MATHEMATICS

SECTION - A

Multiple Choice Questions: This section contains 20 multiple choice questions. Each question has 4 choices (1), (2), (3) and (4), out of which **ONLY ONE** is correct.

Choose the correct answer :

1. The value of $\int_0^1 (2x^3 - 3x^2 - x + 1)^{1/3}$

(1) –1 (2) 1

(3) Zero (4) 2

Answer (3)

Sol. $I = \int_0^1 (2(1-x)^3 - 3(1-x)^2 - (1-x) + 1)^{1/3} dx$

$I = \int_0^1 (2(1-x^3+3x^2-3x) - 3\,(1+x^2-2x) - 1 + x + 1)^{1/3} dx$

$I = \int_0^1 (-1-2x^3+3x^2+x)^{1/3} dx$

$I = -I$

$\Rightarrow \quad 2I = 0$

$I = 0$

2. Probability that Ajay will not go to office is $\frac{1}{5}$ and probability that Ajay and Vijay will go to office is $\frac{2}{7}$, if their visits to office is independent of each other then find probability that Ajay will go to the office but Vijay will not go, is

(1) $\frac{12}{28}$

(2) $\frac{13}{35}$

(3) $\frac{18}{35}$

(4) $\frac{24}{35}$

Answer (3)

Sol. $P(\bar{A}) = \frac{1}{5} \quad \Rightarrow P(A) = \frac{4}{5}$

$P(A \cap V) = P(A) \times P(V) = \frac{2}{7}$

$\Rightarrow \quad P(V) = \frac{2}{7} \times \frac{5}{7} = \frac{5}{14}$

Now, $P(A \cap \bar{V}) = P(A)P(\bar{V})$

$= P(A)[1 - P(V)]$

$= \frac{4}{5}\left[1 - \frac{5}{14}\right] = \frac{9 \times 4}{14 \times 5} = \frac{18}{35}$

3. $\int_0^{\pi/3} \cos^4 x\,dx$ is equal to $a\pi + b\sqrt{3}$ then $a^2 + b$ is equal to

(1) $\frac{1}{2}$

(2) $\frac{1}{8}$

(3) $\frac{1}{4}$

(4) 1

Answer (2)

Sol. $\frac{1}{4}\int_0^{\pi/3} (2\cos^2 x)^2 dx$

$\Rightarrow \frac{1}{4}\int_0^{\pi/3} (1+\cos 2x)^2 dx$

$\Rightarrow \frac{1}{4}\int_0^{\pi/3} (1+\cos^2 2x + 2\cos 2x) dx$

$\Rightarrow \frac{1}{4}\left[\left(\frac{\pi}{3}\right) + \int_0^{\pi/3}\left(\frac{1+\cos 4x}{2}\right)dx + 2\frac{\sin 2x}{2}\Big|_0^{\pi/3}\right]$

$\Rightarrow \frac{1}{4}\left[\frac{\pi}{3} + \frac{1}{2}\left(x + \frac{\sin 4x}{4}\right)\Big|_0^{\pi/3} + \left(\sin\frac{2\pi}{3}\right)\right]$

$\Rightarrow \frac{1}{4}\left[\frac{\pi}{3} + \frac{1}{2}\left(\frac{\pi}{3} + \frac{1}{4}\left(\sin 4\frac{\pi}{3}\right)\right) + \frac{\sqrt{3}}{2}\right]$

$\Rightarrow \frac{1}{4}\left[\frac{\pi}{3} + \frac{\pi}{6} + \frac{1}{8}\left(\frac{-\sqrt{3}}{2}\right) + \frac{\sqrt{3}}{2}\right] = \frac{1}{4}\left[\frac{\pi}{2} + \frac{7}{8}\cdot\frac{\sqrt{3}}{2}\right]$

$= \frac{\pi}{8} + \frac{7\sqrt{3}}{64}$

4. A relation R_1 defined on a set of real number *R* as

$aR, b \Leftrightarrow a^2 + b^2 = 1$

Relation R_2 defined on set of natural number N such that

$(a, b) R_2, (c, d) \Leftrightarrow a + b = c + d$

that

(1) R_1 is reflexive
R_2 is not reflexive

(2) R_1 is not reflexive
R_2 is reflexive

(3) R_1 is reflexive
R_2 is reflexive

(4) R_1 is not reflexive
R_2 is not reflexive

Answer (2)

