

**JEE-Main-31-01-2024 (Memory Based)**  
**[MORNING SHIFT]**

**Mathematics**

**Question:** Solve differential equation:

$$\frac{dx}{dy} = x(\ln x - \ln y + 1)$$

**Answer:**

**Solution:**

$$\frac{dx}{dy} = \frac{x}{y} \left( \ln \frac{x}{y} + 1 \right)$$

Put  $x = vy$

$$\Rightarrow v + y \frac{dv}{dy} = v(\ln v + 1)$$

$$\Rightarrow \int \frac{dv}{v \ln v} = \int \frac{dy}{y}$$

$$\Rightarrow \ln |\ln v| = \ln |y| + \ln c$$

$$\Rightarrow \left| \ln \frac{x}{y} \right| = c |y|$$

**Question:**  $\lim_{x \rightarrow 0} \frac{e^{2|\sin x|} - 2|\sin x| - 1}{x^2} = ?$

**Options:**

- (a) Does not exist
- (b) 2
- (c) 1
- (d) -1

**Answer: (b)**

**Solution:**

$$\lim_{x \rightarrow 0} \frac{e^{2|\sin x|} - 2|\sin x| - 1}{x^2}$$

$$= \lim_{x \rightarrow 0} \frac{e^{2|\sin x|} - 2|\sin x| - 1}{|\sin x|^2} \times \frac{\sin^2 x}{x^2}$$

$$= \lim_{t \rightarrow 0} \frac{e^{2t} - 2t - 1}{t^2} \quad t = |\sin x|$$

$$= \lim_{t \rightarrow 0} \frac{2e^t - 2}{2t} = \lim_{t \rightarrow 0} \frac{2(e^t - 1)}{2t} = 2$$

**Question:**  $|\vec{a}| = 1, |\vec{b}| = 4, \vec{a} \cdot \vec{b} = 2, \vec{c} = (2\vec{a} \times \vec{b} - 3\vec{b})$ . If angle between  $\vec{c}$  and  $\vec{b}$  is  $\alpha$  then find  $192 \sin^2 \alpha$ .

**Answer:** 48.00

**Solution:**

$$|\vec{a}| = 1, |\vec{b}| = 4, \vec{a} \cdot \vec{b} = 2 \rightarrow \cos \theta = \frac{1}{2} \rightarrow \theta = \frac{\pi}{3}$$

$$\vec{c} = 2\vec{a} \times \vec{b} - 3\vec{b}$$

$$|\vec{c}|^2 = 4|\vec{a} \times \vec{b}|^2 + 9|\vec{b}|^2$$

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$$= 4|\vec{a}|^2 |\vec{b}|^2 \sin^2 \theta + 9|\vec{b}|^2$$

$$= 4 \times 1 \times 16 \times \frac{3}{4} + 9 \times 16$$

$$= 48 + 144 = 192$$

$$\vec{b} \cdot \vec{c} = 0 - 3|\vec{b}|^2$$

$$4 \times 8\sqrt{3} \cos \alpha = -3 \times 16$$

$$\cos \alpha = \frac{-\sqrt{3}}{2}$$

$$192 \sin^2 \alpha = 48$$

**Question:** Let 'S' be the set of positive integral values of  $a$  for which  $\frac{ax^2 + 2(a+1)x + 9a + 4}{x^2 + 8x + 32} < 0, \forall x \in R$ . Then, the number of elements in 'S' is

**Answer:**

**Solution:**

$$\frac{a^2 + 2(a+1)x + 4a}{x^2 + 8x + 32} < 0 \forall x$$

$$D^r > 0 \forall x$$

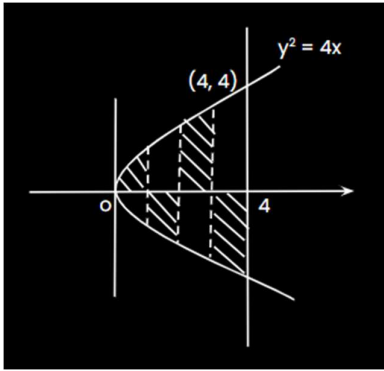
$N^r$  can be -ve for all  $x$  only if

$a$  is -ve so no +ve integral value of  $a$ .

