MATHEMATICS

SECTION - A

Multiple Choice Questions: This section contains 20 multiple choice questions. Each question has 4 choices (1), (2), (3) and (4), out of which **ONLY ONE** is correct.

Choose the correct answer :

1. Sum of series $\frac{1}{1-3\cdot1^2+1^4}+\frac{2}{1-3\cdot2^2+2^4}+\frac{3}{1-3\cdot3^2+3^4}+\ldots$ upto 10 terms is

(1) $\frac{-55}{109}$ (2) $\frac{55}{109}$

(3) $\frac{45}{109}$ (4) $\frac{-45}{109}$

Answer (1)

Sol. General term of series

$$\Rightarrow T_r=\frac{r}{1-3r^2+r^4}$$

$$=\frac{r}{(r^4-2r^2+1)-r^2}=\frac{r}{(r^2-1)^2-r^2}$$

$$=\frac{r}{(r^2-r-1)(r^2+r-1)}=\frac{\frac{1}{2}\left[(r^2+r-1)-(r^2-r-1)\right]}{(r^2-r-1)(r^2+r-1)}$$

$$=\frac{1}{2}\left[\frac{1}{r^2-r-1}-\frac{1}{r^2+r-1}\right]$$

Sum of 10 terms ⇒

$$\sum_{r=1}^{10}T_r=\frac{1}{2}\left[\frac{1}{-1}-\frac{1}{1}\right]+\frac{1}{2}\left[\frac{1}{1}-\frac{1}{5}\right]+\frac{1}{2}\left[\frac{1}{5}-\frac{1}{11}\right]+\ldots+\frac{1}{2}\left[\frac{1}{89}-\frac{1}{109}\right]$$

Telescopic sum

$$\Rightarrow \frac{1}{2}\left[-1-\frac{1}{109}\right]=\frac{1}{2}\left(\frac{-110}{109}\right)=\frac{-55}{109}$$

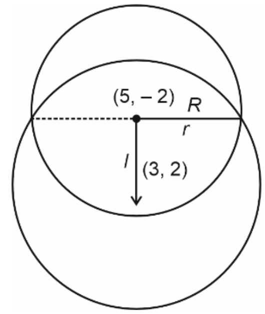
2. If one of the diameter of the circle $x^2 + y^2 - 10x + 4y + 13 = 0$ is a chord of another circle and whose centre is the point of intersection of the lines $2x + 3y = 12$ and $3x - 2y = 5$, then the radius of the circle is

(1) 6 (2) $3\sqrt{2}$

(3) $\sqrt{20}$ (4) 4

Answer (1)

Sol.



$2x + 3y = 12$

$3x - 2y = 5$

Point of intersection ≡ (3, 2)

∴ centre is (3, 2)

$l=\sqrt{4+16}=2\sqrt{5}$

$l^2 + R^2 = r^2$

⇒ $20 + (25 + 4 - 13) = r^2$

⇒ $20 + 16 = r^2$

⇒ $r = 6$

3. An urn contains 15 red, 10 white, 60 orange and 15 green balls. 2 balls are taken with replacement. Find the probability 1 ball is red and other ball is white.

(1) $\frac{2}{27}$ (2) $\frac{3}{22}$

(3) $\frac{1}{33}$ (4) $\frac{1}{29}$

Answer (3)

Sol. Total balls in urn = 15 + 10 + 60 + 15 = 100 balls

2 balls are taken with replacement

So, probability that 1 ball is red and

1 ball is white = $\frac{{}^{15}C_1\times{}^{10}C_1}{{}^{100}C_2}$

$$\frac{15\times10\times2}{100\times99}=\frac{300}{100\times99}=\frac{3}{99}=\frac{1}{33}$$

4. $\lim_{x\to0}\frac{e^{|2\sin x|}-2|\sin x|-1}{x^2}$ is

(1) D.N.E (2) 2

(3) 1 (4) –1

Answer (2)

Sol. $\lim_{x\to 0}\dfrac{e^{|2\sin x|}-2|\sin x|-1}{x^2}$

$= \lim_{x\to 0}\dfrac{1+|2\sin x|+\dfrac{|2\sin x|^2}{2}+...\infty-2|\sin x|-1}{x^2}$

$= \lim_{x\to 0}\dfrac{2\sin^2 x+...}{x^2}$

= 2

5. Number of all 4 letter words formed by the word "DISTRIBUTION' is

(1) 2138 (2) 2976

(3) 3734 (4) 2856

Answer (3)

