MATHEMATICS

SECTION - A

Multiple Choice Questions: This section contains 20 multiple choice questions. Each question has 4 choices (1), (2), (3) and (4), out of which **ONLY ONE** is correct.

Choose the correct answer :

1. In an arithmetic progression if sum of 20 terms is 790 and sum of 10 terms is 145, then $S_{15} - S_5$ is (when S_n denotes sum of n terms)

(1) 400 (2) 395

(3) 385 (4) 405

Answer (2)

Sol. $S_{20} = \frac{20}{2}[2a + 19d] = 790$

$2a + 19d = 79$ ...(1)

$S_{10} = \frac{10}{2}[2a + 9d] = 145$

$2a + 9d = 29$ ...(2)

from (1) and (2) $a = -8$, $d = 5$

$S_{15} - S_5 = \frac{15}{2}[2a + 14d] - \frac{5}{2}[2a + 4d]$

$= \frac{15}{2}[-16 + 70] - \frac{5}{2}[-16 + 20]$

$= 405 - 10$

$= 395$

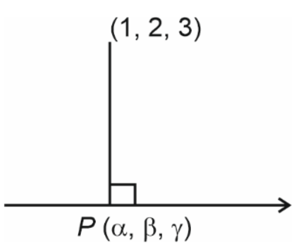
2. If the foot of perpendicular from (1, 2, 3) to the line $\frac{x+1}{2} = \frac{y-2}{5} = \frac{z-4}{1}$ is (α, β, γ) then find $\alpha + \beta + \gamma$

(1) 6 (2) 5.8

(3) 4.8 (4) 5

Answer (2)

Sol.



$(\alpha - 1) \times 2 + (\beta - 2) \times 5 + (\gamma - 3) \times 1 = 0$

$2\alpha + 5\beta + \gamma - 15 = 0$

Also, P lie on line

$\Rightarrow \alpha + 1 = 2\lambda$

$\beta - 2 = 5\lambda$

$\gamma - 4 = \lambda$

$\Rightarrow 2(2\lambda - 1) + 5(5\lambda + 2) + \lambda + 4 - 15 = 0$

$\Rightarrow 4\lambda + 25\lambda + \lambda - 2 + 10 + 4 - 15 = 0$

$30\lambda - 3 = 0$

$\Rightarrow \lambda = \frac{1}{10}$

$\Rightarrow \alpha + \beta + \gamma = (2\lambda - 1) + (5\lambda + 2) + (\lambda + 4)$

$= 8\lambda + 5 = \frac{8}{10} + 5 = 5.8$

3. $\lim_{n\to\infty} \sum_{k=1}^{n} \frac{n^3}{(n^2 + k^2)(n^2 + 3k^2)}$

(1) $\frac{\pi}{2\sqrt{3}} - \frac{\pi}{8}$ (2) $\frac{\pi}{2\sqrt{3}} + \frac{\pi}{8}$

(3) $\frac{\pi}{2} - \frac{\pi}{\sqrt{3}}$ (4) $\frac{\pi}{\sqrt{3}} - \frac{\pi}{4}$

Answer (1)

Sol. $\lim_{n\to\infty} \sum_{k=1}^{n} \frac{n^3}{n^4\left(1 + \frac{k^2}{n^2}\right)\left(1 + \frac{3k^2}{n^2}\right)}$

$= \lim_{n\to\infty} \frac{1}{n} \sum_{k=1}^{n} \frac{1}{\left(1 + \frac{k^2}{n^2}\right)\left(1 + \frac{3k^2}{n^2}\right)}$

$= \int_0^1 \frac{dx}{3(1 + x^2)\left(\frac{1}{3} + x^2\right)}$

$= \int_0^1 \frac{1}{3} \times \frac{3}{2} \frac{(x^2 + 1) - \left(x^2 + \frac{1}{3}\right)}{(1 + x^2)\left(x^2 + \frac{1}{3}\right)} dx$

$= \frac{1}{2}\int_0^1 \left[\frac{1}{x^2 + \left(\frac{1}{\sqrt{3}}\right)^2} - \frac{1}{1 + x^2}\right] dx$

$= \frac{1}{2}\left[\sqrt{3}\tan^{-1}\left(\sqrt{3}x\right)\right]_0^1 - \frac{1}{2}\left(\tan^{-1}x\right)_0^1$

$= \frac{\sqrt{3}}{2}\left(\frac{\pi}{3}\right) - \frac{1}{2}\left(\frac{\pi}{4}\right) = \frac{\pi}{2\sqrt{3}} - \frac{\pi}{8}$

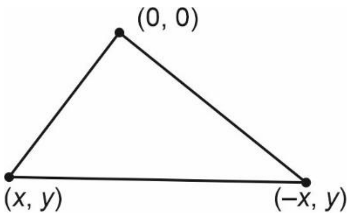
4. The value of maximum area possible of a ΔABC such that $A(0, 0)$ and $B(x, y)$ and $C(-x, y)$ such that $y = -2x^2 + 54x$ is (in sq. unit)

(1) 5800 (2) 5832

(3) 5942 (4) 6008

Answer (2)

Sol.



