JEE Main 29 January 2024 Shift 1 Answer Key

Mathematics

Q.1: What is the rank of the word GTWENTY in the dictionary?
A.1: 553

Q.2: If a die is rolled until 2 is obtained, then what is the probability that 2 is obtained on an even-numbered toss?
A.2: $\frac{5}{11}$

Q.3: A GP has 64 terms such that (Sn) total = 7(Sn) odd. Find the common ratio r.
A.3: R = 6

Q.4: $(C_1^{11}/2) + (C_2^{11}/3) + ... + (C_9^{11}/10) = m/n$. Find m + n.
A.4: 2041

Q.5: If $\frac{dy}{dx} - \left(\frac{sin2x}{1+cos^2x}\right)y = \frac{sinx}{1+cos^2x}$ and $y(0) = 0$, then $y\left(\frac{\pi}{2}\right) = ?$
A.5: 1

Q.6: If $2A^3 = 2^{21}$ and $A = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \alpha & \beta \\ 0 & \beta & \alpha \end{bmatrix}$, then find α if $(\alpha, \beta \in I)$

A.6: 5

Q.7:
$$\lim_{x \to \pi/2} \frac{\int_{x^3}^{\pi/2^3} cos\, t^{1/3} dt}{\left(x - \frac{\pi}{2}\right)^2} = ?$$
A.7: (3π^2)/8

Q.8: If $4\cos\theta + 5\sin\theta = 1$, then find the number of all positive values of $\tan\theta$ where $\theta \in (-\pi/2, \pi/2)$.

A.8: 2

Q.9: If the given data 60, 60, 44, 58, 68, α, β, 56 has a mean of 58 and a variance of 66.2, then find $\alpha^2 + \beta^2$.

A.9: 7181.6

Q.10: If $|z + 1| = \alpha z + \beta (i + 1)$ and $z = (1/2) - 2i$, then find $\alpha + \beta$.

A.10: 6

Q.11: In an increasing arithmetic progression $a_1, a_2,..., a_n$ if $a_6 = 2$ and the product of a_1, a_5 and a_4 is greatest, then the value of d is equal to?

A.11: 1.6

Q.12: If relation R: (a, b) R (c, d) is only if ad - bc is divisible by 5, (a, b, c, d $\in$ Z) then R is:
i. Reflexive
ii. Symmetric, Reflexive but not Transitive
iii. Reflexive, Transitive but not Symmetric
iv. Equivalence Relation

A.12: Symmetric, Reflexive but not Transitive

Q.13: 4. $f(x) = \begin{cases} 2 + 2x & ; x \in (-1, 0) \\ 1 - \frac{x}{3} & ; x \in [0,3) \end{cases}$ and $g(x) = \begin{cases} x & ; x \in [0,1) \\ -x & ; x \in (-3,0) \end{cases}$

Find the range of $fog(x)$.
A.13: (0, 1)

Q.14: $\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \left(\frac{x^2 \cos x}{1 + \pi^x} + \frac{1 + \sin^2 x}{1 + e^{\sin x^{2023}}} \right) dx = \frac{\pi}{4}(\pi + \alpha) - 2$

Find α.
A.14: $\alpha = 3$

Q.15: $\int \frac{(sinx-cosx)\sin^2 x}{sinxcos^2 x+tanxsin^3 x} dx = ?$

A.15: $\frac{\ln|\sin^3 x+\cos^3 x|}{2} + C$

Q.16. $f(x) = \frac{(2^x+2^{-x})(tanx)\sqrt{\tan^{-1}(2x^2-3x+1)}}{(7x^2-3x+1)^3}$

Find $f'(0)$.

A.16: $\sqrt{\pi}$

Q.17: Find the area under the curve $x^2 + y^2 = 169$ and below the line 5x - y = 13.

A.17: $\frac{169\pi}{4} - \frac{65}{2} + \frac{169}{2}sin^{-1}\frac{12}{13}$

Q.18: a, b, c are non-zero vectors and b and c are non-collinear vectors. a +5b is collinear with c and b +6c is collinear with a. If a + αb + βc = 0, then α + β = ?

A.18: 35