Sol. 4 letter words formed by

DISTRIBUTION

D $\rightarrow$ 1

I $\rightarrow$ 3

S $\rightarrow$ 1

T $\rightarrow$ 2

R $\rightarrow$ 1

B $\rightarrow$ 1

U $\rightarrow$ 1

O $\rightarrow$ 1

N $\rightarrow$ 1

(i) 4 alike = 0

(ii) 3 alike + 1 diff = ${}^1C_1 \times {}^8C_1 \times \dfrac{4!}{3!}$

(iii) 2*ABC* type

$\Rightarrow {}^2C_1 . {}^8C_2 . \dfrac{4!}{2!}$

(iv) $2A2B = {}^2C_2 . {}^4C_2$

(v) *ABCD* type : ${}^9C_4 \times 4!$

$\Rightarrow$ Total = 3734

6. Let $\vec{a} = 3\hat{i}+\hat{j}-2\hat{k}, \vec{b} = 4\hat{i}+\hat{j}+7\hat{k}$ and $\vec{c} = \hat{i}-3\hat{j}+4\hat{k}$ be 3 vector. If a vector $\vec{p}$ satisfiers $\vec{p}\times\vec{b} = \vec{c}\times\vec{b}$ and $\vec{p}\cdot\vec{a} = 0$, then $\vec{p}\cdot(\hat{i}-\hat{j}-\hat{k})$ is

(1) 32 (2) 23

(3) 16 (4) 61

Answer (1)

Sol. $\vec{p}\times\vec{b}-\vec{c}\times\vec{b} = 0$

$(\vec{p}-\vec{c})\times\vec{b} = 0$

$\Rightarrow \vec{b} \,||\, \vec{p}-\vec{c}$

$\Rightarrow \vec{b} = \lambda(\vec{p}-\vec{c})$

$\Rightarrow k\vec{b}+\vec{c} = \vec{p}$

$\vec{p} = \hat{i}(4k+1)+\hat{j}(k-3)+\hat{k}(7k+4)$

$\vec{p}\cdot\vec{a} = 0$

$(4k+1)3+(k-3)1+(7k+4)(-2) = 0$

$12k + 3 + k - 3 - 14k - 8 = 0$

$\Rightarrow k = -8$

$\therefore \vec{p} = -31\hat{i}-11\hat{j}-52\hat{k}$

$\vec{p}\cdot(\hat{i}-\hat{j}-\hat{k}) = -31+11+52 = -31+63$

= 32

7. The solution of differential equation $y\dfrac{dx}{dy} = x(\log_e x-\log_e y+1)$, $x > 0$, $y > 0$ and passing through $(e, 1)$ is

(1) $\left|\log_e\left(\dfrac{y}{x}\right)\right| = y^2$ (2) $2\left|\log_e\left(\dfrac{x}{y}\right)\right| = y$

(3) $\left|\log_e\left(\dfrac{y}{x}\right)\right| = x$ (4) $\left|\log_e\left(\dfrac{x}{y}\right)\right| = y$

Answer (4)

Sol. $y\dfrac{dx}{dy} = x(\ln x-\ln y+1),\ x>0, y>0$

$\dfrac{dx}{dy} = \dfrac{x}{y}\left(\ln\left(\dfrac{x}{y}\right)+1\right)$

$\dfrac{x}{y} = t \Rightarrow x = ty$

$\Rightarrow \dfrac{dx}{dy} = t+y\dfrac{dt}{dy}$

$\Rightarrow t+y\dfrac{dt}{dy} = t\ln t+t$

$\Rightarrow \dfrac{dt}{t\ln t} = \dfrac{dy}{y}$

$\Rightarrow \ln(\ln t) = \ln y+c$

$\Rightarrow \ln\left(\ln\dfrac{x}{y}\right) = \ln y+c$

at $x = e$, $y = 1$

$\Rightarrow \ln(\ln e) = \ln 1+c$

$\Rightarrow c = 0$

$\Rightarrow \ln\left(\ln\dfrac{x}{y}\right) = \ln y$

$$\Rightarrow \ln\left(\frac{x}{y}\right) = y$$

$$\left|\log_e\left(\frac{x}{y}\right)\right| = y \qquad \text{as } y > 0$$

8. If $f(x) = \dfrac{4x+3}{6x-4}$ and $g(x) = f(f(x))$,

then $g(g(g(g(x)))) = ?$

(1) x (2) $2x$

(3) $-x$ (4) $-2x$

Answer (1)

