PART : MATHEMATICS

1. What is probability of getting '2' in even number of throws of a die is ?

(1) $\frac{4}{11}$ (2) $\frac{5}{11}$ (3) $\frac{6}{11}$ (4) $\frac{3}{11}$

Ans. (2)

Sol. Let P is probability of getting 2 in single through then $P = \frac{1}{6}$.

Hence probability of getting 2 in even trial = $\bar{P}P + \bar{P}\bar{P}\bar{P}P + \bar{P}\bar{P}\bar{P}\bar{P}\bar{P}P + ... \infty$

$$= \frac{\bar{P}P}{1-\bar{P}\bar{P}} = \frac{\frac{5}{6}\times\frac{1}{6}}{1-\frac{5}{6}\times\frac{5}{6}} = \frac{5}{11}$$

2. A G.P. of 64 terms is such that S_n (total) = $7(S_n)_{\text{odd terms}}$ then common ratio of G.P is

(1) 2 (2) 6 (3) 10 (4) 14

Ans. (2)

Sol. $a + ar + ar^2$ .... 64 terms = 7 ($a + ar^2 + ar^4$ + ... 32 terms)

$$\frac{a(r^{64}-1)}{r-1} = 7\left(\frac{a((r^2)^{32}-1)}{r^2-1}\right)$$

$$1 = \frac{7}{r+1}$$

$r + 1 = 7$

$r = 6$

3. Value of $\lim_{x\to\frac{\pi}{2}} \int_{x^3}^{(\pi/2)^3} \frac{\cos(t^{1/3})}{(x-\pi/2)^2}dt$ is

(1) $3\left(\frac{\pi}{3}\right)^3$ (2) $\frac{3}{2}\left(\frac{\pi}{2}\right)^2$ (3) $\left(\frac{\pi}{2}\right)^3$ (4) $\left(\frac{\pi}{2}\right)^2$

Ans. (2)

Sol. Apply L-H rule

$$\lim_{x\to\pi/2} \frac{\cos(\pi/2)\times 0 - \cos(x)(3x^2)}{2(x-\pi/2)(1)} \text{ (by Leibnitz theorem)}$$

$$\lim_{x\to\pi/2} \frac{-\sin(\pi/2-x)}{-(\pi/2-x)}.\frac{3}{2}x^2 = \frac{3}{2}\left(\frac{\pi}{2}\right)^2$$

4. If all word by using letters of word "GTWENTY" are arranged as in dictionary, then rank of the word "GTWENTY" is

(1) 552 (2) 553 (3) 554 (4) 551

Ans. (2)

Sol. Alphabetical order of letters of the word is

E, G, N, T, T, W, Y

For rank of word

G T W E N T Y

$$\frac{1\,\lfloor 6}{\lfloor 2} + \frac{2\,\lfloor 5}{\lfloor 2} + 3\lfloor 4 + 0\lfloor 3 + 0\lfloor 2 + 0\lfloor 1 + 0\lfloor 0 + 1$$

$= 360 + 120 + 72 + 1 = 553$

5. If $\frac{{}^{11}C_1}{2} + \frac{{}^{11}C_2}{3} + \ldots\ldots \frac{{}^{11}C_9}{10} = \frac{n}{m}$, then value of n+m –8 is

(1) 2033 (2) 2034 (3) 2032 (4) 2034

Ans. (1)

Sol. $\frac{{}^{11}C_1}{2} + \frac{{}^{11}C_2}{3} + \frac{{}^{11}C_3}{4} + \ldots\ldots \frac{{}^{11}C_9}{10} = \sum_{r=1}^{9} \frac{1}{(r+1)} {}^{11}Cr$