Area of Δ

$$= \frac{1}{2}\begin{vmatrix} 0 & 0 & 1 \\ x & y & 1 \\ -x & y & 1 \end{vmatrix}$$

$$\Rightarrow \left|\frac{1}{2}(xy + xy)\right| = |xy|$$

Area $(\Delta) = |xy| = \left|x\left(-2x^2 + 54x\right)\right|$

$\frac{d(\Delta)}{dx} = \left|\left(-6x^2 + 108x\right)\right| \Rightarrow \frac{d\Delta}{dx} = 0$ at $x = 0$ and 18

$\Rightarrow$ at $x = 0$, minima

and at $x = 18$ maxima

Area $(\Delta) = \left|18\left(-2(18)^2 + 54 \times 18\right)\right| = 5832$

5. The range of r for which circles $(x + 1)^2 + (y + 2)^2 = r^2$ and $x^2 + y^2 - 4x - 4y + 4 = 0$ coincide at two distinct points

(1) $3 < r < 7$ (2) $5 < r < 9$

(3) $\frac{1}{2} < r < 4$ (4) $0 < r < 3$

Answer (1)

Sol. If two circles intersect at two distinct points

$\Rightarrow |r_1 - r_2| < C_1C_2 < r_1 + r_2$

$|r - 2| < \sqrt{9 + 16} < r + 2$

$|r - 2| < 5$ and $r + 2 > 5$

$-5 < r - 2 < 5$ $r > 3$ ...(2)

$-3 < r < 7$ ...(1)

From (1) and (2)

$3 < r < 7$

6. An ellipse whose length of minor axis is equal to half of length between foci, then eccentricity is

(1) $\frac{7}{2}$ (2) $\sqrt{17}$

(3) $\frac{2}{\sqrt{5}}$ (4) $\frac{3}{\sqrt{7}}$

Answer (3)

Sol. $\because ae = 2b$

$\therefore \frac{4b^2}{a^2} = e^2$

Or $4(1 - e^2) = e^2$

$\therefore 4 = 5e^2 \Rightarrow e = \frac{2}{\sqrt{5}}$

7. If $g'\left(\frac{3}{2}\right) = g'\left(\frac{1}{2}\right)$ and

$f(x) = \frac{1}{2}\left[g(x) + g(2 - x)\right]$ and $f'\left(\frac{3}{2}\right) = f'\left(\frac{1}{2}\right)$ then

(1) $f''(x) = 0$ has exactly one root in (0, 1)

(2) $f''(x) = 0$ has no root in (0, 1)

(3) $f''(x) = 0$ has at least two roots in (0, 2)

(4) $f''(x) = 0$ has 3 roots in (0, 2)

Answer (3)

Sol.

$$f'(x) = \frac{g'(x) - g'(2 - x)}{2},\ f'\left(\frac{3}{2}\right) = \frac{g'\left(\frac{3}{2}\right) - g'\left(\frac{1}{2}\right)}{2} = 0$$

Also $f'\left(\frac{1}{2}\right) = \frac{g'\left(\frac{1}{2}\right) - g'\left(\frac{3}{2}\right)}{2} = 0,\ f'(1) = 0$

$\Rightarrow f'\left(\frac{3}{2}\right) = f'\left(\frac{1}{2}\right) = 0$

$\Rightarrow$ roots in $\left(\frac{1}{2}, 1\right)$ and $\left(1, \frac{3}{2}\right)$

$\Rightarrow f''(x)$ is zero at least twice in $\left(\frac{1}{2}, \frac{3}{2}\right)$

8. The domain of $y = \cos^{-1}\left|\frac{2 - |x|}{4}\right| + \left(\log(3 - x)\right)^{-1}$ is $[-\alpha, \beta) - \{\gamma\}$, then value of $\alpha + \beta + \gamma = ?$

(1) 9 (2) 12

(3) 11 (4) 10

Answer (3)

