JEE-Main-29-01-2024 (Memory Based) [MORNING SHIFT]

Maths

Question: f(x) = $2^x - x^2$ m = number of solution such that f(x) with x axis
N = number of solutions such that f'(x) with x axis m + n ?
Answer: 5
Solution:

$$f(x) = 2^x - x^2$$
$$f'(x) = 2^x \ell n^2 - 2x$$
$$m = 3$$
$$n = 2$$

Question: $(1 + y^2)(1 + \ln x)\,dx + x\,dy = 0$
Answer:

Question: Find $I = \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \left(\frac{x^2 \cos x}{1+e^x} + \frac{1+\sin^2 x}{1+e^{\sin\left(x^{2023}\right)}} \right)$

Solution:

$$I = \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \frac{x^2 \sin x}{1+e^x} + \frac{1+\cos^2 x}{1+e^{\sin(x2023)}} dx$$

$$I = \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \frac{x^2 \sin x}{1+e^x} - e^x + \frac{1+\cos^2 x}{1+e^{\sin(x2023)}} dx$$

$$\alpha I = \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} x^2 \sin x + 1 + \cos^2 x$$

$$\not{\alpha} I = \not{\alpha} \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} x^2 \sin x + \cos^2 x\,dx$$

$$I = x^2(-\cos x) + (2x)(+\sin x) + 2(\cos x)\Big|_0^{\frac{\pi}{2}} + \frac{\pi}{2} + \frac{\pi}{4}$$

$$I = \left(0 + \not{2}.\frac{\pi}{\not{2}}\right) - (0 + 0 + 2) + \frac{3\pi}{4}$$

$$I = \frac{7\pi}{4} - 2$$

Question: If an AP with terms <ai>, $a_6 = 2$ and a_1, a_4 a_5 is maximum.
Find the common difference.
Solution:

$$a_6 = 2 \& a_1 a_4 a_5 = \text{max.(given)}$$
$$\Rightarrow M = (a_6 - 5d)(a_6 - 2d)(a_6 - d)$$
$$=(2 - sd)(2 - 2d)(2 - d)$$
$$M = 2(-5d^3 + 17d^2 - 16d + 4)$$
$$\frac{dM}{dQ} = 2(-15d^2 + 24d + 10d - 16) = 0$$
$$= 2(-3d(5d - 8) + 2(sd - 8)) = 0$$
$$= -2(3d - 2)(5d - 8) = 0$$
$$d = \frac{8}{5}$$

Question: $\frac{{}^{11}C_1}{2} + \frac{{}^{11}C_2}{3} + - - - - - - - + \frac{{}^{11}C}{9} = \frac{n}{m}$
gcd (M, n) = 1, find m + n.
Solution:

$$(1+x)^{11} = \sum_{r=0}^{r=11} {}^{h}C_r x^r$$
$$\int_0^1 (1+x)^{11}\, dx = \sum_{r=0}^{r=11} \frac{{}^{11}C_r x^r}{r+1}\Bigg|^{l}$$
$$\frac{2^{12} - 1}{12} = \sum_{r=0}^{r=11} \frac{{}^{11}C_r}{r+1}$$
$$= \frac{{}^{11}C_0}{1} + (5) + \frac{{}^{11}C_g}{10} + \frac{{}^{11}C_{10}}{11} + \frac{{}^{11}C_{11}}{12}$$
$$\frac{2^{12} - 1}{12} = 5 + \frac{91}{12} \Rightarrow 5 = \frac{4096 - 91}{12} = \frac{4095}{12} = \frac{1365}{4}$$
$$m + n = 1369$$

Question: $f(x) = \frac{(2^x + 2^{-x})(\tan x)\sqrt{\tan^{-1}(x^2 - x + 1)}}{(x^3 - x^2 + 1)^3}$

Find f′(a) =
Solution:

$$f(x)=\frac{\left(2^x+2^{-x}\right)\tan x\sqrt{\tan^{-1}\left(x^2-x+1\right)}}{\left(x^3-x^2+1\right)^3}$$

$$f(0)=0$$

$$f'(0)=\lim_{x\to 0}\frac{f(x)}{x}=\frac{2\cdot 1\cdot\sqrt{\frac{\pi}{4}}}{1}=\sqrt{\pi}$$

Question:

$$\left.\begin{array}{l} x^2+y^2=46, \\ \frac{x^2}{16}+\frac{y^2}{b^2}=1 \end{array}\right\}$$

POI lives on $y^2 = 3x^2$ find $3\sqrt{3}$ times of areas of rectangle formed by POI of conics

Question:

$$\lim_{x\to\frac{\pi}{2}}\frac{1.\int_{x^3}^{\left(\frac{\pi}{2}\right)^3}\cos\left(x^{\frac{1}{3}}\right)dx}{\left(x-\frac{\pi}{2}\right)^2}$$

Solution:

$$=\frac{3\pi^2}{8}$$

Question: GTWENTY, find rank of GTWENTY

Solution:

$$\begin{array}{cccccc} 2 & 4 & 5 & 1 & 3 & 4 & 6 \end{array}$$

$$\begin{array}{cccccc} G & T & W & N & T & Y \end{array}$$

$$\frac{1}{2}!\ \frac{2}{2}!\ 3\ 0\ 0\ 0\ 0$$

$$\text{Rank}=\frac{1}{2!}\times 6!+\frac{2}{2!}\times 5!+3\times 4!+1$$

$$=360+120+72+1$$

$$=480+73=553$$

Question:

$A\cdot A^T=I$ Value of $\left(\frac{1}{2}A\right)\left[\left(A+A^T\right)^2+\left(A-A^T\right)^2\right]$

Options:

(a) A^3 + AT

(b) $(A^3 + AT)^2$

(c) (A^3+I)

(d) A^3

Solution:

$A \cdot A^T = I$

$\frac{1}{2}A\left(\left(A+A^T\right)^2+\left(A-A^T\right)\right)^2$

$\frac{1}{2}A\left(A^2+\left(A^T\right)^2+2I+A^2\left(AT\right)^2-2I\right)$

$A\left(A^2+\left(A^T\right)^2\right) \qquad A^T = A^{-1}$

$= A^3 + AA^T A^T$

$= A^3 + A^T$

Question:

$\begin{bmatrix} 1 & 0 & 0 \\ 0 & \beta & \alpha \\ 0 & \alpha & \beta \end{bmatrix} = A \det\left(2A\right)^3 = 2^{21}$

Find one of value of

α(or β) α, β both integers.

Options:

(a) 3

(b) 17

(c) 9

(d) 6

Solution:

$\begin{bmatrix} 1 & 0 & 0 \\ 0 & \beta & \alpha \\ 0 & \alpha & \beta \end{bmatrix} = A$

$\text{Net}\left(2A\right)^3 = \left|2A\right|^3$

$= \left(8\left|A\right|\right)^3 = 2^{21}$

$= 2^9 \cdot \left|A\right|^3 = 2^{21}$

$\Rightarrow \left|A\right|^3 = 2^{12}$

$= \beta^2 - \alpha^2 = 2^4 = 16$

Question:

$f\left(x\right) = 4\sqrt{2}x^3 - 2\sqrt{2}x - 1$

$S-1\ f\left[\frac{1}{2};1\right] \to R$; f(x) intersection x axis at 1 point S-2 f(x) intersection x axis at

$x = \cos\frac{\pi}{12}$

Solution:

$S-1$

$f(x)=4\sqrt{2}x^3-2\sqrt{2}x-1$

$f\left(\frac{1}{2}\right)=4\sqrt{2}\times\frac{1}{8}-2\sqrt{2}\times\frac{1}{2}-1=\sqrt{2}-\sqrt{2}-1=-14$

$f(1)=4\sqrt{2}-2\sqrt{2}-1$

$f\left(\frac{1}{2}\right)\cdot f(1)<0$

$f'(x)=12\sqrt{2}x^2-2\sqrt{2}=0$

$=x^2=\frac{1}{6}$

$x=\pm\frac{1}{\sqrt{6}}\notin\left[\frac{1}{2},1\right]$

$=2\sqrt{2}-1>0$

$S-1$ is true

Question: Sum of all 64 terms is 7(sum of terms at odd), find common ratio.
Options:
(a) 6
(b) 7
(c)
(d)
Solution:

$$\frac{a\left(r^{04}-1\right)}{r-1}=7\frac{ar\left(r^{64-1}\right)}{r-1}\Rightarrow r=\frac{1}{7}$$

Question: Event of tossing a dice and setting 2 in even no of throws.
Options:
(a) $\frac{5}{11}$
(b) $\frac{6}{11}$

Question: (1 +y^2) (1 + ln x) dx + x dy = 0 Passes thrown (1, 1) find $f(e)=\dfrac{\alpha\tan-\frac{3}{2}}{\beta+\tan^{-1}\frac{3}{2}}$.

Find $\alpha + 2\beta$

Question: $Z=\frac{1}{2}+2i, |z+1|=\alpha z+\beta(l+i)$

Find $\alpha + 2\beta$ or $2\alpha + \beta$
Solution:

$$Z = \frac{1}{2} + 2i, |z+1| = \alpha z + \beta + \beta i$$

$$Z + 1 = \frac{3}{2} + 2i, \sqrt{\frac{9}{4} + 4} = \alpha\left(\frac{3}{2} + 2i\right) + \beta + \beta i$$

$$\frac{3\alpha}{2} + \beta = \frac{5}{2}, 2\alpha + \beta = 0 \Rightarrow \beta = -2d$$

$$\Rightarrow 3\alpha - 4\alpha = 5 \Rightarrow \alpha = -5, \beta = -10$$

Question: (a, b) R(c, d) a, b, c, d ∈ Z ab - bd is divided by 5.
Options:
(a) S, R not T
(b) Not Transitive
Solution: Not Transitive

Question: $x^2 + y^2 = 169$, $5x - y = 13$, find area inside circle lying below the line.

Question: $4\cos\theta + 5\sin\theta = 1$, x is a solution Find tan x. $\in \left[\frac{-\pi}{2}, \frac{\pi}{2}\right]$

Question: $4\left[\frac{1-f^2}{1+f^2}\right] + 5\left[\frac{2f}{1+f^2}\right] = I$ when $f = \tan\frac{x}{2}$