PART : MATHEMATICS

1. Number of 4 letter words with or without meaning by using letters of the word "DISTRIBUTION" is

Ans. **(3734)**

Sol. $I \to 3$
$T \to 2$
$D \to 1$
$S \to 1$
$R \to 1$
$B \to 1$
$U \to 1$
$O \to 1$
$N \to 1$

(1) no of words whose all letters are distinct = ${}^{9}C_4 \times 4! = 126 \times 24 = 3024$

(2) no of words whose 2 letters are same and 2 letters are distinct = ${}^{2}C_1 \times {}^{8}C_2 \times \frac{4!}{2!} = 672$

(3) no of words whose 2 letters are same and other 2 letters are same = ${}^{2}C_2 \times \frac{4!}{2!2!} = 6$

(4) no of words whose 3 letters are same and 1 letter is distinct = ${}^{1}C_1 \times {}^{8}C_1 \times \frac{4!}{3!} = 32$

total number of words = 3734

2. In the expansion of $(1 - x^2)(1 + x)\left(1+\frac{3}{x}+\frac{3}{x^2}+\frac{1}{x^3}\right)^5$ the sum of the coefficient of x^3 and x^{-13} is

Ans. **(0118)**

Sol. $(1 - x^2)(1 + x)\left(1+\frac{3}{x}+\frac{3}{x^2}+\frac{1}{x^3}\right)^5 = (1 - x^2)(1 + x)\left(1+\frac{1}{x}\right)^{3\times5}$

$= (1 - x)(1 + x)^2\left(\frac{1+x}{x}\right)^{15} = \frac{(1-x)}{x^{15}}(1+x)^{17}$

$= \frac{(1-x)}{x^{15}}\sum_{r=0}^{17} {}^{17}C_r x^r = \sum_{r=0}^{17} {}^{17}C_r x^{r-15} - \sum_{r=0}^{17} {}^{17}C_r x^{r-14}$

so coefficient of $x^3 = -{}^{17}C_{17} = -1$
and coefficient of $x^{-13} = {}^{17}C_2 - {}^{17}C_1 = 119$
sum of coefficient x^3 and x^{-13} is $= -1 + 119 = 118$

3. The sum of series $\frac{1}{1-3.1^2+1^4}+\frac{2}{1-3.2^2+2^4}+\frac{3}{1-3.3^2+3^4}+........$ up to 10 terms, is equal to

(1) $-\frac{55}{109}$ (2) $\frac{55}{109}$ (3) $\frac{-45}{109}$ (4) $\frac{45}{109}$

Ans. **(1)**

Sol. Given sum = $\sum_{r=1}^{10}\frac{r}{1-3r^2+r^4}$

$= \sum_{r=1}^{10}\frac{r}{(r^2-1)^2-r^2}$

$= \sum_{r=1}^{10} \frac{r}{(r^2+r-1)(r^2-r-1)} = \frac{1}{2}\sum_{r=1}^{10} \frac{(r^2+r-1)-(r^2-r-1)}{(r^2+r-1)(r^2-r-1)}$

$= \frac{1}{2}\sum_{r=1}^{10}\left(\frac{1}{r^2-r-1}-\frac{1}{r^2+r-1}\right) = \frac{1}{2}\left[\left(-\frac{1}{1}-\frac{1}{1}\right)+\left(\frac{1}{1}-\frac{1}{5}\right)+\left(\frac{1}{5}-\frac{1}{11}\right)+.....+\left(\frac{1}{89}-\frac{1}{109}\right)\right]$

$= \frac{1}{2}\left[-\frac{1}{1}-\frac{1}{109}\right]$

$= -\frac{1}{2}\left(\frac{110}{109}\right) = -\frac{55}{109}$

4. $\lim_{x\to 0}\frac{e^{2|\sin x|}-2|\sin x|-1}{x^2}$ is equal to

Ans. (2)

