MATHEMATICS

SECTION - A

Multiple Choice Questions: This section contains 20 multiple choice questions. Each question has 4 choices (1), (2), (3) and (4), out of which **ONLY ONE** is correct.

Choose the correct answer :

1. Bag *A* contains 7 white balls and 3 red balls. Bag *B* contains 3 white balls and 2 red balls. A ball is chosen randomly and found to be red then find the probability that it is taken from bag *A*.

(1) $\frac{7}{20}$ (2) $\frac{1}{2}$

(3) $\frac{3}{7}$ (4) $\frac{1}{5}$

Answer (3)

Sol. Bag *A* contains 7 white balls and 3 red balls.

Bag *B* contains 3 white balls and 2 red balls.

Probability that red ball is chosen from bag

$A = P\left(\frac{R}{A}\right) = \frac{3}{10}$

Probability that red ball is chosen from bag *B*

$= P\left(\frac{R}{A}\right) = \frac{2}{5}$

Probability that red ball is chosen from bag *A*

$$= \frac{\frac{3}{10}\times\frac{1}{2}}{\frac{3}{10}\times\frac{1}{2}+\frac{2}{5}\times\frac{1}{2}} = \frac{\frac{3}{20}}{\frac{3}{20}+\frac{2}{10}}$$

$= \frac{3}{7}$

2. Given $|\vec{b}| = 2, |\vec{b}\times\vec{a}| = 2$

Then $|\vec{b}\times\vec{a}-\vec{b}|^2$ is

(1) 0 (2) 8

(3) 1 (4) 10

Answer (2)

Sol. $|\vec{b}\times\vec{a}-\vec{b}|^2 = |\vec{b}\times\vec{a}|^2 + |b^2| - 2(\vec{b}\times\vec{a})\cdot\vec{b}$

$= 4 + 4 \quad \left[\because\ |\vec{b}\times\vec{a}| = 2\ \&\ |\vec{b}| = 2\right]$

$= 8$

3. If $f(x) = \ln\left(\frac{2x+3}{4x^2-x-3}\right) + \cos^{-1}\left(\frac{2x+1}{x+2}\right)$.

If domain of $f(x)$ is $[\alpha,\beta)$, then $5\alpha - 4\beta$ is

(1) –2 (2) 3

(3) –4 (4) 1

Answer (1)

Sol. $\frac{2x+3}{4x^2-x-3} > 0$

$\frac{2x+3}{4x^2-4x+3x-2} > 0$

$\frac{2x+3}{(4x+3)(x-1)} > 0$

$\Rightarrow x \in \left(\frac{-3}{2},\frac{-3}{4}\right)\cup(1,\infty)$

Now

$-1 \le \frac{2x+1}{x+2} \le 1$

$\frac{2x+1}{x+2}+1 \ge 0$ and $\frac{2x+1}{x+2}-1 \le 0$

$\frac{3x+3}{x+2} \ge 0$ and $\frac{x-1}{x+2} \le 0$

$\Rightarrow x \in (-\infty,-2)\cup[-1,\infty)$ ...(2)

$x \in (-2, 1]$ ... (3)

By (2) and (3)

$x \in [-1, 1]$ ... (4)

And By (1) and (4)

$x \in \left[-1, \frac{-3}{4}\right)$

$5\alpha - 4\beta = -2$

4. If $f(x) = \frac{x}{(1+x^4)^{1/4}}$ and $g(x) = f(f(f(f(x))))$

then $\int_0^{\sqrt{2\sqrt{5}}} x^2 g(x)\,dx$ is equal to

(1) $\frac{11}{6}$ (2) $\frac{13}{6}$

(3) $\frac{2}{5}$ (4) $\frac{17}{6}$

Answer (2)

Sol. $f(x) = \dfrac{x}{(1+x^4)^{1/4}}$

$f(f(x)) = \dfrac{x}{(1+2x^4)^{1/4}}$

$f(f(f(x))) = \dfrac{x}{(1+3x^4)^{1/4}}$

$\therefore \quad g(x) = f(f(f(f(x)))) = \dfrac{x}{(1+4x^4)^{1/4}}$

$\therefore \quad I = \displaystyle\int_0^{\sqrt{2\sqrt{5}}} \frac{x^3}{(1+4x^4)^{1/4}}\,dx$

Let $1 + 4x^4 = t^4 \Rightarrow 4x^3\,dx = t^3\,dt$

$\therefore \quad I = \displaystyle\int_1^3 \frac{t^2}{4}\,dt = \frac{1}{12}(3^3 - 1^3) = \frac{13}{6}$

5. If 1st term of a GP is 'a' and 3rd term is 'b' and in 2nd GP 1st term is 'a' and 5th term is 'b' and 11th term of 1st GP common to which term of 2nd GP