Sol. $f(x) = \dfrac{4x+3}{6x-4}$

$g(x) = f(f(x))$

$$= \frac{4\left(\dfrac{4x+3}{6x-4}\right)+3}{6\left(\dfrac{4x+3}{6x-4}\right)-4} = \frac{16x+12+18x-12}{24x+18-24x+16}$$

$$= \frac{34x}{34} = x$$

$g(g(g(g(x)))) = x$

9. If $f(x) = \begin{vmatrix} x^3 & 2x^2+1 & 1+3x \\ 3x^2+2 & 2x & x^3+6 \\ x^3-x & 4 & x^2-2 \end{vmatrix}$, then the value of

$2f(0) + f'(0)$ is equal to

(1) 18 (2) 54

(3) 48 (4) 42

Answer (4)

Sol. $f'(x) = \begin{vmatrix} 3x^2 & 2x^2+1 & 1+3x \\ 6x & 2x & x^3+6 \\ 3x^2-1 & 4 & x^2-2 \end{vmatrix} +$

$$\begin{vmatrix} x^3 & 4x & 1+3x \\ 3x^2+2 & 2 & x^3+6 \\ x^3-x & 0 & x^2-2 \end{vmatrix} + \begin{vmatrix} x^3 & 2x^2+1 & 3 \\ 3x^2+2 & 2x & 3x^2 \\ x^3-x & 4 & 2x \end{vmatrix}$$

Now, $f(0) = \begin{vmatrix} 0 & 1 & 1 \\ 2 & 0 & 6 \\ 0 & 4 & -2 \end{vmatrix} = 12$

and $f'(0) = \begin{vmatrix} 0 & 1 & 1 \\ 0 & 0 & 6 \\ -1 & 4 & -2 \end{vmatrix} + \begin{vmatrix} 0 & 0 & 1 \\ 2 & 2 & 6 \\ 0 & 0 & -2 \end{vmatrix} + \begin{vmatrix} 0 & 1 & 3 \\ 2 & 0 & 0 \\ 0 & 4 & 0 \end{vmatrix}$

$= -6 + 0 + 24 = 18$

$\Rightarrow 2f(0) + f'(0) = 2 \times 12 + 18 = 42$

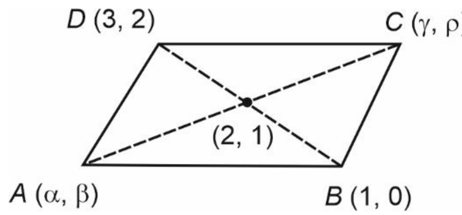
10. $ABCD$ is a parallelogram where $A(\alpha, \beta)$, $B = (1, 0)$, $C(\gamma, \rho)$ and $D(3, 2)$ and $AB = \sqrt{10}$. The value of $2(\alpha + \beta + \gamma + \rho)$ is equal to

(1) 8 (2) 10

(3) 12 (4) 16

Answer (3)

Sol.



Using mid point formula

$\alpha + \gamma = 4$

$\beta + \rho = 2$

$\alpha + \beta + \gamma + \rho = 6$

$\therefore 2(\alpha + \beta + \gamma + \rho) = 12$

11. Let $g(x)$ be a linear function and

$$f(x) = \begin{cases} g(x), & x \le 0 \\ \left(\dfrac{x+1}{x+2}\right)^{\frac{1}{x}}, & x > 0 \end{cases}, \text{ is continuous at } x = 0.$$

If $f'(1) = f(-1)$ then the value of $g(3)$ is

(1) $\dfrac{1}{3}\log_e\left(\dfrac{4}{9}\right)+1$ (2) $\dfrac{1}{3}\log_e\left(\dfrac{4}{9e^{1/3}}\right)$

(3) $\log_e\left(\dfrac{4}{9}\right)-1$ (4) $\log_e\left(\dfrac{4}{9e^{1/3}}\right)$

Answer (4)

Sol. Let $g(x) = px + q$

Since $f(x)$ is continuous at $x = 0$

$$\Rightarrow g(0) = \lim_{x \to 0^+}\left(\frac{x+1}{x+2}\right)^{1/x} = 0$$

$\Rightarrow g(x) = px \qquad (q = 0)$

Now, $f'(1) = f(-1)$

$$y = \left(\frac{1+x}{2+x}\right)^{1/x}$$

$$\Rightarrow \ln y = \frac{1}{x}\ln\left(\frac{1+x}{2+x}\right)$$

$$\frac{1}{y}\frac{dy}{} = \frac{-1}{x^2}\ln\left(\frac{1+x}{2+x}\right) + \frac{1}{x} \times \frac{1}{\left(\dfrac{1+x}{2+x}\right)} \times \frac{(x+2)-(x+1)}{(2+x)^2}$$

at $x = 1$

$$f(1) = \left(\frac{2}{3}\right)\left[-\ln\left(\frac{2}{3}\right) + \frac{3}{2}\left(\frac{1}{9}\right)\right]$$