Sol. $\lim_{x\to 0}\frac{e^{2|\sin x|}-2|\sin x|-1}{x^2}$

since even function so LHL = RHL

Now RHL = $\lim_{x\to 0}\frac{e^{2\sin x}-2\sin x-1}{x^2}$

$= \lim_{x\to 0}\frac{e^{2\sin x}.2\cos x-2\cos x}{2x}$

$= \lim_{x\to 0^+}\cos x\left(\frac{e^{2\sin x}-1}{2\sin x}\right)\times\frac{2\sin x}{x} \qquad = 1\times 1\times 2 = 2$

5. 2 balls are selected with replacement from 10 red, 30 white, 15 orange and 20 blue balls then probability that first ball is red and second ball is white, is.

(1) $\frac{9}{25}$ (2) $\frac{4}{75}$ (3) $\frac{8}{75}$ (4) $\frac{7}{75}$

Ans. (2)

Sol. Probability = $\frac{10}{75}\times\frac{30}{75}=\frac{2}{15}\times\frac{2}{5}=\frac{4}{75}$

6. $\vec{a}=\hat{i}+3\hat{j}+4\hat{k}$, $\vec{b}=2\hat{i}-3\hat{j}+4\hat{k}$, $\vec{c}=5\hat{i}-2\hat{j}+4\hat{k}$ given that $\vec{p}\times\vec{b}=\vec{b}\times\vec{c}$ and $\vec{p}.\vec{a}=0$ then value of $\vec{p}.(\hat{i}-\hat{j}+\hat{k})$ is

Ans. (4)

Sol. $(\vec{p}+\vec{c})\times\vec{b}=\vec{0}$ so, $(\vec{p}+\vec{c})\,||\,\vec{b}$

$\therefore\ \vec{p}+\vec{c}=\lambda\vec{b}$

$\vec{p}=\lambda\vec{b}-\vec{c}$

Now we have $\vec{p}.\vec{a}=0$

$\vec{p}.\vec{a}=\lambda\vec{b}.\vec{a}-\vec{c}.\vec{a}$

$0 = \lambda(2-9+16)-(5-6+16)$

$\lambda = \frac{15}{9}=\frac{5}{3}$

$\therefore\ \vec{p} = \frac{5}{3}\vec{b} - \vec{c}$

and $\vec{p}.(\hat{i} - \hat{j} - \hat{k})$

$= \left[\hat{i}\left(\frac{10}{3} - 5\right) + \hat{j}\left(\frac{-15}{3} + 2\right) + \hat{k}\left(\frac{20}{3} - 4\right)\right] (\hat{i} - \hat{j} + \hat{k})$

$= \frac{-5}{3} + 3 + \frac{8}{3}$

$= 4$

7. If $f(x) = \frac{4x-3}{6x-4}, x \neq \frac{2}{3}$, $g: R - \left\{\frac{2}{3}\right\} \to R - \left\{\frac{2}{3}\right\}$, $g(x) = fof(x)$ then value of gogog(4) is equal to

Ans. (4)

Sol. $g(x) = f\left(\frac{4x-3}{6x-4}\right) = \frac{4\left(\frac{4x-3}{6x-4}\right)-3}{6\left(\frac{4x-3}{6x-4}\right)-4} = \frac{16x-12-18x+12}{24x-18-24x+16} = \frac{-2x}{-2} = x$

gogog(4) = g(g(4)) = g(4) = 4

8. Let 'S' be the set of positive integer values of a for which $\frac{ax^2 + 2(a+1)x + 9a + 4}{x^2 + 8x + 32} < 0\ \forall\ x \in R$, then the number of elements is 'S' is

Ans. (0)