$= \frac{1}{12} \sum_{r=1}^{9} \frac{12}{r+1} {}^{11}Cr$

$= \frac{1}{12} \sum_{r=1}^{9} {}^{12}C_{r+1}$

$\frac{1}{12}({}^{12}C_2 + {}^{12}C_3 + \ldots\ldots + {}^{12}C_{10})$

$\frac{1}{12}(2^{12} - {}^{12}C_0 - {}^{12}C_1 - {}^{12}C_{11} - {}^{12}C_{12})$

$\frac{1}{12}(2^{12} - 2 - 24)$

$\frac{1}{12}(2^{12} - 26)$

$= \frac{1}{6}(2048 - 13) = \frac{2035}{6}$

$n = 2035$, $m = 6$

Hence $n + m - 8 = 2035 + 6 - 8 = 2033$

6. In an A.P $a_6 = 2$ then common difference for which $a_1.a_3.a_6$ is least is

(1) $\frac{16}{15}$ (2) $\frac{8}{15}$ (3) $\frac{15}{8}$ (4) $\frac{15}{16}$

Ans. (2)

Sol. $a_6 = a + 5d = 2 \Rightarrow a = 2 - 5d$

$P = a_1a_3a_6$

$= a\,(a + 2d)\,2$

$= (2 - 5d)\,(2 - 3d)2$

$P = 2(4 - 16d + 15d^2)$

$\frac{dP}{dd} = 2(-16 + 30d)$

So at $d = \frac{16}{30} = \frac{8}{15}$, P will be least

7. $A = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \alpha & \beta \\ 0 & \beta & \alpha \end{bmatrix}$ and $|2A|^3 = 2^{21}$ then, find α ($\alpha, \beta \in I^+$)

(1) 3 (2) 4 (3) 5 (4) 6

Ans. (3)

Sol. $\alpha^2 - \beta^2 = |A|$

And $|2A|^3 = 2^{21}$

Now $(2^3|A|)^3 = 2^{21}$

$|A| = 2^4$

$\alpha^2 - \beta^2 = 16$

$(\alpha-\beta)(\alpha+\beta) = 16$

Case:I $\alpha+\beta = 8$

$\alpha-\beta = 2 \quad \rightarrow \quad \alpha = 5, \quad \beta = 3$

8. If $4\cos\theta + 5\sin\theta = 1$ and α satisfy equation and $\alpha \in \left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$ then, value of $\tan\alpha$ is

(1) $\frac{\sqrt{10}+10}{12}$ (2) $\frac{\sqrt{10}-10}{12}$ (3) $\frac{10-\sqrt{10}}{12}$ (4) $\frac{\sqrt{10}+10}{6}$

Ans. (2)

Sol. $4\cos\theta + 5\sin\theta = 1$

$$4\left(\frac{1-\tan^2(\theta/2)}{1+\tan^2(\theta/2)}\right)+5\left(\frac{2\tan(\theta/2)}{1+\tan^2(\theta/2)}\right)=1$$

$\Rightarrow 5\tan^2(\theta/2) - 10\tan(\theta/2) - 3 = 0$

$$\Rightarrow \tan(\theta/2) = \frac{10 \pm \sqrt{100-4(5)(-3)}}{2\times 5} = \frac{10\pm\sqrt{160}}{10} = \frac{5\pm\sqrt{40}}{5}$$

$\because \alpha \in (-\pi/2, \pi/2)$

$\frac{\alpha}{2} \in (-\pi/4, \pi/4)$

$\tan(\alpha/2) \in (-1, 1)$ Hence $\tan(\alpha/2) = \frac{5-\sqrt{40}}{5}$

$$\tan\alpha = \frac{2\tan(\alpha/2)}{1-\tan^2(\alpha/2)} = \frac{2\left(\frac{5-\sqrt{40}}{5}\right)}{1-\left(1-\frac{\sqrt{40}}{5}\right)^2} = \frac{2\left(\frac{5-\sqrt{40}}{5}\right)}{1-\left(1+\frac{40}{25}-\frac{2\sqrt{40}}{5}\right)}$$

$$= \frac{2\left(\frac{5-\sqrt{40}}{5}\right)}{2\frac{\sqrt{40}}{5}-\frac{8}{5}} = \frac{5-\sqrt{40}}{\sqrt{40}-4}\times\frac{\sqrt{40}+4}{\sqrt{40}+4}$$

$$= \frac{5\sqrt{40}+20-40-4\sqrt{40}}{24}$$

$$= \frac{\sqrt{40}-20}{24} = \frac{\sqrt{10}-10}{12}$$

9. $f(x)=\begin{cases}2+2x & -3\le x\le 0\\ 1-\dfrac{x}{2} & 0<x\le 1\end{cases}$