Sol. $a^2 + a^2 = 1$

$\Rightarrow \quad a^2 = \frac{1}{2}$

$\Rightarrow \quad a = \pm\frac{1}{\sqrt{2}}$

$\Rightarrow \quad (a, a) \notin R_1 \ \forall \ a \in R$

$\therefore \quad R_1$ is not reflexive

$a + b = a + b$

$\Rightarrow \quad (a, b) R_2 (a, b) \ \forall \ a, b \in N$

$\therefore \quad R_2$ is reflexive

5. If the mirror image of *P*(3, 4, 9) in line $\frac{x-1}{3} = \frac{y+1}{2} = \frac{z-2}{1}$ is (α, β, γ) then $14(\alpha + \beta + \gamma)$ is

(1) 132 (2) 105

(3) 102 (4) 108

Answer (4)

Sol. *P*(3, 4, 9)

$$\frac{x-1}{3} = \frac{y+1}{2} = \frac{z-2}{1} = \lambda$$

Any point on line

$Q(3\lambda + 1, 2\lambda - 1, \lambda + 2)$

$(PQ) \cdot \langle 3, 2, 1 \rangle = 0$

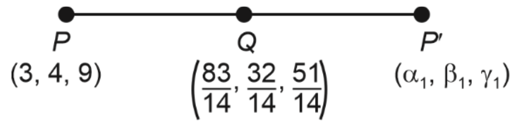
$\langle 3\lambda - 2, 2\lambda - 5, \lambda - 7 \rangle \langle 3, 2, 1 \rangle = 0$

$9\lambda - 6 + 4\lambda - 10 + \lambda - 7 = 0$

$14\lambda - 23 = 0$

$\lambda = \frac{23}{14}$

$\therefore \quad Q\left(\frac{83}{14}, \frac{32}{14}, \frac{51}{14}\right)$



$\frac{3+\alpha_1}{2} = \frac{83}{14} \Rightarrow x_1 = \frac{62}{7}$

$\frac{4+\beta_1}{2} = \frac{32}{14} \Rightarrow y_1 = \frac{4}{7}$

$\frac{9+\gamma_1}{2} = \frac{51}{14} \Rightarrow z_1 = -\frac{12}{7}$

Now $14(\alpha + \beta + \gamma) = 14\left(\frac{62 + 4 - 12}{7}\right) = 108$

6. If α and β are roots of $px^2 + qx - r = 0$, $p \neq 0$, *p*, *q* and *r* are consecutive terms of a non-constant G.P and $\frac{1}{\alpha} + \frac{1}{\beta} = \frac{3}{4}$ then $(\alpha - \beta)^2$ is

(1) $\frac{80}{9}$ (2) $\frac{5}{9}$

(3) $\frac{7}{8}$ (4) $\frac{11}{9}$

Answer (1)

Sol. $px^2 + qx - r = 0$

Let $p = \frac{a}{r_1}$, $q = a$, $r = ar_1$

and $\frac{1}{\alpha} + \frac{1}{\beta} = \frac{3}{4}$

$\frac{\alpha + \beta}{\alpha\beta} = \frac{3}{4}$

$\Rightarrow \quad \frac{-\frac{q}{p}}{-\frac{r}{p}} = \frac{3}{4}$

$\Rightarrow \quad \frac{q}{r} = \frac{3}{4}$

$\Rightarrow \quad \frac{1}{r_1} = \frac{3}{4}$

$\Rightarrow \quad r_1 = \frac{4}{3}$

$(\alpha - \beta)^2 = \alpha^2 + \beta^2 - 2\alpha\beta$

$= (\alpha + \beta)^2 - 4\alpha\beta$

$= \left(\frac{-q}{p}\right)^2 - 4\left(\frac{-r}{p}\right)$

$= \frac{q^2}{p^2} + \frac{4r}{p}$

$= r_1^2 + 4r_1^2$

$= 5r_1^2$

$= 5\left(\frac{4}{3}\right)^2 = \frac{80}{9}$

7. Let m and n be the coefficient of 7th and 13th term in expansion of $\left(\frac{1}{3}x^{1/3} + \frac{1}{2x^{2/5}}\right)^{18}$, then $\left(\frac{m}{n}\right)^{1/3}$ is

