PART : MATHEMATICS

1. Number of integral term in the expansion of $\left(7^{\frac{1}{2}}+11^{\frac{1}{6}}\right)^{824}$

(1) 139 (2) 138 (3) 140 (4) 137

Ans. **(2)**

Sol. $T_{r+1} = {}^{824}C_r\left(7^{\frac{1}{2}}\right)^{824-r}\left(11^{\frac{1}{6}}\right)^{r}$

$\Rightarrow$ r must be multiple of 6

$\Rightarrow$ r = 0,6,12 822

$\Rightarrow$ 138 terms

2. For any real number x, Let [x] denote the largest integer less than or equal to x. The value of $9\int_0^9\left[\sqrt{\frac{10x}{x+1}}\right]dx$

Ans. **(155)**

Sol. $\frac{10x}{x+1}=1 \quad \Rightarrow \quad x=\frac{1}{9}$

$\frac{10x}{x+1}=4 \quad \Rightarrow \quad x=\frac{2}{3}$

$\frac{10x}{x+1}=9 \quad \Rightarrow \quad x=9$

$$I=\int_0^{\frac{1}{9}}0\,dx+\int_{\frac{1}{9}}^{\frac{2}{3}}1\,dx+\int_{\frac{2}{3}}^{9}2\,dx$$

$$I=0+[x]_{\frac{1}{9}}^{\frac{2}{3}}+[2x]_{\frac{2}{3}}^{9}$$

$$I=\frac{2}{3}-\frac{1}{9}+18-\frac{4}{3}$$

So, $I=\frac{155}{9}$

3. If S_n denotes sum of first n terms of an A.P. such that, $S_{20} = 790$, $S_{10} = 145$ then $S_{15} - S_5$

(1) 540 (2) 395 (3) 555 (4) 575

Ans. **(2)**

Sol. $S_{20}=\frac{20}{2}(2a+19d)=790 \Rightarrow 2a + 19d = 79$

$S_{10} = S_{20}=\frac{10}{2}(2a+9d)=145 \Rightarrow 2a + 9d = 29$

10 d = 50

d = 5

$S_{15}-S_5=\frac{15}{2}(2a+14d)-\frac{5}{2}(2a+4d)$

10a + 95d = 395

4. $\vec{a} = a_1\hat{i} + a_2\hat{j} + a_3\hat{k}$ $\quad |\vec{a}| = 1 \quad \vec{a}.\vec{b} = 2$

$\vec{b} = b_1\hat{i} + b_2\hat{j} + b_3\hat{k}$ $\quad |\vec{b}| = 4$

$\vec{c} = 2(\vec{a}\times\vec{b}) - 3\vec{b}$ then angle between $\vec{c}$ and $\vec{b}$

Ans. **(150°)**

Sol. $\because \vec{c} = 2(\vec{a}\times\vec{b}) - 3\vec{b}$ .....(1)

$|\vec{c}|^2 = 4(\vec{a}\times\vec{b}) - 12(\vec{a}\times\vec{b})\vec{b} + 9\vec{b}^2 = 4\left(|\vec{a}|^2|\vec{b}|^2 - (\vec{a}.\vec{b})^2\right) + 9b^2$

$|\vec{c}|^2 = 4(1.16 - 4) + 9.16 = 16\ (3 + 9) = 16 \times 12$

Again, equation .....(1).$\vec{b}$: $\quad \vec{b}.\vec{c} = 0 - 3|\vec{b}|^2$

$|\vec{b}||c\cos\theta| = 3|\vec{b}|^2$

$\cos\theta = \dfrac{3.4}{4.2\sqrt{3}} = \dfrac{\sqrt{3}}{2} \Rightarrow \theta = 30^\circ$

5. A line making an angle 30° with positive x-axis at (4, 0). Now it is rotated by an angle 15° in clockwise direction. The equation of line is

(1) $x + y - 4 = 0$ (2) $x - y - 4 = 0$

(3) $(\sqrt{3} - 2)x + y + 8 - 4\sqrt{3} = 0$ (4) $(2 - \sqrt{3})x - y - 8 + 4\sqrt{3} = 0$

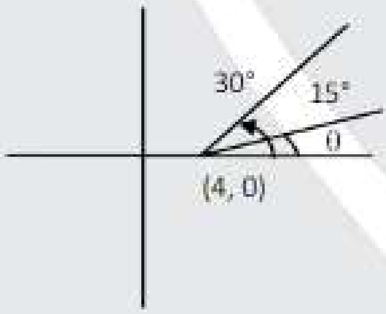
Ans. **(4)**

Sol.