$g(x)=\begin{cases}-x & -1\le x<0\\ x & 0\le x\le 1\end{cases}$

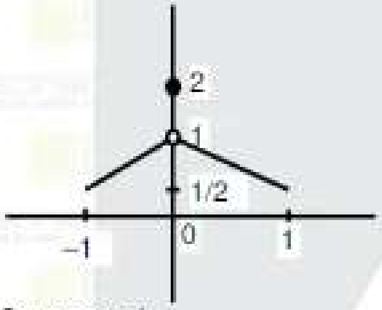
Range of fog(x) is

(1) $\left[\frac{1}{2},4\right]$ (2) $\left(-\infty,\frac{1}{2}\right)\cup[1,2)$ (3) $\left[\frac{1}{2},1\right)\cup\{2\}$ (4) $\left[\frac{1}{2},1\right)\cup[4,\infty]$

Ans. **(3)**

Sol. $f(g(x))=\begin{cases}2+2g(x) & -3\le g(x)\le 0\\ 1-\dfrac{g(x)}{2} & 0<g(x)\le 1\end{cases}$

$=\begin{cases}2-2x & -3\le -x\le 0 \text{ and } -1\le x<0\\ 1+\dfrac{x}{2} & 0<-x\le 1 \text{ and } -1\le x<0\\ 2+2x & -3\le x\le 0 \text{ and } 0\le x\le 1\\ 1-\dfrac{x}{2} & 0<x\le 1 \text{ and } 0\le x\le 1\end{cases}=\begin{cases}1+\dfrac{x}{2} & -1\le x<0\\ 2+2x & x=0\\ 1-\dfrac{x}{2} & 0<x\le 1\end{cases}$



So range of

fog(x) is $\left[\frac{1}{2},1\right)\cup\{2\}$

10. If $AA^T=I$, where A^T is transpose of matrix A then, $\frac{1}{2}A\left[(A+A^T)^2+(A-A^T)^2\right]=?$

(1) A^2 (2) A^3+I (3) A^2+I (4) A^3+A^T

Ans. **(4)**

Sol. $\frac{1}{2}A\left[2A^2+2(A^T)^2\right]$

$=A^3+A(A^T)^2$

$=A^3+A^T$

11. A relation R is defined on I x I such that (a, b)R (c,d) if and only if ad – bc is divisible by 5, then relation R is a

(1) Reflexive and symmetric relation both

(2) symmetric and transitive relation both

(3) Transitive and reflexive relation both

(4) Equivalence relation

Ans. **(1)**

Sol. $\forall$ (a, b) $\in$ I x I

ab – ba = 0, which is divisible by 5 always

$\Rightarrow$ (a, b) R (a, b)

So R is a reflexive relation

If (a, b) R (c,d) $\Rightarrow$ ad–bc is divisible by 5
$\Rightarrow$ cb–da is divisible by 5
$\Rightarrow$ (c,d) R (a, b)

So, R is a symmetric relation.