Sol. $ax^2 + 2(a + 1)x + 9a + 4 < 0$ as $x^2 + 8x + 32$ is always positive

case-I when $a \neq 0$ and $a < 0$

then $D < 0$

$4(a + 1)^2 - 4a(9a + 4) < 0$

$a^2 + 2a + 1 - 9a^2 - 4a < 0$

$0 < 8a^2 + 2a - 1$

$a \in \left(-\infty, -\frac{1}{2}\right) \cup \left(\frac{1}{4}, \infty\right)$

So $a \in \left(-\infty, -\frac{1}{2}\right)$

case –II when $a = 0$

then $2x + 4 < 0$ is not always true

So, $a \neq 0$

So number of positive integer values of 'a' = 0

9. Solution of the differential equation $y\frac{dx}{dy} = x(\ln x - \ln y + 1)$ is given by y = y(x) such that y(e) = 1 then equation of curve is given by

(1) $y = \ln\frac{x}{y}$ (2) $y = \ln\frac{y}{x}$ (3) $x = \ln\frac{x}{y} + e - 1$ (4) $x = \ln\frac{y}{x} + e + 1$

Ans. (1)

Sol. $y\frac{dx}{dy} = x(\ln x - \ln y + 1)$

$\frac{dx}{dy} = \frac{x}{y}\left(\ln\frac{x}{y} + 1\right)$

Let $x = vy \Rightarrow \frac{dx}{dy} = v + y\frac{dv}{dy}$

$\Rightarrow v + y\frac{dv}{dy} = v\ln v + v$

$\Rightarrow y\frac{dv}{dy} = v\ln v$

$\Rightarrow \int\frac{dv}{v\ln v} = \int\frac{dy}{y}$

$\Rightarrow \ln(\ln v) = \ln y + \ln c$

$\Rightarrow \ln v = yc$

$\Rightarrow \ln\left(\frac{x}{y}\right) = yc$

passes through $(e,1) \Rightarrow \ln e = c \Rightarrow c = 1$

$\Rightarrow y = \ln\left(\frac{x}{y}\right)$

10. $525\int_0^{\frac{\pi}{2}} \sin 2x(\cos x)^{\frac{11}{2}}\left(1+(\cos x)^{\frac{5}{2}}\right)^{\frac{1}{2}} dx$

(1) $64 + 176\sqrt{2}$ (2) $176\sqrt{2} - 64$

(3) $64 - 128\sqrt{2}$ (4) $64 + 128\sqrt{2}$

Ans. (2)

Sol. $525\int_2^1 2\sin x.\cos^{13/2}(1+(\cos x)^{5/2})^{\frac{1}{2}}dx$

$1 + (\cos x)^{5/2} = t$

$-\frac{5}{2}(\cos x)^{3/2}\sin x\, dx = dt$

$= -1050\int_2^1 (t-1)^2(t)^{1/2}.\frac{2dt}{5}$

$= -420\int_2^1 \left(t^{5/2} - 2t^{3/2} + t^{1/2}\right)dt$

$$= -420\left[\frac{t^{7/2}}{\frac{7}{2}} - 2\frac{t^{5/2}}{\frac{5}{2}} + \frac{t^{3/2}}{\frac{3}{2}}\right]_2^1 = -420\left[\left(\frac{2}{7}-\frac{4}{5}+\frac{2}{3}\right)-\left(\frac{2}{7}8\sqrt{2}-\frac{4}{5}4\sqrt{2}+\frac{2}{3}2\sqrt{2}\right)\right]$$

$$= -120 + 336 - 280 + 960\sqrt{2} - 1344\sqrt{2} + 560\sqrt{2} = 176\sqrt{2} - 64$$

11. Let $f(x) = \begin{cases} g(x) : x < 0 \\ \left(\frac{1+x}{2+x}\right)^{\frac{1}{x}} : x \geq 0 \end{cases}$ where g(x) is a linear function and f(x) is continuous function and f′(1) = f(–1) then g(3) =

(1) $\frac{3}{2}\ln\left(\frac{3}{2}\right)-\frac{1}{4}$ (2) $\frac{1}{2}\ln\left(\frac{3}{2}\right)+\frac{1}{4}$ (3) $\ln\left(\frac{4}{9}\right)-\frac{1}{3}$ (4) $\ln\left(\frac{4}{9}\right)+\frac{1}{3}$

