

JEE Main 30 January 2024 Shift 1 Answer Key

Mathematics

Q.1: If the foot of the perpendicular from $(1, 2, 3)$ to the line $(x + 1)/2 = (y - 2)/5 = (z - 4)/1$ is (α, β, γ) , then find $\alpha + \beta + \gamma$.

A.1: 5.8

Q.2: In an arithmetic progression, if the sum of 20 terms is 790 and the sum of 10 terms is 145, then $S_{15} - S_5 = ?$

A.2: 395

Q.3: Find the value of the maximum area possible (in sq.units) of ΔABC with vertices $A(0, 0)$, $B(x, y)$ and $C(-x, y)$ such that $y = -2x^2 + 54x$.

A.3: 5832

Q.4: What is the range of r for which circles $(x + 1)^2 + (y + 2)^2 = r^2$ and $x^2 + y^2 - 4x - 4y + 4 = 0$ coincide at two distinct points.

A.4: $3 < r < 7$

Q.5: What is the eccentricity of an ellipse whose length of the minor axis is equal to half of the length between foci?

A.5: $\frac{2}{\sqrt{5}}$

Q.6: $\lim_{n \rightarrow \infty} \sum_{k=1}^n \frac{n^3}{(n^2+k^2)(n^2+3k^2)}$

A.6: $\frac{\pi}{2\sqrt{3}} - \frac{\pi}{8}$

Q.7: The domain of $y = \cos^{-1} \left(\frac{2 - |x|}{4} \right) + \log(3 - x)^{-1}$ is $[\alpha, \beta] - \{\gamma\}$, then find $\alpha + \beta + \gamma$.

A.7: 11

Q.8: A line passes through (9, 0), making angle 30° with positive direction of x-axis. It is rotated by angle of 15° with respect to (9, 0). Find the equation of the new line.

A.8: $y = (2 + \sqrt{3})(x - 9)$

Q.9: For a non-zero complex number z satisfying $z^2 + i\bar{z} = 0$, then value of $|z|^2$ is?

A.9: 1

Q.10: If $|a| = 1$, $|b| = 4$, $a \cdot b = 2$ and $c = 2(a \times b) - 3b$, then what is the angle between b and c ?

A.10: $\theta = \cos^{-1} \left(-\frac{\sqrt{3}}{2} \right)$

Q.11: Set $S = \{0, 1, 2, 3, \dots, 10\}$

If a random ordered pair (x, y) of elements of S is chosen, then find probability that $|x - y| > 5$.

A.11: 30/121

Q.12: Find the number of integral terms in the binomial expansion:
 $(7^{1/2} + 11^{1/6})^{824}$

A.12: 138

Q.13: M. If $y = f(x)$ is the solution of differential equation $(x^2 - 1)dy = ((x^3 + 1) + \sqrt{1 - x^2})dx$ and $y(0) = 2$, then find $y\left(\frac{1}{2}\right)$.

A.13: $\frac{12}{8} + \frac{\pi}{6} - \ln 2$

Q.14: M. Given $x^2 - 70x + 1 = 0$ with positive integral roots α and β where one of the root is less than 10, and of $\frac{\lambda}{2}$ and $\frac{\lambda}{3}$ are not integers, then find value of $\frac{\sqrt{\alpha-1} + \sqrt{\beta-1}}{|\alpha-\beta|}$.

A.14: 1/6

Q.15: M. $9 \int_0^9 \left[\sqrt{\frac{10x}{x+1}} \right] dx = ?$ (Here, $[]$ represents the greatest integer function).

A.15: 155

Q.16: In a class, there are 40 students.

16 students passed in Chemistry.

20 students passed in Physics.

25 students passed in Maths.

15 students passed in both Math and Physics.

15 students passed in both Math and Chemistry

10 students passed in both Physics and Chemistry.

Find the maximum number of students that passed in all the subjects.

A.16: 19

Q.17: Set $A = \{1, 2, 3, 4, 5, 6, 7\}$

If the number of functions from Set A to Power Set A can be expressed as m^n (where m is the least integer), the find $m + n$.

A.17: 51

Q.18: Find the value of $20M$, if M is the median of the following data:

| x_i | f_i | C.F. |
|-------|-------|-------------|
| 0-4 | 2 | 2 |
| 4-8 | 4 | 6 |
| 8-12 | 7 | 13 |
| 12-16 | 8 | 21 |
| 16-20 | 6 | 27 |

A.18: 245