(1) 24 (2) 25

(3) 21 (4) 18

Answer (3)

Sol. First term of 1st GP is a and common ratio be r_1

First term of 2nd GP is a and common ratio be r_2

3rd term of 1st GP = $ar_1^2 = b$

5th term of 2nd GP = $ar_2^4 = b$

$\Rightarrow \quad ar_1^2 = ar_2^4$

$\Rightarrow \quad r_1 = \pm r_2^2$

11th term of 1st GP = ar_1^{10}

$= a(\pm r_2^2)^{10}$

$= ar_2^{20}$

Hence, it will be common to 21st term of 2nd GP

6. $z^{1985} + z^{100} + 1 = 0$ and

$z^3 + 2z^2 + 2z + 1 = 0$

then number of common roots of equation is

(1) 1 (2) 2

(3) 3 (4) 4

Answer (2)

Sol. The roots of equation $z^{1985} + z^{100} + 1 = 0$ be ω & ω^2 and also satisfies $z^3 + 2z^2 + 2z + 1 = 0$

$\therefore$ ω & ω^2 are common solutions.

(ω is cube root of unity)

$\therefore$ 2 solutions

7. If $x^2 - y^2 + 2hxy + 2gx + 2fy + c = 0$ is the locus of points such that it is equidistance from the lines $x + 2y - 8 = 0$ and $2x + y + 7 = 0$, then value of $h + g + f + c$ is

(1) 15 (2) –15

(3) 20 (4) –20

Answer (3)

Sol. Combined equation of angle bisectors of lines is

$$\left[\left(\frac{2x+y+7}{\sqrt{5}}\right) - \left(\frac{x+2y-8}{\sqrt{5}}\right)\right]\left[\left(\frac{2x+y+7}{\sqrt{5}}\right) + \left(\frac{x+2y-8}{\sqrt{5}}\right)\right] = 0$$

$\Rightarrow \quad (2x+y+7)^2 - (x+2y-8)^2 = 0$

$\Rightarrow \quad (3x+3y-1)(x-y+15) = 0$

$\Rightarrow \quad 3x^2 - 3y^2 + 15 + 44x + 46y + 0xy = 0$

$\Rightarrow \quad x^2 - y^2 + \dfrac{44x}{3} + \dfrac{46y}{3} + 5 = 0$

$\Rightarrow \quad h = 0,\ g = \dfrac{22}{3},\ f = \dfrac{23}{3},\ c = 5$

$\Rightarrow \quad h + f + g + c = \dfrac{45}{3} + 5 = 20$

8. $A = \begin{bmatrix} x & 0 & 0 \\ 0 & y & 0 \\ 0 & 0 & z \end{bmatrix}$

$\dfrac{x}{\sin\theta} = \dfrac{y}{\sin\left(\theta + \frac{2\pi}{3}\right)} = \dfrac{z}{\sin\left(\theta + \frac{4\pi}{3}\right)}$

Then

Statement 1: $T_r(A) = 0$

Statement 2: $T_r(\text{adj}(\text{adj } A))$

(1) Statement 1 & 2 are true

(2) Statement 1 is true

(3) Statement 2 is true

(4) None of these

Answer (1)

Sol. $x = k\sin\theta$

$y = k\sin\left(\theta + \dfrac{2\pi}{3}\right)$

$z = k\sin\left(\theta + \dfrac{4\pi}{3}\right)$

$x + y + z = k\left[\sin\theta + \sin\left(\theta + \dfrac{2\pi}{3}\right) + \sin\left(\theta + \dfrac{4\pi}{3}\right)\right]$

$= 0$

$\therefore$ Statement 1 is correct

$$\text{adj } A = \begin{bmatrix} yz & 0 & 0 \\ 0 & xz & 0 \\ 0 & 0 & xy \end{bmatrix}$$

$$(\text{adj}(\text{adj } A)) = \begin{bmatrix} x^2yz & 0 & 0 \\ 0 & y^2xz & 0 \\ 0 & 0 & xyz^2 \end{bmatrix}$$

$Tr(\text{adj}(\text{adj } A)) = xyz[x + y + z] = 0$

$= 0$

$\therefore$ Statement 2 is true

9. If $S_n = 3 + 7 + 11 + \ldots$ upto n terms

And $40 < \dfrac{6}{n(n+1)} \sum_{k=1}^{n} S_k < 45$, then n is

(1) 9 (2) 10

(3) 11 (4) 12

Answer (1)

