

JEE-Main-29-01-2024 (Memory Based) [MORNING SHIFT]

Maths

Question: $f(x) = 2^x - x^2$ $m =$ number of solution such that $f(x)$ with x axis
 $N =$ number of solutions such that $f'(x)$ with x axis $m + n$?

Answer: 5

Solution:

$$f(x) = 2^x - x^2$$

$$f'(x) = 2^x \ln 2 - 2x$$

$$m = 3$$

$$n = 2$$

Question: $(1 + y^2)(1 + \ln x) dx + x dy = 0$

Answer:

Question: Find $I = \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \left(\frac{x^2 \cos x}{1 + e^x} + \frac{1 + \sin^2 x}{1 + e^{\sin(x^{2023})}} \right) dx$

Solution:

$$I = \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \frac{x^2 \sin x}{1 + e^x} + \frac{1 + \cos^2 x}{1 + e^{\sin(x^{2023})}} dx$$

$$I = \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \frac{x^2 \sin x}{1 + e^x} - e^x + \frac{1 + \cos^2 x}{1 + e^{\sin(x^{2023})}} dx$$

$$\alpha I = \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} x^2 \sin x + 1 + \cos^2 x$$

$$\cancel{\alpha} I = \cancel{\alpha} \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} x^2 \sin x + \cos^2 x dx$$

$$I = x^2 (-\cos x) + (2x)(+\sin x) + 2(\cos x) \Big|_0^{\frac{\pi}{2}} + \frac{\pi}{2} + \frac{\pi}{4}$$

$$I = \left(0 + \cancel{2} \cdot \frac{\pi}{2} \right) - (0 + 0 + 2) + \frac{3\pi}{4}$$

$$I = \frac{7\pi}{4} - 2$$

Question: If an AP with terms $\langle a_i \rangle$, $a_6 = 2$ and a_1, a_4, a_5 is maximum. Find the common difference.

Solution:

$$a_6 = 2 \text{ \& } a_1 a_4 a_5 = \text{max. (given)}$$

$$\Rightarrow M = (a_6 - 5d)(a_6 - 2d)(a_6 - d)$$

$$= (2 - 5d)(2 - 2d)(2 - d)$$

$$M = 2(-5d^3 + 17d^2 - 16d + 4)$$

$$\frac{dM}{dQ} = 2(-15d^2 + 24d + 10d - 16) = 0$$

$$= 2(-3d(5d - 8) + 2(sd - 8)) = 0$$

$$= -2(3d - 2)(5d - 8) = 0$$

$$d = \frac{8}{5}$$

Question: $\frac{{}^{11}C_1}{2} + \frac{{}^{11}C_2}{3} + \dots + \frac{{}^{11}C_9}{9} = \frac{n}{m}$

gcd(M, n) = 1, find m + n.

Solution:

$$(1+x)^{11} = \sum_{r=0}^{r=11} {}^h C_r x^r$$

$$\int_0^1 (1+x)^{11} dx = \sum_{r=0}^{r=11} \frac{{}^{11}C_r x^{r+1}}{r+1} \Big|_0^1$$

$$\frac{2^{12} - 1}{12} = \sum_{r=0}^{r=11} \frac{{}^{11}C_r}{r+1}$$

$$= \frac{{}^{11}C_0}{1} + (5) + \frac{{}^{11}C_8}{10} + \frac{{}^{11}C_{10}}{11} + \frac{{}^{11}C_{11}}{12}$$

$$\frac{2^{12} - 1}{12} = 5 + \frac{91}{12} \Rightarrow 5 = \frac{4096 - 91}{12} = \frac{4095}{12} = \frac{1365}{4}$$

$$m + n = 1369$$

Question: $f(x) = \frac{(2^x + 2^{-x})(\tan x)\sqrt{\tan^{-1}(x^2 - x + 1)}}{(x^3 - x^2 + 1)^3}$

Find $f'(a) =$

Solution:

$$f(x) = \frac{(2^x + 2^{-x}) \tan x \sqrt{\tan^{-1}(x^2 - x + 1)}}{(x^3 - x^2 + 1)^3}$$

$$f(0) = 0$$

$$f'(0) = \lim_{x \rightarrow 0} \frac{f(x)}{x} = \frac{2 \cdot 1 \cdot \sqrt{\frac{\pi}{4}}}{1} = \sqrt{\pi}$$

Question:

$$\left. \begin{aligned} x^2 + y^2 &= 46, \\ \frac{x^2}{16} + \frac{y^2}{b^2} &= 1 \end{aligned} \right\}$$

POI lies on $y^2 = 3x^2$ find $3\sqrt{3}$ times of areas of rectangle formed by POI of conics

Question:

$$\lim_{x \rightarrow \frac{\pi}{2}} \frac{1 \cdot \int_{x^3}^{\left(\frac{\pi}{2}\right)^3} \cos\left(x^{\frac{1}{3}}\right) dx}{\left(x - \frac{\pi}{2}\right)^2}$$

Solution:

$$= \frac{3\pi^2}{8}$$

Question: GTWENTY, find rank of GTWENTY

Solution:

