} \right) \right\} dx$$

$$= \frac{\pi}{4}(\pi + \alpha) - 2$$

$$\int_0^{\frac{\pi}{2}} (x^2 \cos x + 1 + \sin^2 x) dx = \frac{\pi}{4}(\pi + \alpha) - 2$$

$$\int_0^{\frac{\pi}{2}} x^2 \cos x dx + \int_0^{\frac{\pi}{2}} (1 + \sin^2 x) dx = \frac{\pi}{4}(\pi + \alpha) - 2 \quad \dots(1)$$

Let $I_1 = \int_0^{\frac{\pi}{2}} (1 + \sin^2 x) dx$

$$I_1 = \int_0^{\frac{\pi}{2}} 1 \cdot dx + \int_0^{\frac{\pi}{2}} \left(\frac{1 - \cos 2x}{2} \right) dx$$

$$I_1 = \frac{\pi}{2} + \frac{1}{2} \left[\frac{\pi}{2} + 0 \right]$$

$$I_1 = \frac{\pi}{2} + \frac{\pi}{4}$$

$$\boxed{I_1 = \frac{3\pi}{4}}$$

Let $I_2 = \int_0^{\frac{\pi}{2}} x^2 \cos x dx$

$$I_2 = \left[x^2(\sin x) - \int 2x \int \cos x dx \right]_0^{\frac{\pi}{2}}$$

$$I_2 = \left[x^2(\sin x) - 2 \int x \sin x \right]_0^{\frac{\pi}{2}}$$

$$I_2 = \left[x^2 \sin x - 2(x(-\cos x) + \int \cos x) \right]_0^{\frac{\pi}{2}}$$

$$I_2 = \left[x^2 \sin x - 2(-x \cos x + \sin x) \right]_0^{\frac{\pi}{2}}$$

$$I_2 = \left(\frac{\pi^2}{4} - 2 \right)$$

∴ Put I_1 and I_2 in (1)

$$\therefore \frac{\pi^2}{4} - 2 + \frac{3\pi}{4}$$

$$\frac{\pi^2}{4} + \frac{3\pi}{4} - 2$$

$$\frac{\pi}{4}(\pi + 3) - 2$$

$$\therefore \boxed{\alpha = 3}$$

12. Area under the curve $x^2 + y^2 = 169$ and below the line $5x - y = 13$ is

(1) $\frac{169\pi}{4} - \frac{65}{2} + \frac{169}{2} \sin^{-1} \frac{12}{13}$

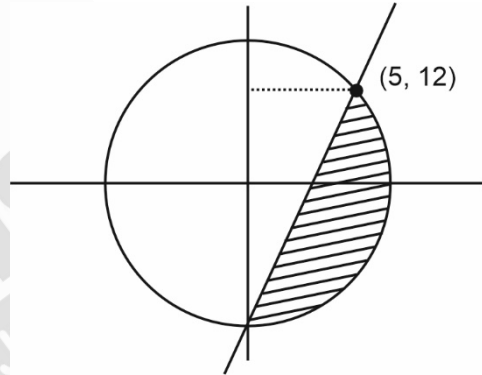
(2) $\frac{169\pi}{4} + \frac{65}{2} - \frac{169}{2} \sin^{-1} \frac{12}{13}$

(3) $\frac{169}{4} - \frac{65}{2} + \frac{169}{2} \sin^{-1} \frac{13}{14}$

(4) $\frac{169\pi}{4} + \frac{65}{2} + \frac{169}{2} \sin^{-1} \frac{13}{14}$

Answer (1)

Sol.



$$\text{Area} = \frac{\pi(13)^2}{2} - \left[\frac{1}{2} \times 25 \times 5 + \int_{12}^{13} \sqrt{169 - y^2} \cdot dy \right]$$

$$= \frac{169\pi}{2} - \left[\frac{125}{2} + \left[\frac{y}{2} \sqrt{169 - y^2} + \frac{169}{2} \sin^{-1} \frac{y}{13} \right]_{12}^{13} \right]$$

$$= \frac{169}{2} \pi - \frac{125}{2} - \left[\frac{169}{2} \times \frac{\pi}{2} - 6 \times 5 - \frac{169}{2} \sin^{-1} \frac{12}{13} \right]$$

$$= \frac{169\pi}{4} - \frac{65}{2} + \frac{169}{2} \sin^{-1} \frac{12}{13}$$

13. If $f(x) = \frac{(2^x + 2^{-x})(\tan x) \sqrt{\tan^{-1}(2x^2 - 3x + 1)}}{(7x^2 - 3x + 1)^3}$, then

$f(0)$ is equal to

(1) $\sqrt{\pi}$

(2) $\sqrt{\frac{\pi}{4}}$

(3) π

(4) $2 \cdot \pi^{3/2}$

Answer (1)