$\theta = 30^\circ - 15^\circ$

Line $y - 0 = \tan 15^\circ\ (x - 4)$

$(2 - \sqrt{3})x - y - 8 + 4\sqrt{3} = 0$



6. Let (α, β, γ) be the foot of perpendicular from the point (1,2,3) on the line $\dfrac{x+3}{5} = \dfrac{y-1}{2} = \dfrac{z+4}{3}$ then $19(\alpha+\beta+\gamma)$

Ans. **(101)**

Sol. $Q(5\lambda-3,\ 2\lambda-1,\ 3\lambda-4) = (\alpha, \beta, \gamma)$

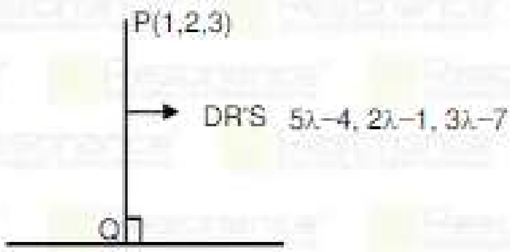
Since lines are perpendicular there for

$a_1a_2 + b_1b_2 + c_1c_2 = 0$

$5(5\lambda-4) + 2(2\lambda-1) + 3(3\lambda-7) = 0$, $38\lambda - 43 = 0$

Now $\quad 19(\alpha+\beta+\gamma) = 19\ (10\lambda-6)$

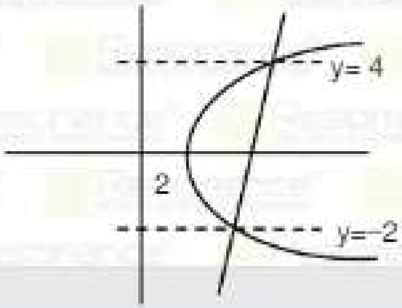
$= 19\left(\dfrac{430}{38} - 6\right) = 101$



7. Area bounded by curves $y^2 = 4(x-2)$ and $y = 2x - 8$ is

Ans. (9)

Sol. $\frac{y+8}{2} = x \Rightarrow y^2 - 2y - 8 = 0 \Rightarrow -2, 4$



Area

$$= \int_{-2}^{4}\left(\frac{y+8}{2} - \left(\frac{y^2}{4} + 2\right)\right)dy = \left.\frac{y^2}{4} + 2y - \frac{y^3}{12}\right)_{-2}^{4}$$

$= 9$

8. Circle $(x+1)^2 + (y+2)^2 = r^2$ & $x^2 + y^2 - 4x - 4y + 4 = 0$
Cuts each other at two different points then value of r is

(1) $\frac{1}{2} < r < 7$ (2) $0 < r < 7$ (3) $3 < r < 7$ (4) $5 < r < 9$

Ans. (3)

Sol. $C_1(-1, -2) : r_1 = r$

$C_2(2, 2) : r_2 = \sqrt{4+4+2} = 2$

Circle intersect each other $\therefore |r_1 - r_2| < C_1C_2 < |r_1 + r_2|$ $|r - 2| < 5 < r + 2$

$|r - 2| < 5$ and $5 < r + 2$

$-5 < r - 2 < 5$ $r > 3$ ______(2)

$-3 < r < 7$ _____(1)

(1) ∩ (2) $r \in (3, 7)$

9. $f(0) = \frac{1}{2}$, find $\lim\limits_{x \to 0} \frac{x\int_0^x f(t)dt}{e^{x^2} - 1} = \alpha$ then find $8\alpha^2$.