Ans. (3)

Sol. Let g(x) = ax + b

f(x) is continuous at x = 0

so $\lim_{x\to 0^+}\left(\frac{1+x}{2+x}\right)^{\frac{1}{x}} = \left(\frac{1}{2}\right)^{\infty} = 0$

$\lim_{x\to 0^-} g(x) = \lim_{x\to 0} ax + b = b$

So b = 0

Now $f(x) = \left(\frac{1+x}{2+x}\right)^{\frac{1}{x}}$

$\ell n f(x) = \frac{1}{x}\left[\ell n(1+x) - \ell n(2+x)\right]$

$\frac{1}{f(x)}f'(x) = \frac{x\left[\frac{1}{1+x}-\frac{1}{2+x}\right] - \ell n\left(\frac{1+x}{2+x}\right)}{x^2}$

$f'(1) = f(1)\times\left[\frac{\left[\frac{1}{2}-\frac{1}{3}\right]-\ell n\frac{2}{3}}{1}\right]$

$f'(1) = \frac{2}{3}\left[\frac{1}{6} - \ell n\frac{2}{3}\right]$

Also $f(-1) = (ax+b)_{\text{at } x=-1} = -a$

So, $-a = \frac{1}{9} - \frac{2}{3}\ln\frac{2}{3}$

$a = \frac{2}{3}\ln\frac{2}{3} - \frac{1}{9}$

g(x) = ax

$g(3) = 3\left(\frac{2}{3}\ln\frac{2}{3} - \frac{1}{9}\right)$

$= \left(2\ln\frac{2}{3} - \frac{1}{3}\right) = \left(\ln\frac{4}{9} - \frac{1}{3}\right)$

12. If $|\vec{a}| = 1$, $|\vec{b}| = 4$, $\vec{a}.\vec{b} = 2$, $\vec{c} = (2\vec{a} \times \vec{b}) - 3\vec{b}$ and $\vec{b} \wedge \vec{c} = \alpha$ then value of $192 \sin^2 \alpha$ is

Ans. (48)

Sol. $\vec{c} = (2\vec{a} \times \vec{b}) - 3\vec{b}$ ..... (1)

taking dot product with $\vec{b}$

$\vec{b}.\vec{c} = -3(\vec{b}.\vec{b})$

$|\vec{b}||\vec{c}|\cos\alpha = -3|\vec{b}|^2$

$|\vec{c}|\cos\alpha = -3 \times 4 = -12$

$|\vec{c}|\cos\alpha = -12$

$|\vec{c}|^2 \cos^2 \alpha = 144$ .... (2)

$\vec{a}.\vec{b} = 2$

$\cos\theta = \frac{1}{2}$

$\theta = \frac{\pi}{3}$ ..... (3)

$|\vec{c}|^2 = ((2\vec{a} \times \vec{b}) - 3\vec{b})^2$

$= (2\vec{a} \times \vec{b})^2 + 9(\vec{b})^2 - 12(\vec{a} \times \vec{b})\vec{b} = 4|\vec{a}|^2|\vec{b}|^2 \sin^2\theta + 9|\vec{b}|^2 = 64 \times \frac{3}{4} + 144$

$|\vec{c}|^2 = 48 + 144 = 192$

By equation (2)

$|\vec{c}|^2 \cos^2 \alpha = 144$

$192\cos^2 \alpha = 144$

$192 - 192\sin^2 \alpha = 144$

$192\sin^2\alpha = 48$

13. Let $\sin^{-1} \alpha + \sin^{-1} \beta + \sin^{-1} \gamma = \pi$ and α, β, γ are non zero real numbers such that $(\alpha + \beta + \gamma)(\alpha + \beta - \gamma) = 3\alpha\beta$ then value of γ

(1) 1 (2) $\frac{1}{2}$ (3) $-\frac{1}{2}$ (4) $\frac{\sqrt{3}}{2}$

Ans. (4)