**Question:** There was on area  $\left\{ y^2 < 4x, x < 4, \frac{xy(x-1)(x-2)}{(x-3)(x-4)} < 0 \right\}$ .

**Answer:**  $\frac{32}{3}$

**Solution:**



$$y > 0$$

$$x(x-1)(x-2)(x-3)(x-4) > 0$$

$$y < 0$$

$$x(x-1)(x-2)(x-3)(x-4) < 0$$

$$\text{Area} = \frac{2}{3} \times 16 = \frac{32}{3}$$

**Question:** If  $f(x) = \begin{vmatrix} x^3 & 2x^2+1 & 1+3x \\ 3x^2+2 & 2x & x^3+6 \\ x^3-x & 4 & x^2-2 \end{vmatrix}$  for all  $x \in \mathbb{R}$ , then  $2f(0) + f'(0)$  is equal to

**Answer: 42.00**

**Solution:**

$$f(0) = \begin{vmatrix} 0 & 1 & 1 \\ 2 & 0 & 6 \\ 0 & 4 & -2 \end{vmatrix} = 8 + 4 = 12$$

$$f'(0) = \begin{vmatrix} 0 & 0 & 3 \\ 2 & 0 & 6 \\ 0 & 4 & -2 \end{vmatrix} + \begin{vmatrix} 0 & 1 & 1 \\ 0 & 2 & 0 \\ 0 & 4 & -2 \end{vmatrix} + \begin{vmatrix} 0 & 1 & 1 \\ 2 & 0 & 6 \\ -1 & 0 & 0 \end{vmatrix}$$

$$= 24 + 0 + (-6) = 18$$

$$2f(0) + f'(0) = 42$$

**Question:** If  $f(x) = \frac{4x+3}{6x-4}, x \neq \frac{2}{3}$ . Find  $(f \circ f)(x) = g(x)$ , where  $g: r - \left[ \frac{2}{3} \rightarrow R \rightarrow \left\{ \frac{2}{3} \right\} \right]$

then  $(g(g(g(4))))$  is equal to

**Answer: 4.00**

**Solution:**

$$f(x) = \frac{4x+3}{6x-4}$$

$$g(x) = f(f(x)) = \frac{4\left(\frac{4x+3}{6x-4}\right) + 3}{6\left(\frac{4x+3}{6x-4}\right) - 4}$$

$$= \frac{16x+12+18x-12}{24x+18-24x+16} = \frac{34x}{34} = x$$

$$g(g(g(4))) = 4$$

**Question:**  $\sum_{r=1}^{10} \frac{r}{1-3r^2+r^4} = S$ . Find S.

**Answer:**  $\frac{-55}{109}$

**Solution:**

$$\sum_{r=1}^{10} \frac{r}{1-3r^2+r^4} = \sum_{r=1}^{10} \frac{r}{(r^2-1)^2 - r^2}$$

$$= \sum_{r=1}^{10} \frac{r}{(r^2+r-1)(r^2-r-1)}$$

$$= \sum_{r=1}^{10} \frac{1}{2} \left[ \frac{1}{r^2-r-1} - \frac{1}{r^2+r-1} \right]$$

$$= \frac{1}{2} \left[ \left( \frac{1}{-1} - \frac{1}{1} \right) + \left( \frac{1}{1} - \frac{1}{5} \right) + \dots + \left( -\frac{1}{109} \right) \right]$$

$$= \frac{1}{2} \left[ -1 - \frac{1}{109} \right] = \frac{-110}{2 \times 109} = \frac{-55}{109}$$