(1) $\frac{1}{4}$ (2) $\frac{4}{7}$

(3) $\frac{1}{9}$ (4) $\frac{4}{9}$

Answer (4)

Sol. $T_7 = m = {}^{18}C_6\left(\frac{1}{3}\right)^{12}\left(\frac{1}{2}\right)^6$

$T_{13} = n = {}^{18}C_{12}\left(\frac{1}{3}\right)^6\left(\frac{1}{2}\right)^{12}$

$$\left(\frac{m}{n}\right)^{1/3} = \left[\frac{{}^{18}C_6\left(\frac{1}{3}\right)^{12}\left(\frac{1}{2}\right)^6}{{}^{18}C_{12}\left(\frac{1}{3}\right)^6\left(\frac{1}{2}\right)^{12}}\right]^{1/3}$$

$$= \frac{\left(\frac{1}{3}\right)^6}{\left(\frac{1}{2}\right)^6} = \left[\left(\frac{2}{3}\right)^6\right]^{1/3}$$

$= \frac{4}{9}$

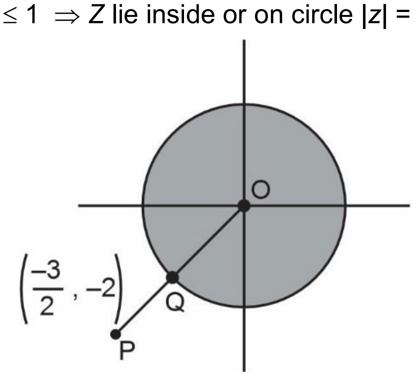
8. The minimum value of $\left|z + \frac{3+4i}{2}\right|$; $|z| \le 1$ is

(1) $\frac{3}{2}$ (2) $\frac{5}{2}$

(3) 3 (4) 5

Answer (1)

Sol. If $|z| \le 1 \Rightarrow Z$ lie inside or on circle $|z| = 1$



$\left|z + \frac{3+4i}{2}\right| \Rightarrow$ distance of z from $\left(\frac{-3}{2}, -2\right)$

Minimum value (PQ)

$= (PO) - (QO) = \sqrt{\frac{9}{4} + 4} - 1 = \frac{5}{2} - 1 = \frac{3}{2}$

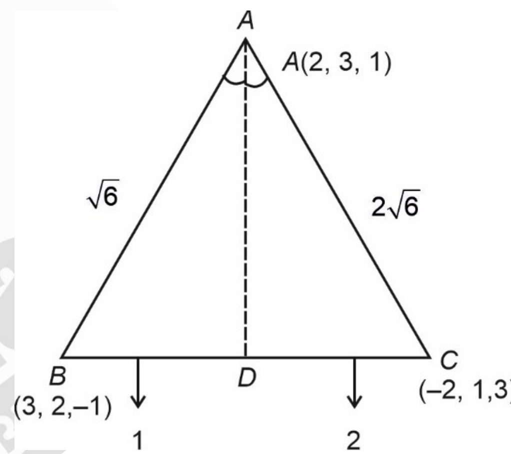
9. Let vertex A (2, 3, 1), B (3, 2, –1), C (–2, 1, 3). If AD is angle bisector of angle A, then projection of $\overrightarrow{AD}$ on $\overrightarrow{AC}$ is equal to

(1) $\frac{\sqrt{3}}{2}$ (2) $\sqrt{\frac{2}{3}}$

(3) $\sqrt{\frac{3}{2}}$ (4) $\frac{2}{\sqrt{3}}$

Answer (2)

Sol.