Sol. $-1 \le \left|\frac{2-|x|}{4}\right| \le 1$

$\Rightarrow \left|\frac{2-|x|}{4}\right| \le 1$

$\Rightarrow -1 \le \frac{2-|x|}{4} \le 1$

$-4 \le 2 - |x| \le 4$

$-6 \le -|x| \le 2$

$-2 \le |x| \le 6$

$|x| \le 6$

$\Rightarrow x \in [-6, 6]$ ...(1)

Now, $3 - x \ne 1$

And $x \ne 2$ ...(2)

and $3 - x > 0$

$x < 3$ ...(3)

From (1), (2) and (3)

$\Rightarrow x \in [-6, 3] - \{2\}$

$\alpha = 6$

$\beta = 3$

$\gamma = 2$

$\alpha + \beta + \gamma = 11$

9. If $y = f(x)$ is solution of differential equation $(x^2 - 1)\,dy = \left((x^3+1) + \sqrt{1-x^2}\right)dx$ and $y(0) = 2$ then find $y\left(\frac{1}{2}\right)$.

(1) $\frac{13}{7} - \frac{\pi}{2} + \ln 5$ (2) $\frac{15}{7} + \frac{\pi}{3} + \ln 2$

(3) $\frac{17}{8} + \frac{\pi}{6} - \ln 2$ (4) $\frac{18}{7} - \frac{\pi}{6} + \ln 3$

Answer (3)

Sol. $\frac{dy}{dx} = \frac{(x+1)(x^2-x+1) + \sqrt{(1-x)(1+x)}}{(x-1)(x+1)}$

$\Rightarrow \frac{dy}{dx} = \frac{x(x-1)+1}{(x-1)} + \sqrt{\frac{(1-x)(1+x)}{(x-1)^2(x+1)^2}}$

$\frac{dy}{dx} = x + \frac{1}{x-1} + \frac{1}{\sqrt{(1-x)(1+x)}}$

$\Rightarrow dy = x\,dx + \frac{1}{(x-1)}dx + \frac{dx}{\sqrt{1-x^2}}$

$\Rightarrow y = \frac{x^2}{2} + \ln|x-1| + \sin^{-1}x + c$

at $x = 0$, $y = 2 \Rightarrow 2 = c$

$\Rightarrow y = \frac{x^2}{2} + \ln|x-1| + \sin^{-1}x + 2$

$y\left(\frac{1}{2}\right) = \frac{17}{8} + \frac{\pi}{6} - \ln 2$

10. Given $x^2 - 70x + \lambda = 0$ with positive roots α and β where one of the root is less than 10 and $\frac{\lambda}{2}$ and $\frac{\lambda}{3}$ are not integers then find value of $\frac{\sqrt{\alpha-1}+\sqrt{\beta-1}}{|\alpha-\beta|}$ is equal to

(1) $\frac{1}{5}$ (2) $\frac{1}{12}$

(3) $\frac{1}{60}$ (4) $\frac{1}{70}$

Answer (1)

Sol. Given : $x^2 - 70x + \lambda = 0$

$\Rightarrow$ Let roots be α and β

$\Rightarrow \beta = 70 - \alpha$

$\lambda = \alpha(70 - \alpha)$

λ is not divisible by 2 and 3

$\Rightarrow \alpha = 5, \beta = 65$

$\Rightarrow \frac{\sqrt{5-1}+\sqrt{65-1}}{|60|} = \left|\frac{4+8}{60}\right| = \frac{1}{5}$

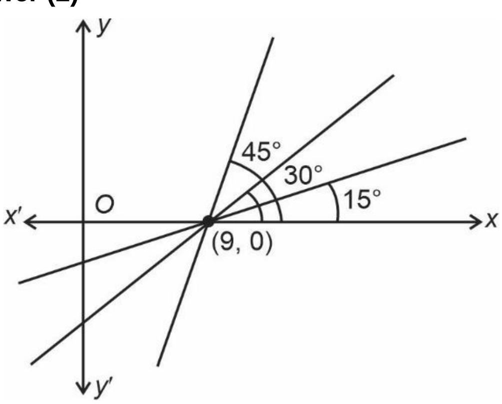
11. A line passes through (9, 0), making angle 30° with positive direction of x-axis. It is rotated by angle of 15° with respect to (9, 0). Then one of the equation of new line is

(1) $y = (2+\sqrt{3})(x-9)$ (2) $y = (2-\sqrt{3})(x-9)$

(3) $y = 2(x-9)$ (4) $y = -(x-9)$

Answer (2)

Sol.