If (a,b) R (c,d) and (c,d) R(e, f) $\Rightarrow$ ad – bc = 5λ and cf–de = 5μ

$\Rightarrow$ af – be need not be multiple of 5 $\Rightarrow$ (a, b) R (e, f)

Let (a, b) = (6, 1), (c, d) = (0, 0), (e, f) = (1, 7)

ad – bc = 0 and cf – de = 0

but af – be = 42–1=41 is not multiple of 5

so R is not a transitive relation

12. $\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}}\left(\frac{x^2\cos x}{1+\pi^x}+\frac{\sin^2 x+1}{e^{\sin^{2023} x}+1}\right)dx$

(1) $\frac{\pi}{2}+\frac{3\pi^2}{4}-2$ (2) $\frac{\pi}{4}+\frac{3\pi^2}{4}+2$ (3) $\frac{\pi^2}{4}+\frac{3\pi}{4}-2$ (4) $\frac{\pi^2}{4}+\frac{3\pi}{4}+2$

Ans. (3)

Sol. By even odd property

$$\int_0^{\frac{\pi}{2}}\left(x^2\cos x+\sin^2 x+1\right)dx$$

$$\int_0^{\frac{\pi}{2}}\left(x^2\cos x+\frac{3}{2}-\frac{\cos 2x}{2}\right)dx$$

$$\left(x^2\sin x-\int 2x\sin x dx+\frac{3x}{2}-\frac{\sin 2x}{4}\right)_0^{\frac{\pi}{2}}$$

$$=\left[x^2\sin x-\left[2x(-\cos x)-\int 2(-\cos x)dx\right]+\frac{3x}{2}-\frac{\sin 2x}{4}\right]_0^{\frac{\pi}{2}}=\left[x^2\sin x+2x\cos x-2\sin x+\frac{3x}{2}-\frac{\sin 2x}{4}\right]_0^{\frac{\pi}{2}}$$

$$=\left[\frac{\pi^2}{4}+0-2+\frac{3\pi}{4}-0\right]-[0]=\frac{\pi^2}{4}+\frac{3\pi}{4}-2$$

13. If $|z+1| = \alpha z + \beta (1 + i)$ and $z=\frac{1}{2}-2i$, then the value of $|\alpha + \beta|$ is equal to, where $\alpha, \beta \in R$

(1) 4 (2) 3 (3) 2 (4) 1

Ans. (2)

Sol. $\left|\frac{3}{2}-2i\right|=\left(\frac{\alpha}{2}-2\alpha i\right)+\beta i+\beta$

$2\alpha = \beta$ and $\frac{\alpha}{2}+\beta=\sqrt{\frac{9}{4}+4}$

$\frac{\alpha}{2}+2\alpha=\frac{5}{2}$

$\alpha = 1$

$\beta = 2$ Ans. $|\alpha + \beta| = 3$

14. If mean and variance of observations 60, 60, 44, 58, 68, 56, α, β are 58 and 66.2 respectively then $\alpha^2 + \beta^2$ is equal to

(1) 6150.2 (2) 7181.6 (3) 9532.8 (4) 3252.6

Ans. **(2)**

Sol. $\sigma^2 = \frac{\sum x_i^2}{8} - (\bar{x})^2 = 66.2$

$\frac{60^2+60^2+44^2+58^2+68^2+56^2+\alpha^2+\beta^2}{8} - (58)^2 = 66.2$

$\frac{3600+3600+1936+3364+4624+3136+\alpha^2+\beta^2}{8} = 66.2+3364$

$20260 + \alpha^2 + \beta^2 = 8 \times (3430.2)$

$\alpha^2 + \beta^2 = 27441.6 - 20260$

$\alpha^2 + \beta^2 = 7181.6$

15. $\vec{a}, \vec{b}, \vec{c}$ are three pairwise non-collinear vectors. $\vec{a}+6\vec{b}$ is collinear with $\vec{c}$, $\vec{b}+5\vec{c}$ is collinear with $\vec{a}$, then $\vec{a}+\alpha\vec{b}+\beta\vec{c}=\vec{0}$, then $\alpha+\beta = ?$

(1) 36 (2) 30 (3) 25 (4) 39

Ans. **(1)**

Sol. $\vec{a}+6\vec{b} = \lambda\vec{c}$

$\vec{b}+5\vec{c} = \mu\vec{a} \quad \Rightarrow 6\vec{b}+30\vec{c} = 6\mu\vec{a}$