Sol. Let $\sin^{-1}\alpha = A$, $\sin^{-1}\beta = B$ and $\sin^{-1}\gamma = C$ then $A + B + C = \pi$ .... (1)

also $(\alpha + \beta + \gamma)(\alpha + \beta - \gamma) = 3\alpha\beta$

$\Rightarrow \alpha^2 + \beta^2 - \gamma^2 = \alpha\beta$

$\Rightarrow \sin^2 A + \sin^2 B - \sin^2 C = \sin A \sin B$

$\Rightarrow \sin^2 A + \sin(B + C)\sin(B - C) = \sin A \sin B$

$\Rightarrow \sin^2 A + \sin A \sin(B - C) = \sin A \sin B$

$\Rightarrow \sin A [\sin A + \sin(B - C) - \sin B] = 0$

$\Rightarrow \sin A [\sin(B + C) + \sin(B - C) - \sin B] = 0$

$\Rightarrow \sin A [2\sin B \cos C - \sin B] = 0$

$\Rightarrow \sin A \sin B(2\cos C - 1) = 0$

$\Rightarrow \sin A = 0$ or $\sin B = 0$ or $2\cos C = 1$

but α, β are non zero $\Rightarrow \cos C = \frac{1}{2} \Rightarrow \sin C = \frac{\sqrt{3}}{2} \Rightarrow \gamma = \frac{\sqrt{3}}{2}$

14. If system of linear equations

$x - 2y + z = -4$

$2x + \alpha y + 3z = 5$

$3x - y + \beta z = 3$

has infinite solution then the value of $12\alpha + 13\beta$ is equal to

Ans. (58)

Sol. for infinite solution

$D = 0$

$$\begin{vmatrix} 1 & -2 & 1 \\ 2 & \alpha & 3 \\ 3 & -1 & \beta \end{vmatrix} = 0$$

$1(\alpha\beta + 3) + 2(2\beta - 9) + 1(-2 - 3\alpha) = 0$

$\alpha\beta + 4\beta - 3\alpha - 17 = 0$ _____ (1)

Also for infinite solution

Compare $P_1 + \lambda P_2 = 0$

& $P_3 = 0$

$x(1 + 2\lambda) + y(-2 + \alpha\lambda) + z(1 + 3\lambda) = -4 + 5\lambda$

$3x - y + \beta z = 3$

$$\frac{1+2\lambda}{3} = \frac{-2+\alpha\lambda}{-1} = \frac{1+3\lambda}{\beta} = \frac{-4+5\lambda}{3}$$

Solving first and last

$3\lambda = +5$

$\lambda = \frac{+5}{3}$

Now $$\frac{1+\frac{10}{3}}{3} = \frac{-2+\frac{5\alpha}{3}}{-1} = \frac{1+5}{\beta} = \frac{-4+\frac{25}{3}}{3}$$

$$\frac{6}{\beta} = \frac{13}{9}$$

$\alpha = \frac{1}{3}$ and $\beta = \frac{54}{13}$

So $12\alpha + 13\beta = 4 + 54 = 58$

15. Let A = {1, 2, 3, 4} and R is relation defined on set A such that R= {(1, 2), (1, 4), (2, 3)} then minimum number of elements added to R so that it becomes equivalence

(1) 10 (2) 11 (3) 13 (4) 14

Ans. (2)

Sol. for reflexive add elements

(1, 1), (2, 2), (3, 3), (4, 4)

For symmetric add elements

(2, 1), (4, 1), (3, 2)

For Transitive add

(1, 3), (3, 1)

(4, 2), (2, 4)

So Total min. 11 elements should be added to form equivalence

16. Q and R are foot of perpendicular drawn from point P (a, a, a) to the lines x =y, z = 1 and x = –y, z = – 1 such that ∠QPQ is 90º then find the value of $12a^2$

Ans. (0012)