Eq^n : $y - 0 = \tan 15° (x - 9) \Rightarrow y = (2-\sqrt{3})(x-9)$

Eq^n : $y - 0 = \tan 45° (x - 9) \Rightarrow y = (x - 9)$

Option (B) is correct

12. For a non-zero complex number z satisfying $z^2 + i\bar{z} = 0$, then value of $|z|^2$ is

(1) 1 (2) 2

(3) 3 (4) 4

Answer (1)

Sol. $z^2 = -i\bar{z}$

$|z^2| = |-i\bar{z}|$

$|z^2| = |z|$

$|z|^2 - |z| = 0$

$|z|(|z| - 1) = 0$

$|z| = 0$ (not acceptable)

$\therefore |z| = 1$

$\therefore |z|^2 = 1$

13. If $|\vec{a}| = 1,\ |\vec{b}| = 4$

$\vec{a}\cdot\vec{b} = 2$ and $\vec{c} = 2(\vec{a}\times\vec{b}) - 3\vec{b}$

Then the angle between $\vec{b}$ and $\vec{c}$ is

(1) $\theta = \cos^{-1}\left(\frac{-\sqrt{3}}{2}\right)$ (2) $\theta = \cos^{-1}\left(\frac{\sqrt{3}}{2}\right)$

(3) $\theta = \cos^{-1}\left(\frac{1}{2}\right)$ (4) $\theta = \cos^{-1}\left(\frac{-1}{2}\right)$

Answer (1)

Sol. Given $|\vec{a}| = 1,\ |\vec{b}| = 4,\ \vec{a}\cdot\vec{b} = 2$

$\vec{c} = 2(\vec{a}\times\vec{b}) - 3\vec{b}$

Dot product with $\vec{a}$ on both sides

$\vec{c}\cdot\vec{a} = -6$ ... (1)

Dot product with $\vec{b}$ on both sides

$\vec{b}\cdot\vec{c} = -48$ ... (2)

$\vec{c}\cdot\vec{c} = 4|\vec{a}\times\vec{b}|^2 + 9|\vec{b}|^2$

$|\vec{c}|^2 = 4\left[|\vec{a}|^2\cdot|\vec{b}|^2 - (\vec{a}\cdot\vec{b})^2\right] + 9|\vec{b}|^2$

$|\vec{c}|^2 = 4\left[(1)(4)^2 - (4)\right] + 9(16)$

$|\vec{c}|^2 = 4[12] + 144$

$|\vec{c}|^2 = 48 + 144$

$|\vec{c}|^2 = 192$

$\therefore\quad \cos\theta = \frac{\vec{b}\cdot\vec{c}}{|\vec{b}||\vec{c}|}$

$\cos\theta = \frac{-48}{\sqrt{192}\cdot 4}$

$\cos\theta = \frac{-48}{8\sqrt{3}\cdot 4}$

$\cos\theta = \frac{-3}{2\sqrt{3}}$

$\cos\theta = \frac{-\sqrt{3}}{2} \quad \Rightarrow \quad \theta = \cos^{-1}\left(\frac{-\sqrt{3}}{2}\right)$

14. Given set S = {0, 1, 2, 3, ….., 10}. If a random ordered pair (x, y) of elements of S is chosen, then find probability that $|x - y| > 5$

(1) $\frac{30}{121}$ (2) $\frac{31}{121}$

(3) $\frac{62}{121}$ (4) $\frac{64}{121}$

Answer (1)

Sol. If x = 0, y = 6, 7, 8, 9, 10

If x = 1, y = 7, 8, 9, 10

If x = 2, y = 8, 9, 10

If x = 3, y = 9, 10

If x = 4, y = 10

If x = 5, y = no possible value

Total possible ways = (5 + 4 + 3 + 2 + 1) × 2

= 30

Required probability $= \frac{30}{11\times 11} = \frac{30}{121}$

15.

16.

17.

18.

19.

20.

SECTION - B

Numerical Value Type Questions: This section contains 10 questions. In Section B, attempt any five questions out of 10. The answer to each question is a **NUMERICAL VALUE.** For each question, enter the correct numerical value (in decimal notation, truncated/rounded-off to the second decimal place; e.g. 06.25, 07.00, –00.33, –00.30, 30.27, –27.30) using the mouse and the on-screen virtual numeric keypad in the place designated to enter the answer.