(1) 3 (2) 1 (3) 2 (4) 0

Ans. (3)

Sol. $$\lim_{x\to 0}\frac{\int_0^x f(t)dt + xf(x)}{e^{x^2}(2x)} = \lim_{x\to 0}\frac{\int_0^x f(t)dt}{e^{x^2}(2x)} + \lim_{x\to 0}\frac{f(x)}{e^{x^2}(2)}$$

$$= \lim_{x\to 0}\frac{f(x)}{2\left(e^{x^2} + xe^{x^2}(2x)\right)} + \frac{1}{4} = \frac{1}{2}$$

$\Rightarrow 8\alpha^2 = 2$

10. Let A (2, 3, 5) and C (–3, 4, –2) be opposite vertices of a parallelogram ABCD. If the diagonal $\overrightarrow{BD} = \hat{i} + 2\hat{j} + 3\hat{k}$, then area of parallelogram is equal to

Ans. $\frac{1}{2}\sqrt{474}$

Sol. $\frac{1}{2}\left|\overrightarrow{AC} \times \overrightarrow{BD}\right| = \frac{1}{2}\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 5 & -1 & 7 \\ 1 & 2 & 3 \end{vmatrix}$

$= \frac{1}{2}\left|-17\hat{i} - 8\hat{j} + 11\hat{k}\right|$

$= \frac{1}{2}\sqrt{289 + 64 + 121}$, $\quad = \frac{1}{2}\sqrt{474}$

11. If $\alpha = 1^2 + 4^2 + 8^2 + 13^2 + \ldots\ldots$ up to 10 terms and $\beta = \sum_{N=1}^{10} N^4$ such that $4\alpha - \beta = 55k + 40$, then find k.

Ans. (353)

Sol. 1, 4, 8, 13, ...
Difference 3, 4, 5, ... A.P.
$t_n = an^2 + bn + c$
$n = 1, 1 = a + b + c$; $n = 2, 4 = 4a + 2b + c$;
$n = 3 \quad 8 = 9a + 3b + c$

$a = \frac{1}{2}, \; b = \frac{3}{2}, \; c = -1$

$\alpha = \sum_{N=1}^{10}\left(\frac{N^2}{2} + \frac{3N}{2} - 1\right)^2$

$4\alpha - \beta = \sum_{N=1}^{10}\left(6N^3 + 5N^2 - 12N + 4\right)$

$= 6(55)^2 + 5\,(5.11.7) - 12.5.11 + 40$
$= 55(353) + 40$
$\therefore k = 353$

12.

Class	Frequency
0-4	2
4-8	9
8-12	10
12-16	8
16-20	7
Total	**36**

If median is M then find the value of 20M

(1) 208 (2) 104 (3) 52 (4) 216

Ans. (4)

Sol. $N = \sum f_i = 36$

$\frac{N}{2} = 18$

Class	fi	cfi
0-4	2	2
4-8	9	11
8-12	10	21
12-16	8	29
16-20	7	36
	N= 36	

$\ell = 8$
$f_m = 10$
$h = 12 - 8 = 4$
$c.f_{m-1} = 11$

$$\text{Median} = \ell + \left(\frac{\frac{N}{2} - cf_{m-1}}{f_m}\right)h$$

$$= 8 + \frac{18-11}{10} \times 4$$

$$M = 8 + \frac{7}{5} \times 2 = 8 + \frac{14}{5} = \frac{54}{5}$$

$$20M = 20 \times \frac{54}{5} = 216$$

13. $\sec x \, dy - \{2(1-x)\tan x + x(2-x)\}dx = 0$

Ans. $y = (\sin x)(2x - x^2) + c$

Sol. $dy = \frac{2(1-x)\tan x + x(2-x)}{\sec x} dx$

$dy = (2(1-x)\sin x + x(2-x)\cos x)dx$

$y = (\sin x)(2x - x^2) + c$

14. The value of $\underset{n\to\infty}{Lt} \sum_{k=1}^{n} \frac{n^3}{(n^2+k^2)(n^2+3k^2)}$ is

(1) $\frac{\pi}{2\sqrt{2}} - \frac{\pi}{4}$ (2) $\frac{\pi}{2\sqrt{3}} - \frac{\pi}{8}$ (3) $\frac{\pi}{2\sqrt{3}} + \frac{\pi}{4}$ (4) $\frac{\pi}{\sqrt{3}} - \frac{\pi}{8}$

Ans. (2)