**Question:** If the system of linear equation  $x - 2y + z = -4$ ;  $2x + \alpha y + 3z = 5$  and  $3x - y + \beta z = 3$  has infinitely many solutions then  $12\alpha + 13\beta$  is equal

**Answer: 58.00**

**Solution:**

$$x - 2y + z = 4$$

$$2x + \alpha y + 3z = 5$$

$$3x = y + \beta z = 3$$

$$\begin{vmatrix} 1 & -2 & 1 \\ 2 & \alpha & 3 \\ 3 & -1 & \beta \end{vmatrix} = \alpha - 18 - 2 - 3\alpha + 3 - 4\beta = 0$$

$$\alpha\beta - 3\alpha + 4\beta - 17 = 0$$

$$(\alpha + 4)(\beta - 3) = 5$$

$$\text{Also, } D_2 = \begin{vmatrix} 1 & -4 & 1 \\ 2 & 5 & 3 \\ 3 & 3 & \beta \end{vmatrix} = 0$$

$$\Rightarrow s\beta - 36 + 6 - 15 - 9 + 8\beta = 0$$

$$13\beta - 54 = 0$$

$$D_3 = \begin{vmatrix} 1 & -2 & -4 \\ 2 & \alpha & 5 \\ 3 & -1 & 3 \end{vmatrix} = 0$$

$$\rightarrow 3\alpha - 30 + 8 + 12\alpha + 5 + 12 = 0$$

$$15\alpha - 5 = 0 \Rightarrow \alpha = \frac{1}{3}$$

$$12\alpha + 13\beta = 4 + 54 = 58$$

$$\Delta = \left(\frac{1}{3} + 4\right) \left(\frac{54}{13} - 3\right)$$

$$= \frac{13}{3} \times \frac{15}{13} = 5$$

Also,

$$D_1 = \begin{vmatrix} -4 & -2 & 1 \\ 5 & \alpha & 3 \\ 3 & -1 & \beta \end{vmatrix}$$

$$= -4\alpha\beta - 18 - 5 - 3\alpha - 12 + 10\beta$$

$$= -4\alpha\beta - 3\alpha + 10\beta - 35$$

$$= -\frac{4\beta}{3} - 1 + 10\beta - 35$$

$$= \frac{26\beta}{3} - 36 = 0$$

**Question:**  $f(x) = \begin{cases} g(x), & x \leq 0 \\ \left(\frac{x+1}{x+2}\right)^{\frac{1}{x}}, & x > 0 \end{cases}$ , where  $g(x)$  is linear function and  $f(x)$  is

continuous at  $x = 0$ . Also  $f'(1) = g(-1)$ ,  $g(0) = f(0)$  then find the value of  $g(3)$ ?

If ans is  $\frac{a}{b} \ln\left(\frac{\alpha}{\beta} e^{\frac{1}{3}}\right)$  find  $\alpha + \beta + a + b$

**Answer: 15.00**

**Solution:**

$$f(x) = \begin{cases} g(x), & x \leq 0 \\ \left(\frac{x+1}{x+2}\right)^{\frac{1}{x}}, & x > 0 \end{cases}$$

$$f(0^+) = 0; f(0^-) = g(0^-) = 0$$

$$\Rightarrow g(x) = mx$$

$$\begin{aligned} f'(1) &= \frac{d}{dx} e^{x^{\frac{1}{x}[\ln(x+1) - \ln(x+2)]}} \\ &= \frac{2}{3} \times \left[ 1 \cdot \left( \frac{1}{2} - \frac{1}{3} \right) - 1 \left( \ln \frac{2}{3} \right) \right] \\ &= \frac{2}{3} \left[ \frac{1}{6} - \ln \frac{2}{3} \right] = \frac{1}{9} - \frac{2}{3} \ln \frac{2}{3} \\ &= g(-1) = -m \end{aligned}$$

$$g(3) = 3m = 2 \ln \frac{2}{3} - \frac{1}{3}$$

$$= 2 \left[ \ln \frac{2}{3} - \ln e^{\frac{1}{3}} \right]$$

$$= 2 \left[ \ln \frac{2}{2e^{\frac{1}{3}}} \right]$$

$$= \ln \frac{4}{9e^{\frac{1}{3}}}$$

$$a = b = 1, \alpha = 4, \beta = 9$$

$$\alpha + \beta + a + b = 15$$

**Question:** 3 rotten apples are mixed with 15 normal apples. Let the random variable be defined as number of rotten apples on picking 3 apples with replacement. Find variance of x