$D: \left(\frac{-2+6}{3}, \frac{4+1}{3}, \frac{-2+3}{3}\right)$

$= \left(\frac{4}{3}, \frac{5}{3}, \frac{1}{3}\right)$

$\overrightarrow{AD} = \left(\frac{4}{3} - 2\right)\hat{i} + \left(\frac{5}{3} - 3\right)\hat{j} + \left(\frac{1}{3} - 1\right)\hat{k}$

$= \frac{-2}{3}\hat{i} - \frac{4}{3}\hat{j} - \frac{2}{3}\hat{k}$

$\overrightarrow{AC} = -4\hat{i} - 2\hat{j} + 2\hat{k}$

Projection of $\overrightarrow{AD}$ on $\overrightarrow{AC}$

$$= \frac{\frac{8}{3} + \frac{8}{3} - \frac{4}{3}}{\sqrt{16+4+4}} = \frac{4}{2\sqrt{6}} = \frac{2}{\sqrt{6}}$$

$= \sqrt{\frac{2}{3}}$

10. If system of equation

$x + 2y + 3z = 5$

$3x + 3y + z = 9$

$x + 4y + \lambda z = \mu$

have infinitely many solutions then the value of $3\lambda + \mu$ equals to

(1) 17 (2) 21

(3) 43 (4) 34

Answer (4)

Sol. $\Delta = \begin{vmatrix} 1 & 2 & 3 \\ 3 & 3 & 1 \\ 1 & 4 & \lambda \end{vmatrix} = 0$

$(3\lambda - 4) - 2(3\lambda - 1) + 3(12 - 3) = 0$

$3\lambda - 4 - 6\lambda + 2 + 27 = 0$

$-3\lambda + 25 = 0$

$\lambda = \frac{25}{3}$

$\Delta_1 = \begin{vmatrix} 5 & 2 & 3 \\ 9 & 3 & 1 \\ \mu & 4 & \lambda \end{vmatrix} = 0$

$= 5(3\lambda - 4) - 2(9\lambda - \mu) + 3(36 - 3\mu) = 0$

For $\lambda = \frac{25}{3}$ $\mu = 9$

Now for $\lambda = \frac{25}{3}$ & $\mu = 9 \Delta = \Delta_1 = \Delta_2 = \Delta_3 = 0$

$3\lambda + \mu = \frac{25}{3} \times 3 + 9 = 25 + 9 = 34$

11. let P be a point on ellipse $\frac{x^2}{9} + \frac{y^2}{4} = 1$. Let the line passing through P and parallel to y-axis meet the circle $x^2 + y^2 = 9$ at Q such that. P and Q are on the same side of x-axis then the eccentricity of the focus of the point R on RQ such that $PR : RQ = 4 : 3$ as P moves on ellipse is

(1) $\frac{\sqrt{117}}{21}$

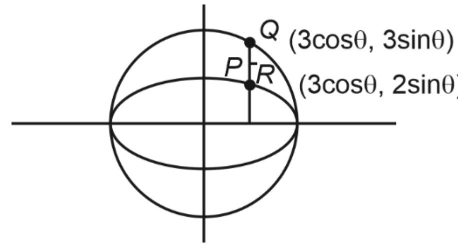
(2) $\frac{\sqrt{139}}{23}$

(3) $\frac{13}{21}$

(4) $\frac{\sqrt{13}}{7}$

Answer (1)

Sol.



$h = \frac{12\cos\theta + 9\cos\theta}{7} \left(\frac{7h}{21}\right)^2 + \left(\frac{7k}{18}\right)^2 = 1$

$k = \frac{12\sin\theta + 6\sin\theta}{7}$

$e^2 = 1 - \frac{18^2}{21^2}$

$e^2 = \frac{21^2 - 18^2}{21^2}$

$e^2 = \frac{117}{21^2}$

$e = \frac{\sqrt{117}}{21}$

12. A function satisfying $F(x) = \int_0^x tf(t)dt$ and it is given that $F(x^2) = x^4 + x^5$, then $\sum_{r=1}^{12} r^2 f(r^2)$ is equal to

(1) 1637

(2) 1540

(3) 1363

(4) 1247

Answer (1)