$\vec{a}-30\vec{c} = \lambda\vec{c} - 6\mu\vec{a}$

$(1+6\mu)\ \vec{a} = (\lambda+30)\vec{c}$

$\vec{a}$ and $\vec{c}$ are non-collinear

Sol $\lambda = -30$ and $\mu = -\frac{1}{6}$

So $\vec{a}+6\vec{b} = -30\vec{c}$

$\vec{a}+6\vec{b}+30\vec{c} = \vec{0}$

$\therefore \alpha+\beta = 6+30 = 36$

16. If $\frac{dy}{dx} - \left(\frac{\sin 2x}{1+\cos^2 x}\right)y = \frac{\sin x}{1+\cos^2 x}$ and y (0) = 0 than $y\left(\frac{\pi}{2}\right)$ is

(1) 2 (2) 1 (3) 5 (4) 3

Ans. **(2)**

Sol. $\frac{dy}{dx} - \left(\frac{\sin 2x}{1+\cos^2 x}\right)y = \frac{\sin x}{1+\cos^2 x}$

$\text{If} = e^{-\int \frac{\sin 2x}{1+\cos^2 x}dx} = e^{\ln(1+\cos^2 x)} = 1+\cos^2 x$

So general solution of DE

$y(1+\cos^2 x) = \int \frac{\sin x}{1+\cos^2 x}.(1+\cos^2 x)dx$

$y(1+\cos^2 x) = -\cos x + c$

$y(1+\cos^2 x) + \cos x = c$

were $y(0) = 0$

$0 + 1 = c$

$y(1+\cos^2 x) + \cos x = 1$

$y = \frac{1-\cos x}{1+\cos^2 x}$

$y\left(\frac{\pi}{2}\right) = \frac{1-0}{1+0} = 1$

17. If $\int \frac{(\text{cosec}x - \sin x)}{\text{cosec}x\sec x + \tan x \sin^2 x} dx = f(x)$ and $f(\pi/2) = 1$ then $f(\pi/4)$

(1) $1+\frac{1}{\sqrt{2}}\tan^{-1}\frac{1}{2}$ (2) $1-\frac{1}{\sqrt{2}}\tan^{-1}\frac{1}{\sqrt{2}}$

(3) $1-\frac{1}{\sqrt{2}}\tan^{-1}\frac{1}{2}$ (4) $1+\frac{1}{\sqrt{2}}\tan^{-1}\frac{1}{\sqrt{2}}$

Ans. (3)

Sol. $\int\left(\frac{1+\sin^2 x}{\frac{1}{\cos x}+\frac{\sin^4 x}{\cos x}}\right)dx$

$= \int \frac{(1+\sin^2 x)\cos x}{1+\sin^4 x}dx$

$\sin x = t,$ $\cos x dx = dt$

$\int \frac{1+t^2}{1+t^4}dt = \int\left(\frac{1+\frac{1}{t^2}}{t^2+\frac{1}{t^2}}\right)dt$

$= \int \frac{du}{u^2+2}$ $t-\frac{1}{t}=4$

$= \frac{1}{\sqrt{2}}\tan^{-1}\frac{u}{\sqrt{2}}+c$ $\left(1+\frac{1}{t^2}\right)dt = dx$

$f(x) = \frac{1}{\sqrt{2}}\tan^{-1}\frac{\left(\sin x - \frac{1}{\sin x}\right)}{\sqrt{2}}+c$

$f\left(\frac{\pi}{2}\right) = 1 \Rightarrow c = 1$

$f\left(\frac{\pi}{4}\right) = \frac{1}{\sqrt{2}}\tan^{-1}\left(\frac{\frac{1}{\sqrt{2}}-\sqrt{2}}{\sqrt{2}}\right)+1$