Sol. $= \underset{n\to\infty}{Lt} \sum_{k=1}^{n} \frac{\frac{1}{n}}{\left(1+\frac{k^2}{n^2}\right)\left(1+\frac{3k^2}{n^2}\right)}$

$$= \int_0^1 \frac{1}{(1+x^2)(1+3x^2)} dx$$

$$= \frac{1}{2}\int_0^1 \left(\frac{3}{1+3x^2} - \frac{1}{1+x^2}\right) dx$$

$$= \frac{1}{2}\left(\sqrt{3}\tan^{-1}x\sqrt{3} - \tan^{-1}x\right)_0^1 = \frac{1}{2}\left(\frac{\sqrt{3}\pi}{3} - \frac{\pi}{4}\right)$$

15. If $z = x + iy$, $xy \neq 0$ satisfy the equation $z^2 + i\bar{z} = 0$, then $|z^2|$ equal to

Ans. **(1)**

Sol. $z^2 = -i\bar{z}$

$|z^2| = |\bar{z}| \Rightarrow |z| = 1$

$|z^2| = 1$

16. If the length of the minor axis of an ellipse is equal to half of the distance between the foci then the eccentricity of the ellipse is

(1) $\frac{2}{\sqrt{5}}$ (2) $\frac{3}{\sqrt{5}}$ (3) $\frac{2}{\sqrt{7}}$ (4) $\frac{3}{\sqrt{7}}$

Ans. (1)

Sol. $2b = \frac{1}{2}(2ae)$

$\frac{b}{a} = \frac{e}{2} \Rightarrow \frac{b^2}{a^2} = \frac{e^2}{4}$

$1 - e^2 = \frac{e^2}{4} \Rightarrow e^2\left(\frac{5}{4}\right) = 1 \Rightarrow e = \frac{2}{\sqrt{5}}$

17. If $f(x) = \begin{vmatrix} 2\cos^4 x & 2\sin^4 x & 3+\sin^2 2x \\ 3+2\cos^4 x & 2\sin^4 x & \sin^2 2x \\ 2\cos^4 x & 3+2\sin^4 x & \sin^2 2x \end{vmatrix}$ then $\frac{1}{5}f'(0)$ is equal to

Ans. **(0)**

Sol. on expanding

$f(x) = 45$, $\Rightarrow f'(x) = 0$

18. If $x, y \in \{0,1,2,3, \ldots\ldots, 10\}$ then the probability that $|x - y| > 5$ is

(1) $\frac{30}{121}$ (2) $\frac{31}{121}$ (3) $\frac{60}{121}$ (4) $\frac{62}{121}$

Ans. **(1)**

Sol. Total number of ways = $11 \times 11 = 121$

$x = 0$, $|y| > 5 \Rightarrow y = 6, 7, 8, 9, 10 \Rightarrow 5$ ways

$x = 1$, $|1 - y| > 5 \Rightarrow y = 7, 8, 9, 10 \Rightarrow 4$ ways

So on

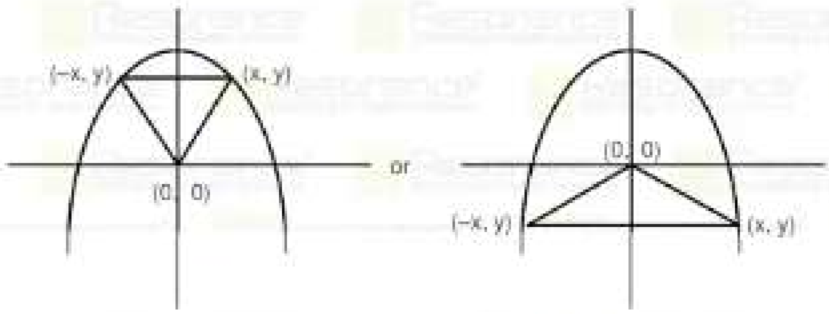
Required probability $= \frac{2(5+4+\ldots.+2+1)}{11\times 11} = \frac{30}{121}$

19. A triangle is formed by vertices (0, 0), (x, y), (–x, y) on xy – plane. If the point (x, y) and (–x, y) lies on $y = -x^2 + 54$, then maximum area of triangle is

(1) $18\sqrt{2}$ (2) $108\sqrt{2}$ (3) $36\sqrt{2}$ (4) $54\sqrt{2}$

Ans. **(2)**

Sol.