21. Number of integral terms in the binomial expansion of $(7^{1/2} + 11^{1/6})^{824}$ is

Answer (138)

Sol. $T_{n+1} = {}^nC_r 11^{\frac{r}{6}} \cdot 7^{\frac{824-r}{2}}$

For integral term

6 should divide r

and $\dfrac{824-r}{2}$ must be integer

$\Rightarrow$ 2 most divide r

$\Rightarrow$ r divisible by 6

$\Rightarrow$ possible values of $r \in \{0, 1, 2, \ldots 824\}$

$\Rightarrow$ For integer terms

$r \in \{0, 6, 12, \ldots 822\}$ $(822 = 0 + (n-1)6 \Rightarrow n = 138)$

= 138 terms

22. $9\int_0^9 \left[\sqrt{\dfrac{10x}{x+1}}\right] dx$ is equal to (where [ ] represents greatest integer function)

Answer (155)

Sol. $I = 9\int_0^9 \left[\sqrt{\dfrac{10x}{x+1}}\right] dx$

$= 9\left[\int_0^{1/9} 0\,dx + \int_{1/9}^{2/3} dx + \int_{2/3}^{9} 2dx\right]$

$= 9\left[\dfrac{2}{3} - \dfrac{1}{9} + 2\left[9 - \dfrac{2}{3}\right]\right]$

$= 9\left[\dfrac{5}{9} + 2\times\dfrac{25}{3}\right]$

= 5 + 6 × 25

= 5 + 150

= 155

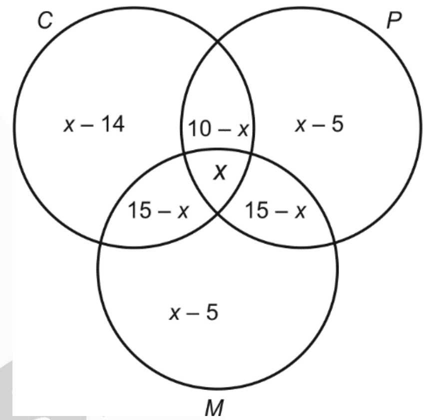
23. In a class there are 40 students. 16 passed in Chemistry, 20 passed in Physics, 25 passed in Mathematics. 15 students passed in both Mathematics and Physics. 15 students passed in both Mathematics and Chemistry and 10 students passed in both Physics and Chemistry. Find the maximum number of students that passed in all the subjects.

Answer (19)

Sol. $n(C) = 16$, $n(P) = 20$, $n(M) = 25$

$n(M \cap P) = n(M \cap C) = 15$, $n(P \cap C) = 10$,

$n(M \cap C \cap P) = x.$



$n(C \cup P \cup M) \le n(U) = 40$

$n(C \cup P \cup M) = n(C) + n(P) + n(M) - n(C \cup M) - n(P \cup M) - n(C \cap P) + n(C \cap P \cap M)$

$40 \ge 16 + 20 + 25 - 15 - 15 - 10 + x$

$40 \ge 61 - 40 + x$

$19 \ge x$

So maximum number of students that passed all the exams is 19.

24. For the following data table

x_i	f_i
0 – 4	2
4 – 8	4
8 – 12	7
12 – 16	8
16 – 20	6

Find the value of 20 M (where M is median of the data)

Answer (245)

Sol.

x_i	f_i	c.f.
0 – 4	2	2
4 – 8	4	6
8 – 12	7	13
12 – 16	8	21
16 – 20	6	27

$$N = \sum f = 27$$

$$\left(\frac{N}{2}\right) = \frac{27}{2} = 13.5$$

So, we have median lies in the class 12 – 16

$l_1 = 12$, $f = 8$, $h = 4$, $c.f. = 13$

So, here we apply formula

$$M = l_1 + \frac{\frac{N}{2} - c.f.}{f} \times h = 12 + \frac{13.5 - 13}{8} \times 4$$

$$= 12 + \frac{.5}{2}$$

$$M = \frac{24.5}{2} = 12.25$$

20 M = 20 × 12.25

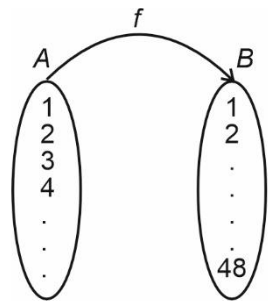
= 245

25. Set $A = \{1, 2, 3, 4, 5, 6, 7\}$

If number of functions from set A to power set of A can be expressed as m^n (m is least integer). Find $m + n$.

Answer (51)

Sol. $n\,P(A) = 2^7 = 128$



$f : A \to B$

Number of function = 128 × 128….128 = 128^7

$$= \left(2^7\right)^7 = 2^{49}$$

$\Rightarrow \quad m^n = 2^{49}$

$\therefore \quad m + n = 49 + 2 = 51$

26.

27.

28.

29.

30.

❑ ❑ ❑