Sol. $S_n = n(2n + 1)$

$$\sum_{k=1}^{n} S_k = \sum_{k=1}^{n} (2k^2 + k)$$

$$= 2 \cdot \frac{n(n+1)(2n+1)}{6} + \frac{n(n+1)}{2}$$

$$\therefore \frac{6}{n(n+1)} \sum_{k=1}^{n} S_k$$

$$= \frac{6}{n(n+1)} \cdot n(n+1)\left(\frac{2n+1}{3} + \frac{1}{2}\right)$$

$= 4n + 2 + 3$

$= 4n + 5$

$$\because 40 < \frac{6}{n(n+1)} \sum_{k=1}^{n} S_k < 45$$

$40 < 4n + 5 < 45$

$3s < 4n < 40$

$\therefore n = 9$

10. In a paper there are 3 sections A, B and C which has 8, 6 and 6 questions each. A student have to attempt 15 questions such that they have to attempt atleast 4 questions out of each sections, then number of ways of attempting these questions are

(1) 11300 (2) 11376

(3) 12576 (4) 13372

Answer (2)

Sol.

A	B	C	$\Rightarrow$	No. of ways	
4	5	6	$\to$	${}^8C_4\,{}^6C_5\,{}^6C_6$	$= 6 \times {}^8C_4$
4	6	5	$\to$	${}^8C_4\,{}^6C_6\,{}^6C_5$	$= 6 \times {}^8C_4$
7	4	4	$\to$	${}^8C_7\,{}^6C_4\,{}^6C_4$	$= 8 \times (15)^2$
6	5	4	$\to$	${}^8C_6\,{}^6C_5\,{}^6C_4$	$= 28 \times 6 \times 15$
6	4	5	$\to$	${}^8C_6\,{}^6C_4\,{}^6C_5$	$= 28 \times 15 \times 6$
5	5	5	$\to$	${}^8C_5\,{}^6C_5\,{}^6C_5$	$= {}^8C_5 \times 36$
5	6	4	$\to$	${}^8C_5\,{}^6C_6\,{}^6C_4$	$= {}^8C_5 \times 15$
5	4	6	$\to$	${}^8C_5\,{}^6C_4\,{}^6C_6$	$= {}^8C_5 \times 15$

$= {}^8C_5\,[66] + 28 \times 15 \times 12 + 8 \times 15^2 + 12 \times {}^8C_4$

$= 11376$

11.

12.

13.

14.

15.

16.

17.

18.

19.

20.

SECTION - B

Numerical Value Type Questions: This section contains 10 Numerical based questions. The answer to each question should be rounded-off to the nearest integer.

21. If $f(x) = (x-2)^2(x-3)^3$ and $x \in [1, 4]$. If M and m denotes maximum and minimum value respectively, then $M - m$ is

Answer (12)

Sol. $f'(x) = 2(x-2)(x-3)^3 + 3(x-2)^2(x-3)^2 = 0$

$(x-2)(x-3)^2[2(x-3) + 3(x-2)] = 0$

$(x-2)(x-3)^2[5x - 12] = 0$

Now $f\left(\dfrac{12}{5}\right) = \dfrac{4}{25} \times \left(-\dfrac{27}{125}\right)$

$f(1) = -8$ (minimum)

$f(4) = 4$ (maximum)

$\therefore \quad M - m = 12$

22. If $\vec{a}=\hat{i}+\alpha\hat{j}+\beta\hat{k}$ $|\vec{b}|^2=6$ and angle between $\vec{a}$ and $\vec{b}$ is $\frac{\pi}{4}$. If $\vec{a}\cdot\vec{b}=3$ then $(\alpha^2+\beta^2)|\vec{a}\times\vec{b}|^2$ is

Answer (18)

Sol. $\vec{a}\cdot\vec{b}=|\vec{a}||\vec{b}|\cos\theta=3$

$=\sqrt{1+\alpha^2+\beta^2}\,.\,\sqrt{6}\frac{1}{\sqrt{2}}=3$

$\Rightarrow$ $1+\alpha^2+\beta^2=3$

$\Rightarrow$ $\alpha^2+\beta^2=2$

Also $|\vec{a}|=\sqrt{1+\alpha^2+\beta^2}=\sqrt{3}$

$\Rightarrow$ $|\vec{a}\times\vec{b}|=|\vec{a}||\vec{b}|\sin\theta$

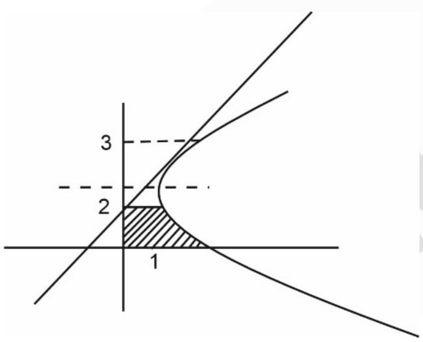
$=\sqrt{3}\times\sqrt{6}\times\frac{1}{\sqrt{2}}=3 \Rightarrow |\vec{a}\times\vec{b}|^2=9$

$\Rightarrow$ $(\alpha^2+\beta^2)\,|\vec{a}\times\vec{b}|^2=2\times 9=18$

23. If $(y-2)^2=(x-1)$ and $x-2y+4=0$, then find the area bounded by the curves between the coordinate axis in first quadrant (in sq. unit).