$= \frac{-2}{3}\ln\left(\frac{2}{3}\right) + \frac{1}{9}$

$\Rightarrow f(-1) = -P = \frac{-2}{3}\ln\left(\frac{2}{3}\right) + \frac{1}{9}$

$\Rightarrow p = \frac{2}{3}\ln\left(\frac{2}{3}\right) - \frac{1}{9}$

$g(3) = 3p = 2\ln\left(\frac{2}{3}\right) - \frac{1}{3} = \ln\left(\frac{4}{9}\right) + \log e^{\left(-1/3\right)}$

$\Rightarrow \ln\left(\frac{4}{9e^{1/3}}\right)$

12. The distance of the point $Q(0, 2, -2)$ from the line passing through the $P(5, -4, 3)$ and perpendicular to the lines $\vec{r} = (-3\hat{i} + 2\hat{k}) + \lambda(2\hat{i} + 3\hat{j} + 5\hat{k}), \lambda \in R$ and $\vec{r} = (\hat{i} - 2\hat{j} + \hat{k}) + \mu(-\hat{i} + 3\hat{j} + 2\hat{k}), \mu \in R$ is

(1) $\sqrt{66}$

(2) $\sqrt{74}$

(3) $\sqrt{56}$

(4) $\sqrt{46}$

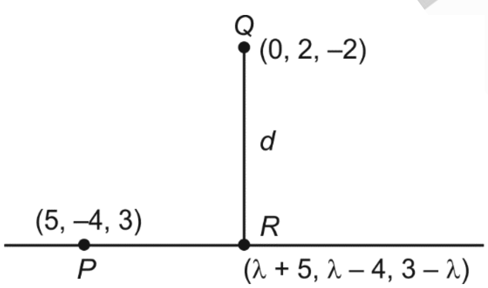
Answer (2)

Sol. $\vec{\tau} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & 3 & 5 \\ -1 & 3 & 2 \end{vmatrix} = \hat{i}(6-15) - \hat{j}(4+5) + \hat{k}(6+3)$

$= -9\hat{i} - 9\hat{j} + 9\hat{k}$

$\therefore$ Required line (L) : $\frac{x-5}{1} = \frac{y+4}{1} = \frac{z-3}{-1} = \lambda$

$x = \lambda + 5, y = \lambda - 4, z = 3 - \lambda$



$\overrightarrow{QR} \cdot \vec{l} = (\lambda+5)\cdot 1 + (\lambda-6)\cdot 1 + (3-\lambda+2)(-1) = 0$

$\Rightarrow \lambda + 5 + \lambda - 6 - 5 + \lambda = 0$

$\Rightarrow 3\lambda = 6$

$\Rightarrow \lambda = 2$

$\therefore R(7, -2, 1)$

$\therefore QR = \sqrt{49 + 16 + 9}$

$= \sqrt{74}$

13. If α denotes the number of real solutions of $(1 - i)^x = 2^x$ and $\beta = \frac{|z|}{\arg z}$,

where $z = \frac{\pi}{4}(1+i)^4\left[\frac{1-\sqrt{\pi}i}{\sqrt{\pi}+i} + \frac{\sqrt{\pi}-i}{1+\sqrt{\pi}i}\right]$, $i = \sqrt{-1}$

then distance of (α, β) from the line $4x - 3y - 7 = 0$ is

(1) 2 (2) 3

(3) 7 (4) 4

Answer (2)

Sol. $z = \frac{\pi}{4}(1+i)^4\left[\frac{1+\pi+\pi+1}{i(1+\pi) - \sqrt{\pi} + \sqrt{\pi}}\right]$

$= \frac{\pi}{4}(1+i)^4 \frac{2}{(i)} = \frac{\pi}{2}\frac{(1+i)^4}{i} = \frac{-\pi}{2}i(1+i)^4$

$|z| = \left|-\frac{\pi}{2}\right| |i| |1+i|^4 = \left(\frac{\pi}{2}\right)(1)(\sqrt{2})^4 = 2\pi$