Sol. $f(x) = \frac{(2^x + 2^{-x}) \tan x \sqrt{\tan^{-1}(2x^2 - 3x + 1)}}{(7x^2 - 3x + 1)^3}$

$f(x) = (2^x + 2^{-x}) \cdot \tan x \cdot \sqrt{\tan^{-1}(2x^2 - 3x + 1)} \cdot (7x^2 - 3x + 1)^{-3}$

$f'(x) = (2^x + 2^{-x}) \cdot \sec^2 x \cdot \sqrt{\tan^{-1}(2x^2 - 3x + 1)} \cdot (7x^2 - 3x + 1)^{-3} + \tan x \cdot (Q(x))$

$\therefore f'(0) = 2 \cdot 1 \cdot \sqrt{\frac{\pi}{4}} \cdot 1$

$= \sqrt{\pi}$

14. $\int \frac{(\sin x - \cos x) \sin^2 x}{\sin x \cos^2 x + \tan x \sin^3 x} dx$ is equal to

(1) $\frac{\ln |\sin^3 x - \cos^3 x|}{3} + c$

(2) $\frac{\ln |\sin^3 x + \cos^3 x|}{3} + c$

(3) $\frac{\ln |\sin^3 x - \cos^3 x|}{2} + c$

(4) $\frac{\ln |\sin^3 x + \cos^3 x|}{4} + c$

Answer (2)

Sol. $\int \frac{(\sin x - \cos x) \sin^2 x}{\tan x (\sin^3 x + \cos^3 x)} dx$

$\int \frac{(\sin x - \cos x) \sin x \cos x}{\sin^3 x + \cos^3 x} dx$, put $\sin^3 x + \cos^3 x = t$

$(3 \sin^2 x \cdot \cos x - 3 \cos^2 x \cdot \sin x) dx = dt$

$\Rightarrow \frac{1}{3} \int \frac{dt}{t}$

$= \frac{\ln t}{3} + c$

$= \frac{\ln |\sin^3 x + \cos^3 x|}{3} + c$

15.

16.

17.

18.

19.

20.

SECTION - B

Numerical Value Type Questions: This section contains 10 Numerical based questions. The answer to each question should be rounded-off to the nearest integer.

21. $\frac{{}^{11}C_1}{2} + \frac{{}^{11}C_2}{3} + \dots + \frac{{}^{11}C_9}{10} = \frac{m}{n}$

Then $m + n$ is

Answer (2041)

Sol. $(1+x)^{11} = {}^{11}C_0 + {}^{11}C_1 x + {}^{11}C_2 x^2 + \dots + {}^{11}C_{11} x^{11}$

$\int_0^1 (1+x)^{11} dx = {}^{11}C_0 x + \frac{{}^{11}C_1 x^2}{2} + \frac{{}^{11}C_2 x^3}{3} + \dots + \frac{{}^{11}C_9 x^{10}}{10} + \frac{{}^{11}C_{10} x^{11}}{11} + \frac{{}^{11}C_{11} x^{12}}{12} \Big|_0^1$

$\left. \frac{(1+x)^{12}}{12} \right|_0^1 = {}^{11}C_0 + \frac{{}^{11}C_1}{2} + \frac{{}^{11}C_2}{3} + \dots + \frac{{}^{11}C_9}{10} + \frac{{}^{11}C_{10}}{11} + \frac{{}^{11}C_{11}}{12}$

$\frac{2^{12} - 1}{12} - 1 - 1 - \frac{1}{12} = \frac{{}^{11}C_1}{2} + \frac{{}^{11}C_2}{3} + \dots + \frac{{}^{11}C_{10}}{11}$

$= \frac{2^{12} - 2 - 24}{12}$

$= \frac{2^{12} - 26}{12} = \frac{4070}{12} = \frac{2035}{6} = \frac{m}{n}$

$m + n = 2035 + 6 = 2041$

22. Rank of the word 'GTWENTY' in dictionary is

Answer (553)