$f\left(\frac{\pi}{4}\right) = 1-\frac{1}{\sqrt{2}}\tan^{-1}\frac{1}{2}$

18. $f(x)=\dfrac{\left(2^x+2^{-x}\right)\tan x\sqrt{\tan^{-1}\left(2x^2-3x+1\right)}}{\left(7x^2-3x+1\right)^3}$, then find $f'(0)$

(1) $\sqrt{3\pi}$ (2) $\sqrt{2\pi}$ (3) $\sqrt{\pi}$ (4) $2\sqrt{\pi}$

Ans. **(3)**

Sol. RHD = $\lim\limits_{h\to0}\dfrac{f(0+h)-f(0)}{h}$

$= \lim\limits_{h\to0}\dfrac{\dfrac{\left(2^h+2^{-h}\right)\tan h\sqrt{\tan^{-1}\left(2h^2-3h+1\right)}}{\left(7h^2-3h+1\right)^3}-0}{h} = \left(2^h+2^{-h}\right)\dfrac{\tan h}{h}\dfrac{\sqrt{\tan^{-1}\left(2h^2-3h+1\right)}}{\left(7h^2-3h+1\right)^3}$

$= \dfrac{2\times1\times\sqrt{\tan^{-1}1}}{1} = \dfrac{2\sqrt{\pi}}{2} = \sqrt{\pi}$

similarly LHD = $\sqrt{\pi}$

19. Statement- I: $f(x) = 4\sqrt{2}x^3-3\sqrt{2}x-1=0$ has only one solution in $x\in\left(\dfrac{1}{2},1\right)$

Statement- II: $\cos\dfrac{\pi}{12}$ is a solution of $f(x) = 0$

(1) Statement- 1 is true & Statement -2 is true (2) Statement- 1 is true & Statement -2 is false
(3) Statement- 1 is false & Statement -2 is true (4) Statement- 1 is false & Statement -2 is false

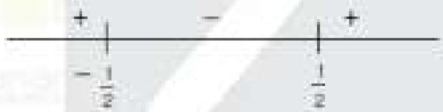
Ans. **(1)**

Sol. $f(x) = 4\sqrt{2}x^3-3\sqrt{2}x-1$

$f'(x) = 12\sqrt{2}x^2-3\sqrt{2}$

$f'(x) = 3\sqrt{2}\left(4x^2-1\right)$

$f'(x) = 3\sqrt{2}(2x+1)(2x-1)$



In $x\in\left(\dfrac{1}{2},1\right)$, f(x) is increasing

and $f\left(\dfrac{1}{2}\right)<0$ and $f(1)>0$ so only one solution in $\left(\dfrac{1}{2},1\right)$

$f(x) = \sqrt{2}\left(4x^3-3x\right)-1=0$

Put $x = \cos\dfrac{\pi}{12}$

$\sqrt{2}\left(4\cos^3\dfrac{\pi}{12}-3\cos\dfrac{\pi}{12}\right)-1=0$

$\sqrt{2}\cos\dfrac{\pi}{4}-1=0$

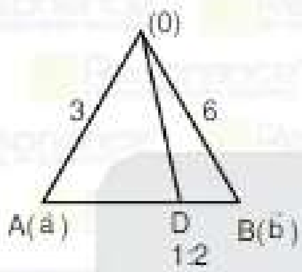
It is true.

So both statement are true

20. If $A(\vec{a}) = 2\hat{i} + 2\hat{j} + \hat{k}$ & $B(\vec{b}) = 2\hat{i} + 4\hat{j} + 4\hat{k}$, find the length of angle bisector of $\angle AOB$.