$z = \frac{-\pi}{2}(-4i) = 2\pi i \Rightarrow \arg(z) = \frac{\pi}{2}$

$\Rightarrow \beta = \frac{2\pi}{\frac{\pi}{2}} = 4$

Also, $(1 - i)^x = 2^x$

$\Rightarrow$ Taking modulus both sides

$\Rightarrow \left(\sqrt{2}\right)^x = 2^x \Rightarrow \left(\sqrt{2}\right)^x = 0$

$\Rightarrow$ at $x = 0$

$\Rightarrow$ 1 solution $\Rightarrow \alpha = 1$

$\Rightarrow$ Perpendicular distance from (1, 4)

$\left|\frac{4(1) - 3(4) - 7}{5}\right| = \left|\frac{4 - 12 - 7}{5}\right| = 3$

14. Let S be the set of positive integral values of 'a' for which $\frac{ax^2 + 2(a+1)x + 9a + 4}{x^2 - 8x + 32} < 0 \quad \forall x \in \mathbb{R}$

Then the number of elements in S is

(1) 1 (2) 3

(3) 0 (4) ∞

Answer (3)

Sol. Given $\frac{ax^2 + 2(a+1)x + 9a + 4}{x^2 - 8x + 32} < 0 \quad \forall x \in \mathbb{R}$

In $x^2 - 8x + 32$, we have $D = 64 - 128 < 0$

$\therefore \quad x^2 - 8x + 32 > 0 \ \forall x \in \mathbb{R}$

$\Rightarrow$ $ax^2 + 2(a + 1)x + 9a + 4 < 0 \ \forall x \in \mathbb{R}$

$\Rightarrow$ $a < 0$ and $D < 0$

$\because$ Question has asked for positive integral values of a

$\therefore$ $|S| = 0$

15. For α, β $\gamma > 0$, if $\sin^{-1}\alpha + \sin^{-1}\beta + \sin^{-1}\gamma = \pi$ and $(\alpha + \beta + \gamma)(\alpha - \gamma + \beta) = 3\alpha\beta$, then γ is

(1) $-\left(\frac{\sqrt{3}-1}{2\sqrt{2}}\right)$ (2) $\frac{-1}{\sqrt{2}}$

(3) $-\sqrt{3}$ (4) $\frac{\sqrt{3}}{2}$

Answer (4)

Sol. Let $\sin A = \alpha$

$\sin B = \beta$

$\sin C = \gamma$

$A + B + C = \pi$

$\Rightarrow$ $(\alpha + \beta)^2 - \gamma^2 = 3\alpha\beta$

$\Rightarrow$ $\alpha^2 + \beta^2 - \alpha\beta = \gamma^2$

$\Rightarrow$ $\alpha^2 + \beta^2 - \gamma^2 = \alpha\beta$

$$\Rightarrow \frac{\alpha^2+\beta^2-\gamma^2}{2\alpha\beta} = \frac{1}{2}$$

$$\Rightarrow \cos C = \frac{1}{2} \Rightarrow C = 60^o$$

$$\Rightarrow \sin C = \frac{\sqrt{3}}{2} = \gamma$$

16.

17.

18.

19.

20.

SECTION - B

Numerical Value Type Questions: This section contains 10 Numerical based questions. The answer to each question should be rounded-off to the nearest integer.

21. In the expansion of

$(1+x)(1-x^2)\left(1+\frac{3}{x}+\frac{3}{x^2}+\frac{1}{x^3}\right)^5$, the sum of coefficients of x^3 and x^{-13} is

Answer (118)

Sol. $(1+x)(1-x^2)\left(1+\frac{3}{x}+\frac{3}{x^2}+\frac{1}{x^3}\right)^5$

$$\Rightarrow (1+x)^2(1-x)\frac{\left(x^3+3x^2+3x+1\right)^5}{x^{15}}$$

$$\Rightarrow \frac{(1+x)^2(1-x)\left[(x+1)^3\right]^5}{x^{15}} = \frac{(1-x)(1+x)^{17}}{x^{15}}$$

$\Rightarrow$ Coefficient of $x^3 \Rightarrow x^{18}$ in $(1 - x)(1 + x)^{17}$

$\Rightarrow$ $(1 + x)^{17} - x(1 + x)^{17}$

$$\Rightarrow 0 - x\left({}^{17}C_{17}\, x^{17}\right) = -{}^{17}C_{17} = -1$$

and coefficient of $x^{-13} \Rightarrow x^2$ in $(1 - x)(1 + x)^{17}$

$\Rightarrow$ $(1 + x)^{17} - x(1 + x)^{17}$

$$\Rightarrow {}^{17}C_2 - {}^{17}C_1 = 17\times 8 - 17$$

$= 17\times 7 = 119$

$\Rightarrow$ sum = 119 – 1= 118

22. $A = \{1, 2, 3, 4\}$, $R = \{(1, 2), (2, 3), (2, 4)\}$ $R \subseteq S$ and S is an equivalence relation, then minimum number of elements to be added to R is n then value of n?