**Answer:**  $\frac{5}{12}$

**Solution:**

3 rotten apples, 15 good apples

$$p = \frac{1}{6}, q = \frac{5}{6}$$

$$\text{Variance} = npq = 3 \times \frac{1}{6} \times \frac{5}{6} = \frac{5}{12}$$

**Question:** A box contains 10 red balls, 30 White, 15 orange balls, 10 or 20 blue balls, 2 balls drawn with replacement. Find the probability such that the first drawn ball is red and second drawn ball is white?

**Answer:**  $\frac{4}{75}$

**Solution:**

10 R, 30 W, 15 O, 10 or 20 B

$$P(1^{\text{st}} \text{ Red } 2^{\text{nd}} \text{ W}) = \frac{10}{65} \times \frac{30}{65} = \frac{12}{169}$$

$$\text{or } = \frac{10}{75} \times \frac{30}{75} = \frac{12}{225} = \frac{4}{75}$$

**Question:** Using the word “DISTRIBUTION “. Find the no of ways of selection of 4 letters

**Answer: 191.00**

**Solution:**

D, 3I, 2T, R, B, U, O, N, S

4 different:  ${}^9C_4 = 126$

2 alike, 2 different:  ${}^2C_1 \times {}^8C_2 = 2 \times 28 = 56$

3 alike, 1 different =  $1 \times {}^8C_1 = 8$

2 alike, 2 alike = 1

Total = 191

**Question:**  $\sum_{r=0}^n \frac{{}^nC_r \cdot {}^nC_r}{r+1} = \alpha$ ,  $\sum_{r=0}^n \frac{{}^nC_r \cdot {}^{n+1}C_r}{r+1} = \beta$ . If  $4\beta = 7\alpha$ , then find  $n$ .

**Options:**

(a) 2

(b) 4

(c) 6

(d) 5

**Answer: (c)**

**Solution:**

$$\sum_{r=0}^n \frac{{}^nC_r \cdot {}^nC_r}{r+1} = \alpha$$

$$\sum_{r=0}^n \frac{{}^nC_r \cdot {}^{n+1}C_r}{r+1} = \beta$$

$$\alpha = \frac{1}{n+1} \sum {}^{n+1}C_{r+1} \cdot {}^nC_r = \sum \frac{{}^{2n+1}C_n}{n+1}$$

$$\beta = \frac{1}{n+1} \sum {}^{n+1}C_{r+1} \cdot {}^{n+1}C_r = \sum \frac{{}^{2n+2}C_n}{n+1}$$

$$\frac{7}{n+1} \left( \sum {}^{2n+2}C_n \right) = \frac{4 \left( \sum {}^{2n+2}C_n \right)}{n+1}$$

$$7^{2n+1} C_n = \frac{4 \times 3n + 2}{n} (\sum^{2n+2} C_{n-1})$$

$$\frac{7}{8} \frac{n}{n+1} \times \frac{{}^{2n+1} C_n}{{}^{2n+1} C_{n-1}} = 1$$

$$\left( \frac{2n+1-(n)+1}{n} \right) = \frac{8(n+1)}{7n} \frac{{}^n C_r}{{}^n C_{r-1}} = \frac{n-r+1}{r}$$

$$\frac{n+2}{r} = \frac{8(n+1)}{7r}$$

$$7n+14 = 8n+8$$

$$n = 6$$