Area $= \frac{1}{2}(2x)(54 - x^2)$

$A = 54x - x^3 \Rightarrow \frac{dA}{dx} = 54 - 3x^2$

$x = \pm\sqrt{18}$

Maximum area $A = (54 - 18)\ \sqrt{18}\ = 108\sqrt{2}$

20. If $x^2 - 70x + \lambda = 0$ have roots $\alpha, \beta \in N$, $\frac{\lambda}{2}, \frac{\lambda}{3} \notin N$. Find minimum value of λ.

(1) 320 (2) 325 (3) 330 (4) 335

Ans. (2)

Sol. $\alpha + \beta = 70$

$\alpha\beta = \lambda$

λ Minimum when $\alpha = 5$, $\beta = 65$

$\Rightarrow \lambda = 325$

21. g(x) is non constant differentiable functions $g'\left(\frac{1}{2}\right) = g'\left(\frac{3}{2}\right)$ and $f(x) = \frac{1}{2}[g(x) + g(2-x)]$

(1) $f'\left(\frac{1}{2}\right) + f'\left(\frac{3}{2}\right) = 1$

(2) $f''(x) = 0$, for at least 1 value of $x \in (0, 2)$

(3) $f'''(x) = 0$, for number of values of $x \in (0, 1)$

(4) $f''(x) = 0$, for exactly one value of $x \in (0, 1)$

Ans. (2)

Sol. $f'(x) = \frac{1}{2}(g'(x) - g'(2-x))$

$f'\left(\frac{1}{2}\right) + f'\left(\frac{3}{2}\right) = 0 + 0 = 0$

Since $f'\left(\frac{1}{2}\right) = f'\left(\frac{3}{2}\right) = 0$ (Rolle theorem)

$\Rightarrow f''(x) = 0$ for atleast 1 value of $x \in (0, 2)$

22. $\lim\limits_{x \to 0} \frac{ae^{x^2} + b\cos x}{x^2} = \frac{1}{2}$ then

(1) $a = \frac{1}{3}, b = \frac{1}{3}$ (2) $a = \frac{1}{2}, b = -\frac{1}{2}$

(3) $a = -\frac{1}{3}, b = -\frac{1}{2}$ (4) $a = \frac{1}{3}, b = -\frac{1}{3}$

Ans. (4)

Sol. $a + b = 0$

$\lim\limits_{x \to 0} \frac{2xae^{x^2} - b\sin x}{2x} = \frac{1}{2}$

$\lim\limits_{x \to 0} ae^{x^2} - \frac{b}{2}\frac{\sin x}{x} = \frac{1}{2}$

$a - \frac{b}{2} = \frac{1}{2} \Rightarrow 2a - b = 1$

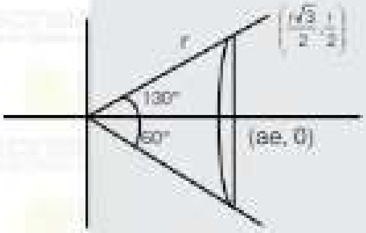
$a + b = 0 \qquad a = \frac{1}{3}, b = -\frac{1}{3}$

23. If latus rectum of the hyperbola $\frac{x^2}{9} - \frac{y^2}{b^2} = 1$ subtends $60°$ at centre of hyperbola and $b^2 = \frac{\ell}{m}\left(1 + \sqrt{n}\right)$, ℓ, m, $n \in N$, ℓ, m being co-prime, then $\ell^2 + m^2 + n^2$ is

(1) 180 (2) 181 (3) 182 (4) 183

Ans. (3)

Sol.



$r = \frac{2b^2}{3} \Rightarrow \frac{r}{2} = \frac{b^2}{3}$

$ae = \frac{r}{2}\sqrt{3} = \frac{b^2}{\sqrt{3}}$

$\therefore a^2e^2 = \frac{b^4}{3} = a^2 + b^2 = 9^2 + b^2$

$b^4 - 3b^2 - 27 = 0$

$+ve\ b^2 = \frac{3 + \sqrt{9 + 108}}{2} = \frac{3}{2}\left(1 + \sqrt{13}\right)$

$\therefore \ell + m^2 + n^2$

$3^2 + 2^2 + 13^2$

$= 182$