Answer (13)

Sol. $R = \{(1, 2), (2, 3), (2, 4)\}$

for reflexive, we need to add,

(1, 1), (2, 2), (3, 3), (4, 4)

for symmetric

if $(1, 2) \in R$

then $(2, 1) \in R$

if $(2, 3) \in R$

then $(3, 2) \in R$

if $(2, 4) \in R$

then $(4, 2) \in R$

So set becomes

{(1, 1), (2, 2), (3, 3), (4, 4), (1, 2), (2, 1), (2, 3), (3, 2), (2, 4), (4, 2)}

for transitive

As $(1, 2) \in R$

$(2, 3) \in R$

then $(1, 3) \in R$ then $(3, 1) \in R$ (for symmetric)

& $(1, 2) \in R$

$(2, 4) \in R$

then $(1, 4) \in R$ then $(4, 1) \in R$ (for symmetric)

(3, 2) $\in R$

(2, 4) $\in R$

then (3, 4) $\in R$ then (4, 3) $\in R$ (for symmetric)

so set S = {(1, 1), (2, 2), (3, 3), (4, 4), (1, 2), (2, 1), (2, 3), (3, 2), (2, 4), (4, 2), (1, 3), (3, 1), (1, 4), (4, 1), (3, 4), (4, 3)}

so 13 new elements are added

$\Rightarrow \quad n = 13$

23. If $|\vec{a}| = 1, |\vec{b}| = 4$ are $\vec{a}\cdot\vec{b} = 2$

also, $\vec{c} = (3\vec{a}\times\vec{b}) - \vec{b}$ and α is the angle between $\vec{b}$ and $\vec{c}$ then the value of $192\sin^2\alpha$.

Answer (167)

Sol. $|\vec{c}|^2 = 9(|\vec{a}\times\vec{b}|)^2 + |\vec{b}|^2 + 0$

$|\vec{c}|^2 = 9(16-4) + 16$

$\{\because \ |\vec{a}\times\vec{b}|^2 = |\vec{a}|^2|\vec{b}|^2 - (\vec{a}\cdot\vec{b})^2\}$

$|\vec{c}|^2 = 124$

$|\vec{c}| = \sqrt{124}$

$\vec{c} = (3(\vec{a}\times\vec{b})) - \vec{b}$

$\vec{c}\cdot\vec{b} = -|\vec{b}|^2 = -16$

$4\times\sqrt{124}\cos\alpha = -16$

$\cos\alpha = \frac{-4}{\sqrt{124}} = \frac{-2}{\sqrt{31}}$

$\sin\alpha = \sqrt{1-\frac{4}{31}}$

$\sin\alpha = \sqrt{\frac{27}{31}}$

Then, $192\sin^2\alpha = 192\times\frac{27}{31}$

≈ 167.2

24. If the system of linear equation x – 2y + z = – 4, $2x + \alpha y + 3z = 5$ & $3x - y + \beta z = 3$ has infinitely many solutions then $12\alpha + 13\beta$ is equal to

Answer (58)

Sol. $x - 2y + z = -4$

$2x + \alpha y + 3z = 5$

$3x - y + \beta z = 3$

$$\Delta_2 = \begin{vmatrix} 1 & -4 & 1 \\ 2 & 5 & 3 \\ 3 & 3 & \beta \end{vmatrix} = 0$$

$(5\beta - 9) + 4(2\beta - 9) - 9 = 0$

$13\beta = 54$

$\beta = \frac{54}{13}$

$$\Delta_3 = \begin{vmatrix} 1 & -2 & -4 \\ 2 & \alpha & 5 \\ 3 & -1 & 3 \end{vmatrix} = 0$$

$(3\alpha + 5) + 2(-9) - 4(-2 - 3\alpha) = 0$

$3\alpha + 5 - 18 + 8 + 12\alpha = 0$

$\Rightarrow \alpha = \frac{1}{3}$

$12\alpha + 13\beta = 4 + 13\times\frac{54}{13}$

$= 58$

25.

26.

27.

28.

29.

30.

❑ ❑ ❑