Sol. $F(x) = \int_0^x tf(t)dt$

and $F(x^2) = x^4 + x^5 = (x^2)^2 + (x^2)^{5/2}$

$F(x) = x + x^{5/2}$

$F'(x) = 1 + \frac{5}{2}x^{3/2} = xf(x)$

$\Rightarrow \quad f(x) = \frac{1}{x} + \frac{5}{2}\sqrt{x}$

$f(x^2) = \frac{1}{x^2} + \frac{5x}{2}$

$x^2 f(x^2) = 1 + \frac{5}{2}x^3$

$\sum_{x=1}^{12} x^2 f(x^2) = \sum_{x=1}^{12}\left(1 + \frac{5}{2}x^3\right)$

$= 12 + \frac{5}{2} \times \frac{12 \times 13 \times 25}{6}$

$= 12 + 5 \times 13 \times 25 = 1637$

13.
14.
15.
16.
17.
18.
19.
20.

SECTION - B

Numerical Value Type Questions: This section contains 10 Numerical based questions. The answer to each question should be rounded-off to the nearest integer.

21. $\frac{dx}{dy} = \frac{1+x-y^2}{y}$ and $x(1) = 1$, then $5x(2)$ is equal to

Answer (5)

Sol. $\Rightarrow \frac{dx}{dy} - \frac{1}{y}(1+x) = -y$

$\text{I.F.} = e^{-\int \frac{1}{y} dy} = \frac{1}{y}$

$\frac{1+x}{y} = \int \frac{1}{y}(-y)\lambda y + c$

$\frac{1+x}{y} = -y + c$

$x(1) = 1$

$\frac{2}{1} = -1 + c \Rightarrow c = 3$

$\frac{1+x}{y} = -y + 3$

For $y = 2$

$\frac{1+x}{y} = -2 + 3$

$x = 1$

Then $5x(2) = 5$

22. If $y = \frac{(\sqrt{x}+1)\left(x^2-\sqrt{x}\right)}{x\sqrt{x}+x+\sqrt{x}} + \frac{1}{15}\left(3\cos^5 x - 5\cos^3 x\right)$

then $96y'\left(\frac{\pi}{6}\right)$ equals to

Answer (105)

Sol. $y = \frac{\left(\sqrt{x}+1\right)\left(x^2-\sqrt{x}\right)\left(\sqrt{x}-1\right)}{\sqrt{x}\left(x+\sqrt{x}+1\right)\left(\sqrt{x}-1\right)}$

$+\frac{1}{15}[3\cos^5 x - 5\cos^3 x]$

$= \frac{(x-1)\sqrt{x}\left(x^{3/2}-1\right)}{\sqrt{x}\left(x^{3/2}-1\right)} + \frac{1}{15}[3\cos^5 x - 5\cos^3 x]$

$y = x - 1 + \frac{1}{15}[3\cos^5 x - 5\cos^3 x]$

$y' = 1 + [(\cos x)^4 (-\sin x) + \cos^2 x \sin x]$

$y'\left(\frac{\pi}{6}\right) = 1 + \frac{9}{16} \times \left(-\frac{1}{2}\right) + \frac{3}{4} \times \frac{1}{2}$

$y'\left(\frac{\pi}{6}\right) = 1 - \frac{9}{32} + \frac{3}{8}$

$96y'\left(\frac{\pi}{6}\right) = 96 - 27 + 36 = 105$

23. If the domain of the function $f(x) = \frac{\sqrt{x^2-25}}{4-x^2} + \log\left(x^2+2x-15\right)$ is $(-\infty, \alpha) \cup [\beta, \infty)$, then $\alpha^2 + \beta^2$ is equal to

Answer (50)

Sol. $x^2 - 25 \geq 0 \Rightarrow x \in (-\infty, -5) \cup [5, \infty)$

$4 - x^2 \neq 0$

$\Rightarrow x \neq -2, 2$

$x^2 + 2x - 15 > 0$

$(x + 5)(x - 3) > 0$

$x \in (-\infty, -5) \cup (3, \infty)$

So,

$x \in (-\infty, -5) \cup [5, \infty)$

$\alpha = -5$

$\beta = 5$

$\alpha^2 + \beta^2 = 50$

24. There are 20 lines numbered as 1, 2, 3 … 20. All the odd numbered lines intersect at a point and all the even numbered lines are parallel. Find the maximum number of point of intersections.