(1) $\frac{1}{2}\sqrt{136}$ (2) $\frac{1}{2}\sqrt{236}$ (3) $\frac{1}{3}\sqrt{236}$ (4) $\frac{1}{3}\sqrt{136}$

Sol. (4)



Position vector of $D = \left(\frac{6\hat{i} + 8\hat{j} + 6\hat{k}}{3}\right)$

So $|\vec{oD}| = \frac{1}{3}\sqrt{36 + 64 + 36} = \frac{1}{3}\sqrt{136}$

21. Area inside the curve $x^2 + y^2 = 169$ and below the line $5x - y = 13$ is

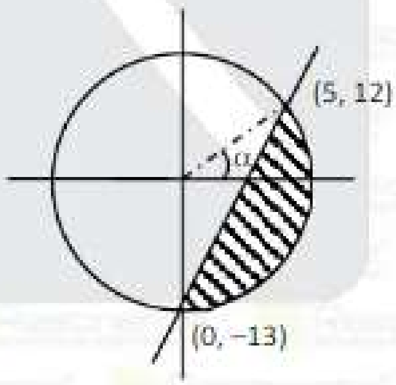
(1) $\frac{169\pi}{4} - \frac{169}{2}\sin^{-1}\frac{12}{13} + \frac{65}{2}$ (2) $\frac{169\pi}{4} + \frac{169}{2}\sin^{-1}\frac{12}{13} - \frac{65}{2}$

(3) $\frac{169\pi}{4} - \frac{169}{2}\sin^{-1}\frac{12}{13} - \frac{65}{2}$ (4) $\frac{169\pi}{4} + \frac{169}{2}\sin^{-1}\frac{12}{13} + \frac{65}{2}$

Ans. (2)

Sol.

Solving line & circle x = 0, 5



$\tan\alpha = \frac{12}{5}$

$\sin\alpha = \frac{12}{13}$

$\sin\left(\frac{\pi}{2} + \alpha\right) = \cos\alpha = \frac{5}{13}$

Now area of segment

$= \frac{1}{2}(r^2)(\theta - \sin\theta)$

$= \frac{1}{2} \times 169 \times \left[\frac{\pi}{2} + \sin^{-1}\alpha - \sin\left(\frac{\pi}{2} + \alpha\right)\right]$

$= \frac{169}{2}\left[\frac{\pi}{2} + \sin^{-1}\frac{12}{13} - \frac{5}{13}\right]$

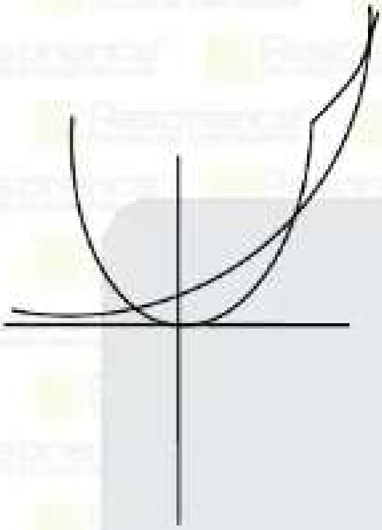
$= \frac{169\pi}{4} + \frac{169}{2}\sin^{-1}\frac{12}{13} - \frac{65}{2}$

22. If $f(x) = 2^x - x^2$ and $f(x) = 0$ has m solutions. $f'(x) = 0$ has n solutions then $m + n =$

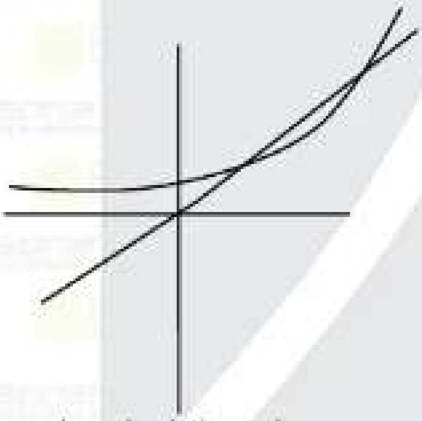
Ans. (5)

Sol. $f(x) = 0 \Rightarrow 2^x = x^2$

number of solutions $= 3$



and



number of solutions of

$f'(x) = 0$

$2^x \ln 2 = 2x$

2 solutions

So total 5 solutions