JEE MAIN 29 JANUARY 2024 SHIFT 1 QUESTION PAPER

MATHEMATICS

1. $(C_1{}^{11}/2) + (C_2{}^{11}/3) + ... + (C_9{}^{11}/10) = m/n$. Find m + n.
2. $\int \frac{(sinx-cosx)\sin^2 x}{sinxcos^2x+tanxsin^3x}dx = ?$
3. $\lim_{x\to\pi/2} \frac{\int_{x^3}^{\pi/2^3} cos\, t^{1/3}dt}{\left(x-\frac{\pi}{2}\right)^2} = ?$
4. $\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \left(\frac{x^2cosx}{1+\pi^x} + \frac{1+\sin^2 x}{1+e^{sinx^{2023}}}\right)dx = \frac{\pi}{4}(\pi+\alpha) - 2$
5. A GP has 64 terms such that $(S_n)_{total} = 7(S_n)_{odd}$. Find the common ratio r.
6. a, b, c are non-zero vectors and b and c are non-collinear vectors. a +5b is collinear with c and b +6c is collinear with a. If $a + \alpha b + \beta c = 0$, then $\alpha + \beta = ?$
7. Find $f'(0)$.
8. Find the area under the curve $x^2 + y^2 = 169$ and below the line 5x - y = 13.
9. Find the range of $fog(x)$.
10. Find α.
11. If $\frac{dy}{dx} - \left(\frac{sin2x}{1+cos^2x}\right)y = \frac{sinx}{1+cos^2x}$ and $y(0) = 0$, then $y\left(\frac{\pi}{2}\right) = ?$
12. If $|z + 1| = \alpha z + \beta (i + 1)$ and $z = (1/2) - 2i$, then find $\alpha + \beta$.
13. If $2A^3 = 2^{21}$ and $A = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \alpha & \beta \\ 0 & \beta & \alpha \end{bmatrix}$, then find α if $(\alpha, \beta \in I)$
14. If $4\cos\theta + 5\sin\theta = 1$, then find the number of all positive values of $\tan\theta$ where $\theta \in (-\pi/2, \pi/2)$.
15. If a die is rolled until 2 is obtained, then what is the probability that 2 is obtained on an even-numbered toss?
16. If relation R : (a, b) R (c, d) is only if ad - bc is divisible by 5, (a, b, c, d ∈ Z) then R is:
    i. Reflexive
    ii. Symmetric, Reflexive but not Transitive
    iii. Reflexive, Transitive but not Symmetric
    iv. Equivalence Relation

17. If the given data 60, 60, 44, 58, 68, α, β, 56 has a mean of 58 and a variance of 66.2, then find $\alpha^2 + \beta^2$.

18. In an increasing arithmetic progression $a_1, a_2, \ldots, a_n$ if $a_6 = 2$ and the product of a_1, a_5 and a_4 is greatest, then the value of d is equal to?

19. What is the rank of the word GTWENTY in the dictionary?

20. $f(x) = \frac{(2^x+2^{-x})(tanx)\sqrt{\tan^{-1}(2x^2-3x+1)}}{(7x^2-3x+1)^3}$

21. $f(x) = \begin{cases} 2+2x & ; x \in (-1,0) \\ 1-\frac{x}{3} & ; x \in [0,3) \end{cases}$ and $g(x) = \begin{cases} x & ; x \in [0,1) \\ -x & ; x \in (-3,0) \end{cases}$