JEE-Main-29-01-2024 (Memory Based)
[EVENING SHIFT]

Maths

Question: $\int_{\frac{\pi}{6}}^{\frac{\pi}{3}} \sqrt{1-\sin 2x}\, dx = \alpha + \beta\sqrt{2} + \gamma\sqrt{3}$

Find r - 3β; find $\alpha\beta\gamma$

Solution:

$$\int_{\frac{\pi}{6}}^{\frac{\pi}{3}} \sqrt{1-\sin 2x}\, dx$$

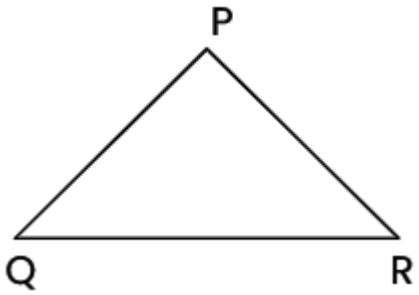
$$= \int_{\frac{\pi}{6}}^{\frac{\pi}{3}} |\cos x - \sin x|\, dx + \int_{\frac{\pi}{6}}^{\frac{\pi}{3}} |(\sin x - \cos x)|\, dx$$

$$= +\sin x + \cos x \Big|_{\frac{\pi}{6}}^{\frac{\pi}{4}} + -\cos x - \sin x \Big|_{\frac{\pi}{4}}^{\frac{\pi}{3}}$$

$$= \left(\frac{1}{\sqrt{2}} - \frac{1}{2} + \frac{1}{\sqrt{2}} - \frac{\sqrt{3}}{2}\right) + \left(\frac{1}{\sqrt{2}} + \frac{\sqrt{3}}{2} + \frac{1}{\sqrt{2}} - \frac{1}{2}\right)$$

$$= 2\sqrt{2} - \sqrt{3} - 1$$

Question: If P(3, 2, 3) Q(4, 6, 2) R(7, 3, 2) are the vertices of Δ PQR, then find $\angle$QPR =



Options:

(a) $\cos^{-1}\frac{1}{18}$

(b) $\frac{\pi}{6}$

(c) $\frac{\pi}{3}$

(d) $\cos^{-1}\frac{7}{18}$

Solution:

$\overline{PQ} = (1, 4, -1)$

$\overline{PR} = (4, 1, -1)$

$\cos x = \frac{4+4+1}{18}$

$x = \frac{\pi}{3}$

Question: Area bounded by $0 \le y \le \min\{x2 + 2, 2x + 2\}$, $x \in [0, 3]$, then 12A is

Solution:

$\int_0^2 x^2 + 2 + \int_0^3 2x + 2\frac{x^3}{3} + 2x$

$\left|0^2 + x^2 + 2x\right| 2^3 \frac{8}{3} + 4 + 9 + 6 - (4+4)$

$A = \frac{41}{2}$

$12A = 164$

Question: The remainder when $64^{32^{32}}$ is divided by 9 is _____.

Solution:

$64^{32^{32}} \to 9$

$64 = 1(mod\ 9)$

$64^{32} = 1(mod\ 9)$

$64^{32^{32}} = 1(mod\ 9)$

Question: $\cos\left(2\sin^{-1} x\right) = \frac{1}{9}$ holds for $x = \frac{m}{n}$ and $\alpha\beta$ are the roots of the equation $mx^2 - nx - m + n = 0 (\alpha > \beta)$ then α, β lies on the line

Options:

(a) 5x - 8y = 9

(b) 5x + 8y = 9

(c) 8x + 5y = 9

(d) 8x - 5y = 9

Solution:

$$\cos\left(\sin^{-1}x\right)=\frac{1}{9}$$

$$1-2\sin^2\left(\sin^{-1}x\right)=\frac{1}{9}$$

$$2x^2=\frac{8}{9}\rightarrow x=\pm\frac{2}{3}=-\frac{2}{3}$$

$$\frac{2}{3}:2x^2-3x+1=9$$

$$\alpha=1,\beta=\frac{1}{2}$$

$$5x+8y=9$$

Question: If α & β are roots of the equation $x^2-\sqrt{6}x+3$ and where $\text{lm}(\beta)<0$ if $\frac{\alpha^{99}}{\beta}+\beta^{98}=3^n\left(a+ib\right)$ then find a, b, n.