Answer (05.00)

Sol. We have to find shaded area



$$\Rightarrow \int_0^2\left[(y-2)^2+1\right]dy+\int_2^3\left[\left((y-2)^2+1\right)-\left(\frac{2y-4}{2}\right)\right]dy$$

$$=\frac{(y-2)^3}{3}+y\Big|_0^2+\frac{(y-2)^3}{3}+y-\left(\frac{y^2}{2}-2y\right)\Big|_2^3$$

$$=\left(2+\frac{8}{3}\right)+\left[\left(\frac{1}{3}+3\right)-\left(\frac{9}{2}-6\right)\right]-\left[2-(2-4)\right]=5$$

24. If $3\sin(A+B)=4\sin(A-B)$ and

If $\tan A=k\tan B$, then value of k is ______

Answer (7)

Sol. $\frac{\sin(A+B)}{\sin(A-B)}=\frac{4}{3}$

$$\frac{\sin(A+B)+\sin(A-B)}{\sin(A+B)-\sin(A-B)}=\frac{7}{1}$$

[$\because$ Using componendo and dividendo]

$$\frac{2\sin A\cos B}{2\cos A\sin B}=7$$

$$\frac{\tan A}{\tan B}=7$$

$k=7$

25. If $x(x^2+3|x|+5|x-1|+6|x-2|)=0$ then, find the number of solutions of the given equation.

Answer (1)

Sol. $x=0$ is the solution

(I) $x<0$

$(x^2-3x-5(x-1)-6(x-2))=0$

$x^2-14x+17=0$

All the roots are greater than 0

(II) $0<x<1$

$x^2+3x-5(x-1)-6(x-2)=0$

$x^2-8x+17=0$

$D<0$

No solution in this interval

(III) $1<x<2$

$x^2+3x+5(x-1)-6(x-2)=0$

$x^2+2x+7=0$

$D<0$

No Solution

(IV) $x>2$

$x^2+3x+5(x-1)+6(x-2)=0$

$x^2+14x-17=0$

All the roots is less than 2

Hence, $x=0$ is the only solution.

26. A set $R=\{1, 2, 3, 4\}$ is given then find the number of symmetric relation which are not reflexive relation.

Answer (960)

Sol. $R=\{1, 2, 3, 4\}$

here number of elements $n=4$

Number of relations which are symmetric but not reflexive $=2^{\frac{n(n+1)}{2}}-2^{\frac{n^2-n}{2}}$

$= 2^{\frac{4.5}{2}} - 2^6$

$= 2^{10} - 2^6$

$= 1024 - 64 = 960$

27. If $f(x) = ae^{2x} + be^x + cx$, $f(0) = -1$, $f'(\ln 2) = 4$, if $\int_0^{\ln 4} (f(x) - cx)\, dx = \frac{39}{2}$ find $|a + b + c|$

Answer (25)

Sol. $\because\ f(x) = ae^{2x} + be^x + cx$

$\Rightarrow\ f'(x) = 2ae^{2x} + be^x + c$

$\because\ f'(\ln 2) = 4$

$\Rightarrow\ 4 = 2a(4) + b(2) + c$

$\Rightarrow\ 8a + 2b + c = 4$ ...(i)

$\because\ \int_0^{\ln 4} (ae^{2x} + be^x)\, dx = \frac{39}{2}$

$\Rightarrow\ \frac{a}{2}\left[e^{2x}\right]_0^{\ln 4} + b(e^x)_0^{\ln 4} = \frac{39}{2}$

$\Rightarrow\ \frac{a}{2}[16-1] + b(4-1) = \frac{39}{2}$

$\Rightarrow\ \frac{15a}{2} + 3b = \frac{39}{2}$

$\Rightarrow\ \frac{5a}{2} + b = \frac{13}{2}$

$\Rightarrow\ 5a + 2b = 13$ ...(ii)

Also $f(0) = -1$

$\Rightarrow\ -1 = a + b$ ...(iii)

From (ii) & (iii)

$5a + 5b = -5$

$5a + 2b = 13$

$3b = -18$

$\Rightarrow\ \boxed{b = -6}$

$\Rightarrow\ \boxed{a = 5}$

$\therefore\ \boxed{c = -24}$

$|a + b + c| = 25$

28.

29.

30.

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