2 4 5 1 3 4 6

G T W N T Y

$$\frac{1}{2}! \frac{2}{2}! 3 0 0 0 0$$

$$\text{Rank} = \frac{1}{2!} \times 6! + \frac{2}{2!} \times 5! + 3 \times 4! + 1$$

$$= 360 + 120 + 72 + 1$$

$$= 480 + 73 = 553$$

Question:

$$A \cdot A^T = I \text{ Value of } \left(\frac{1}{2}A\right) \left[(A + A^T)^2 + (A - A^T)^2 \right]$$

Options:

(a) $A^3 + AT$

(b) $(A^3 + AT)^2$

(c) $(A^3 + I)$

(d) A^3

Solution:

$$A \cdot A^T = I$$

$$\frac{1}{2} A \left((A + A^T)^2 + (A - A^T)^2 \right)$$

$$\frac{1}{2} A \left(A^2 + (A^T)^2 + 2I + A^2 (AT)^2 - 2I \right)$$

$$A \left(A^2 + (A^T)^2 \right) \quad A^T = A^{-1}$$

$$= A^3 + AA^T A^T$$

$$= A^3 + A^T$$

Question:

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & \beta & \alpha \\ 0 & \alpha & \beta \end{bmatrix} = A \det(2A)^3 = 2^{21}$$

Find one of value of α (or β) α, β both integers.

Options:

- (a) 3
- (b) 17
- (c) 9
- (d) 6

Solution:

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & \beta & \alpha \\ 0 & \alpha & \beta \end{bmatrix} = A$$

$$\text{Net}(2A)^3 = |2A|^3$$

$$= (8|A|)^3 = 2^{21}$$

$$= 2^9 \cdot |A|^3 = 2^{21}$$

$$\Rightarrow |A|^3 = 2^{12}$$

$$= \beta^2 - \alpha^2 = 2^4 = 16$$

Question:

$$f(x) = 4\sqrt{2}x^3 - 2\sqrt{2}x - 1$$

$S-1 f\left[\frac{1}{2}; 1\right] \rightarrow R; f(x)$ intersection x axis at 1 point S-2 $f(x)$ intersection x axis at

$$x = \cos \frac{\pi}{12}$$

Solution:

S-1

$$f(x) = 4\sqrt{2}x^3 - 2\sqrt{2}x - 1$$

$$f\left(\frac{1}{2}\right) = 4\sqrt{2} \times \frac{1}{8} - 2\sqrt{2} \times \frac{1}{2} - 1 = \sqrt{2} - \sqrt{2} - 1 = -1$$

$$f(1) = 4\sqrt{2} - 2\sqrt{2} - 1$$

$$f\left(\frac{1}{2}\right) \cdot f(1) < 0$$

$$f'(x) = 12\sqrt{2}x^2 - 2\sqrt{2} = 0$$

$$= x^2 = \frac{1}{6}$$

$$x = \pm \frac{1}{\sqrt{6}} \notin \left[\frac{1}{2}, 1\right]$$

$$= 2\sqrt{2} - 1 > 0$$

S-1 is true

Question: Sum of all 64 terms is 7(sum of terms at odd), find common ratio.

Options:

- (a) 6
- (b) 7
- (c)
- (d)

Solution:

$$\frac{a(r^{64} - 1)}{r - 1} = 7 \frac{ar(r^{64-1})}{r - 1} \Rightarrow r = \frac{1}{7}$$

Question: Event of tossing a dice and getting 2 in even no of throws.

Options:

- (a) $\frac{5}{11}$
- (b) $\frac{6}{11}$

Question: $(1 + y^2)(1 + \ln x) dx + x dy = 0$ Passes through $(1, 1)$ find $f(e) = \frac{\alpha \tan^{-1} \frac{3}{2}}{\beta + \tan^{-1} \frac{3}{2}}$.

Find $\alpha + 2\beta$

Question: $Z = \frac{1}{2} + 2i, |z + 1| = \alpha z + \beta(1 + i)$

Find $\alpha + 2\beta$ or $2\alpha + \beta$

Solution:

$$Z = \frac{1}{2} + 2i, |z+1| = \alpha z + \beta + \beta i$$

$$Z+1 = \frac{3}{2} + 2i, \sqrt{\frac{9}{4} + 4} = \alpha \left(\frac{3}{2} + 2i \right) + \beta + \beta i$$

$$\frac{3\alpha}{2} + \beta = \frac{5}{2}, 2\alpha + \beta = 0 \Rightarrow \beta = -2d$$

$$\Rightarrow 3\alpha - 4\alpha = 5 \Rightarrow \alpha = -5, \beta = -10$$

Question: (a, b) R(c, d) a, b, c, d $\in \mathbb{Z}$ ab - bd is divided by 5.

Options:

(a) S, R not T

(b) Not Transitive

Solution: Not Transitive

Question: $x^2 + y^2 = 169$, $5x - y = 13$, find area inside circle lying below the line.

Question: $4\cos\theta + 5\sin\theta = 1$, x is a solution Find $\tan x \in \left[\frac{-\pi}{2}, \frac{\pi}{2} \right]$

Question: $4 \left[\frac{1-f^2}{1+f^2} \right] + 5 \left[\frac{2f}{1+f^2} \right] = I$ when $f = \tan \frac{x}{2}$