Sol. Start with

(1) $\overline{E} : \frac{6!}{2!} = 360$

(2) $\overline{GE} : \frac{5!}{2!}, \overline{GN} : \frac{5!}{2!}$

(3) $\overline{GTE} : 4!, \overline{GTN} : 4!, \overline{GTT} : 4!$

(4) $\overline{GTWENTY} = 1$

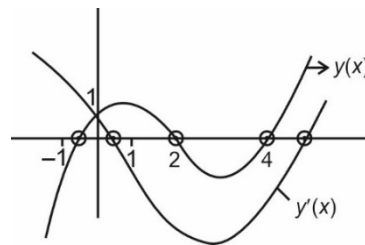
$\Rightarrow 360 + 60 + 60 + 24 + 24 + 24 + 1 = 553$

23. Curve $y = 2^x - x^2$, $y(x)$ & $y'(x)$ cut x-axis in M & N number of points respectively, find $M + N$.

Answer (5)

Sol. $y(x) = 2^x - x^2$

$y'(x) = 2^x \log 2 - 2x$



$M = 3$

$N = 2$

$M + N = 5$

24. Given data

60, 60, 44, 58, 68, α , β , 56 has mean 58, variance = 66.2 then find $\alpha^2 + \beta^2$

Answer (7182)

Sol. Variance = $\frac{\sum x^2}{n} - (\bar{x})^2$

$$\frac{60^2 + 60^2 + 44^2 + 58^2 + 68^2 + \alpha^2 + \beta^2 + 56^2}{8}$$

$$- (58)^2 = 66.2$$

$$\frac{7200 + 1936 + 3364 + 4624 + 3136 + \alpha^2 + \beta^2}{8}$$

$$- 3364 = 66.2$$

$$2532.5 + \frac{\alpha^2 + \beta^2}{8} - 3364 = 66.2$$

$$\alpha^2 + \beta^2 = 897.7 \times 8$$

$$= 7181.6$$

25. If $|z + 1| = \alpha z + \beta (i + 1)$ and $z = \frac{1}{2} - 2i$, find $\alpha + \beta$.

Answer (3)

Sol. $\left| \frac{1}{2} - 2i + 1 \right| = \alpha \left(\frac{1}{2} - 2i \right) + \beta(1 + i)$

$$\sqrt{\frac{9}{4} + 4} = \alpha \left(\frac{1}{2} - 2i \right) + \beta(1 + i)$$

$$\frac{5}{2} = \alpha \left(\frac{1}{2} \right) + \beta + i(-2\alpha + \beta)$$

$$\frac{\alpha}{2} + \beta = \frac{5}{2} \quad \dots(1)$$

$$-2\alpha + \beta = 0 \quad \dots(2)$$

Solving (1) and (2)

$$\frac{\alpha}{2} + 2\alpha = \frac{5}{2}$$

$$\frac{5}{2}\alpha = \frac{5}{2}$$

$$\alpha = 1$$

$$\beta = 2$$

$$\Rightarrow \alpha + \beta = 3$$

26. If $\vec{a}, \vec{b}, \vec{c}$ are non-zero and \vec{b} and \vec{c} are non-collinear. $\vec{a} + 5\vec{b}$ is collinear with \vec{c} and $\vec{b} + 6\vec{c}$ is collinear with \vec{a} . If $\vec{a} + \alpha\vec{b} + \beta\vec{c} = 0$, then find $\alpha + \beta$.

Answer (35)

Sol. $\because \vec{a} + 5\vec{b}$ is collinear with \vec{c}

$$\Rightarrow \vec{a} + 5\vec{b} = \lambda \vec{c} \quad \dots(1)$$

$\vec{b} + 6\vec{c}$ is collinear with \vec{a}

$$\Rightarrow \vec{b} + 6\vec{c} = \mu \vec{a} \quad \dots(2)$$

From (1) and (2)

$$\vec{b} + 6\vec{c} = \mu(\lambda \vec{c} - 5\vec{b})$$

$$\Rightarrow (1 + 5\mu)\vec{b} + (6 - \lambda\mu)\vec{c} = 0$$

$\because \vec{b}$ and \vec{c} are non-collinear

$$\Rightarrow 1 + 5\mu = 0 \Rightarrow \mu = \frac{-1}{5} \text{ and}$$

$$6 - \lambda\mu = 0 \Rightarrow \lambda\mu = 6$$

$$\Rightarrow \lambda = -30$$

Now,

$$\vec{b} + 6\vec{c} = \frac{-1}{5}\vec{a}$$

$$5\vec{b} + 30\vec{c} = -\vec{a}$$

$$\vec{a} + 5\vec{b} + 30\vec{c} = 0$$

$$\vec{a} + \alpha\vec{b} + \beta\vec{c} = 0$$

On comparing

$$\alpha = 5, \beta = 30 \Rightarrow \alpha + \beta = 35$$

27.

28.

29.

30.

