

JEE MAIN 31 JANUARY 2024 SHIFT 1 QUESTION PAPER

MATHEMATICS

1. $f(x) = \begin{vmatrix} x^3 & 2x^2 + 1 & 1 + 3x \\ 3x^2 + 2 & 2x & x^3 + 6 \\ x^3 - x & 4 & x^2 - 2 \end{vmatrix}$

Find $2f(0) + f'(0)$.

2. $f(x) = \frac{(4x+3)}{(6x-4)}$ and $g(x) = f(f(x))$, then find $g(g(g(x)))$.

3. Find the sum of the coefficients of x^3 and x^{-13} in the following expansion:

$$(1+x)(1-x^2) \left(1 + \frac{3}{x} + \frac{3}{x^2} + \frac{1}{x^3}\right)^5$$

4. $[1 / (1 - 3(1)^2 + 1^4)] + [2 / (2 - 3(2)^2 + 2^4)] + [3 / (3 - 3(3)^2 + 3^4)] + \dots$ (up to $n = 10$) = ?

5. $A = \{1, 2, 3, 4\}$, $R = \{(1, 2), (2, 3), (2, 4)\}$, $R \subseteq S$ and S is an equivalence relation, then the minimum number of elements to be added to R is n . Find the value of n .

6. ABCD is a parallelogram where $A(\alpha, \beta)$, $B(1,0)$, $C(\gamma, \delta)$, and $D(3,2)$ and $AB = \sqrt{10}$. Find the value of $2(\alpha + \beta + \gamma + \delta)$.

7. An urn contains 15 red, 10 white, 60 orange, and 15 green balls. If 2 balls are taken with replacement, find the probability 1 ball is red and the other ball is white.

8. $f(x) = (4x+3)/(6x-4)$ and $g(x) = f(f(x))$, then find $g(g(g(x)))$.

9. Find the solution of the differential equation $y \, dy/dx = x (\log_e x - \log_e y + 1)$, $x > 0$, $y > 0$ and passes through $(e, 1)$.

10. For $\alpha, \beta, \gamma \neq 0$, if $\sin^{-1}\alpha + \sin^{-1}\beta + \sin^{-1}\gamma = \pi$ and $(\alpha + \beta + \gamma)(\alpha - \gamma + \beta) = 3\alpha\beta$, then find the value of γ .

11. If $|a| = 1$, $|b| = 4$ and $a \cdot b = 2$. Also, $c = (3a \times b) - b$ and α is the angle between b and c , then what is the value of $192 \sin^2\alpha$?

12. If one of the diameters of the circle $x^2 + y^2 - 10x + 4y + 13 = 0$ is a chord of another circle and whose centre is the point of intersection of the lines $2x + 3y = 12$ and $3x - 2y = 5$. then the radius of the circle is?

13. If the system of linear equation $x - 2y + z = -4$, $2x + \alpha y + 3z = 5$ & $3x - y + \beta z = 3$ has infinitely many solutions then find the value of $12\alpha + 13\beta$.

14. If three vectors are:

$$a = 3i + j - 2k$$

$$b = 4i + j + 7k$$

$$c = i - 3j + 4k$$

If p is a vector such that $p \times b = c \times b$ and $p \cdot a = 0$, then find $p \cdot (i - j - k)$.

15. Let S be the set of positive integral value of a for which

$$[(ax^2 + 2(a+1)x + 9a + 4) / (x^2 + 8x + 32)] < 0 \quad \forall x \in \mathbb{R}.$$

Find the number of elements in S .

16. $\lim_{x \rightarrow 0} [(e^{2|\sin x|} - 2|\sin x| - 1) / x^2] = ?$

17. The distance of the point $Q(0, 2, -2)$ from the line passing through the point $P(5, -4, 3)$ and perpendicular to the line $r = (-3i + 2k) + \lambda(2i + 3j + 5k)$, $\lambda \in \mathbb{R}$ and $r = (i - 2j + k) + \mu(-i + 3j + 2k)$, $\mu \in \mathbb{R}$ is?