Solution:

$$x^2-\sqrt{6}x+3=0$$

$$\beta,\alpha=\frac{\sqrt{6}\pm\sqrt{6}}{2}=\frac{\sqrt{6}}{\sqrt{2}}\frac{(1\pm i)}{\sqrt{2}}=\sqrt{3}e^{\pm i\frac{\pi}{4}}$$

$$\alpha=\sqrt{3}e^{i\frac{\pi}{4}};\frac{\pi}{\beta}=i\alpha\beta=3$$

$$\frac{\alpha^{99}}{\beta}+\beta^{98}=\alpha^{98}i+\beta^{98}$$

$$=3^{49}[i\cdot i+(-j)]3^{49}(-1-j)$$

$$n=49,a=-1,B=-1$$

Other method $\frac{\alpha^{99}+\beta^{99}}{\beta}=\frac{\alpha^{100}}{3}+\beta^{98}=3^{49}(-1)+3^{49}(-i)$

Question: Distance of (2,4) from the line 2x + y +2 =0 measured parallel to the line $\sqrt{3}x+y+2=0$:

Solution:

$$\left(2\frac{\sqrt{3}r}{2},4+\frac{r}{2}\right)$$

$$2\left(2-\frac{\sqrt{3}r}{2}\right)+4+\frac{r}{2}+2=0$$

$$10-\sqrt{3}r+\frac{r}{2}=0$$

$$10=\left(\sqrt{3}-\frac{1}{2}\right)r$$

$$r=\frac{20}{2\sqrt{3}-1}$$

Question: a_1, a_2 …..are in G.P. such that a,

$= \frac{1}{8}, a_1 \neq a_2$

and every term is equal to arithmetic mean of it's two successive terms. Find S_{20} - S_{18}.

Solution:

$$a_1 = \frac{a_2 + a_3}{2}$$

$$a = \frac{ar + ar^2}{2}$$

$$r^2 + r = 2$$

$$r = -2$$

$$S_{20} - S_{18}$$

$$\frac{a\left[r^{20} - 1\right] - a\left[r^{10} - 1\right]}{r - 1}$$

$$\frac{a}{r-1}\left[r^{20} - r^{18}\right]$$

$$\frac{1}{8(-3)}\left[2^{20} - 2^{18}\right]$$

$$\frac{2^{15}}{-3}[3] = -2^{15}$$

Question: $f(x) = 2x + 3(x)^{\frac{2}{3}}$. x ∈ R.

Options:

(a) it has one maxima no minima

(b) it has one minima no maxima

(c) it has 2 maxima and 1 minima

(d) it has 1 maxima and 1 minima

Solution:

$$f(x) = 2x + 3(x)^{\frac{2}{3}}$$

$$f'(x) = 2 + 2x^{-1/3} = 0 = 2\frac{\left(x^{1/3} + 1\right)}{x^{1/3}}$$

$$x = -1$$

$x = -1$ Maxima

$x = 0$ Minima

Question: R : {1, 2, 3, 4} → {1, 2, 3, 4} if (1, 4), (1, 2) ∈ R
Find the number of possible equivalence relations.
Solution: R = {(1, 1), (2, 2), (3, 3), (4, 4), (1, 4), (4, 1), (1, 2), (2, 1), (4, 2), (2, 4)}
Other {(1, 2), (3, 1), (2, 3), (3, 2), (4, 3), (3, 4)}
Either they all will come on not so 2.

Question: There are 8 identical book and 4 identical shelfs Find the number of ways to arrange the book such that any shelf may be empty, every shelf can accommodate all the books
Options:
(a) 13
(b) 14
(c) 15
(d) 16
Solution:

(8, 0, 0, 0)	(5, 3, 0, 0)	(4, 3, 1, 0)
(7, 1, 0, 0)	(5, 2, 1, 0)	(4, 2, 2, 0)
(6, 2, 0, 0)	(5, 2, 1, 0)	(4, 2, 2, 0)
(6, 1, 1, 0)	(4, 4, 0, 0)	(3, 3, 1, 1)
(2, 2, 2, 2)	(3, 3, 2, 0)	(3, 2, 2, 1)

Total 15 ways.

Question: Find the probability that number selected from 1 to 50 such that number is divisible by at least one out of 4, 6 or by 7
Options:
(a) $\frac{21}{25}$
(b) $\frac{18}{50}$
(c) $\frac{8}{25}$
(d) $\frac{21}{25}$
Solution: n (A ∪ B ∪ C)
= 12 + 8 +7 - 4 - 1 - 1 +0
= 21
Probability = $\frac{21}{25}$

Question: A.B point of L_1, & L_2 of shortest distance points from origin OA.OB