Answer (101)

Sol. 10 lines are concurrent, 10 lines are parallel.

Odd lines $\in \{l_1, l_3 ..., l_{19}\}$

Even lines $\in \{l_2, l_4 ..., l_{20}\}$

For maximum intersection:

$^{(\text{Even lines})}C_2 \times$ (zero point of intersection)

+ (One line from odd lines) (one line from even lines) + 1 point of intersection of concurrent lines

$\Rightarrow {}^{10}C_2(0) + {}^{10}C_1 \cdot {}^{10}C_1 + 1$

$\Rightarrow (101)$

25. The number of solutions of the equation $4\sin^2 x - 4\cos^3 x + 9 - 4\cos x = 0$ in $x \in (-2\pi, 2\pi)$

Answer (0)

Sol. $f(x) = -4\cos^3 x - 4\cos^2 x - 4\cos x + 13$

Let $\cos x = t, \quad t \in [-1, 1]$

$f(t) = -4t^3 - 4t^2 - 4t + 13$

$f'(t) = -2t^2 - 8t - 4 = 0$

$\Rightarrow f'(t)$ is always –ve

So, decreasing function

$3t^2 + \underset{+\text{ve}}{2t} + 1 = 0$

$f(-1) = 4 - 4 + 4 + 13 = 17$

$f(1) = -4 - 4 - 4 + 13 = 1$

$f(x)_{\min} = 1$

$\therefore$ $f(x)$ can not be zero for any value of x zero solutions.

26. If $\lim\limits_{x\to-\infty} f(x) = 1$, and $f'(x) = \alpha f(x) + 3$, and $f : R \to R$ with $f(0) = 2$ then $|\alpha|$ is equal to

Answer (03.00)

Sol. $\dfrac{dy}{dx} = (\alpha y + 3)$

$\Rightarrow$ $\ln|\alpha y + 3| = x + c$

$\Rightarrow$ $\alpha y + 3 = ke^x$

$\Rightarrow$ $f(0) = 2$

$\Rightarrow$ $2\alpha + 3 = ke^0 \Rightarrow k = (2\alpha + 3)$

$\Rightarrow \quad y = \dfrac{(2\alpha + 3)e^x - 3}{\alpha}$

$\lim\limits_{x\to-\infty} \dfrac{(2\alpha+3)e^x - 3}{\alpha} = \dfrac{-3}{\alpha} = 1 \Rightarrow \alpha = -3$

27. Let $a_1, a_2, a_3 \ldots a_n$ be in A.P. and S_n denotes the sum of first n terms of this A.P. If $S_{10} = 390$, $\dfrac{a_{10}}{a_{50}} = \dfrac{15}{7}$, then $S_{15} - S_5$ is equal to

Answer (468)

Sol. $S_{10} = 390$

$\dfrac{a_{10}}{a_{50}} = \dfrac{15}{7}$

$\Rightarrow \quad \dfrac{a + 9d}{a + 49d} - \dfrac{15}{7}$

$\Rightarrow$ $8a = 63d - 735d$

$\Rightarrow$ $8a = -672d$

$a = -84d$

$S_{10} = \dfrac{10}{2}(-168d + 9d) = 390$

$\Rightarrow$ $5(-159d) = 390$

$\Rightarrow \quad d = -\dfrac{78}{159}$

$\Rightarrow$ $S_{15} - S_5$

$\Rightarrow \dfrac{15}{2}(2a - 14d) - \dfrac{5}{2}(2a + 4d)$

$\Rightarrow$ $15(a - 7d) - 5(a + 2d)$

$\Rightarrow$ $10a - 115d$

$\Rightarrow$ $-840d - 115d$

$= -955d \Rightarrow \; -955\left(-\dfrac{78}{159}\right) \approx 468.49$

28.

29.

30.

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