Sol. Foot of perpendicular from P(a, a, a) on $\frac{x}{1}=\frac{y}{1}=\frac{z-1}{0}$ is Q (a, a, 1)

Also foot of perpendicular from P(a, a,a) on $\frac{x}{1}=\frac{y}{-1}=\frac{z+1}{0}$ is R (0, 0, –1)

D' ratios of QP are < 0,0, 1 – a >

D' ratios of PR are < a,a, a + 1 >

Since ∠QPR = 90º ⇒ 0(a) + 0(a) + (1– a) (a + 1) = 0 ⇒ $a^2 = 1$ ⇒ $12a^2 = 12$

17. If $f(x)=\begin{vmatrix} x^3 & 2x^2+1 & 1+3x \\ 3x^2+2 & 2x & x^3+6 \\ x^3-x & 4 & x^2-2 \end{vmatrix}$ for all x ∈ R then 2f(0)+ f'(0) is equal to

(1) 12 (2) 24 (3) 42 (4) 36

Ans. (3)

Sol. $f(0)=\begin{vmatrix} 0 & 1 & 1 \\ 2 & 0 & 6 \\ 0 & 4 & -2 \end{vmatrix}=-2(-2-4)=12$

$$f'(x)=\begin{vmatrix} 3x^2 & 4x & 3 \\ 3x^2+2 & 2x & x^3+6 \\ x^3-x & 4 & x^2-2 \end{vmatrix}+\begin{vmatrix} x^3 & 2x^2+1 & 1+3x \\ 6x & 2 & 3x^2 \\ x^3-x & 4 & x^2-2 \end{vmatrix}+\begin{vmatrix} x^3 & 2x^2+1 & 1+3x \\ 3x^2+2 & 2x & x^3+6 \\ 3x^2-1 & 0 & 2x \end{vmatrix}$$

$$f'(0)=\begin{vmatrix} 0 & 0 & 3 \\ 2 & 0 & 6 \\ 0 & 4 & -2 \end{vmatrix}+\begin{vmatrix} 0 & 1 & 1 \\ 0 & 2 & 0 \\ 0 & 4 & -2 \end{vmatrix}+\begin{vmatrix} 0 & 1 & 1 \\ 2 & 0 & 6 \\ -1 & 0 & 0 \end{vmatrix}=3(8)+0-1(6)=18$$

So 2f(0) +f'(0) = 2(12) + 18 =24 + 18 = 42

18. $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ has foci (±5, 0) and latus rectum is $\sqrt{50}$, find square of eccentricity of $\frac{x^2}{a^2}-\frac{y^2}{a^2b^2}=1$

Ans. (26)

Sol. ae = 5

$\frac{2b^2}{a}=\sqrt{50}$

$b^2=\frac{5\sqrt{2}a}{2}$

$b^2 = a^2(1-e^2)=\frac{5\sqrt{2}a}{2} \Rightarrow a(1-e^2)=\frac{5\sqrt{2}}{2} \Rightarrow \frac{5}{e}(1-e^2)=\frac{5}{\sqrt{2}}$

$\sqrt{2}(5-5e^2)=5e \Rightarrow 5\sqrt{2}e^2+5e-5\sqrt{2}=0$

$\sqrt{2}e^2+e-\sqrt{2}=0 \Rightarrow e=-\sqrt{2}$. (rejected) and $e=\frac{1}{\sqrt{2}}$

from ae = 5 ⇒ $a=5\sqrt{2}$ and $b^2 = 25$

So, square of eccentricity of new curve = $1+\frac{a^2b^2}{a^2}=1+b^2=26$

19. If one of the diameter of the circle $x^2 + y^2 - 10x + 4y + 13 = 0$ is a chord of another circle and whose centre is the point of intersection of the lines $2x + 3y = 12$ and $3x - 2y = 5$, then the radius of the circle is

(1) 6 (2) $3\sqrt{2}$ (3) $\sqrt{20}$ (4) 4

Ans. (1)

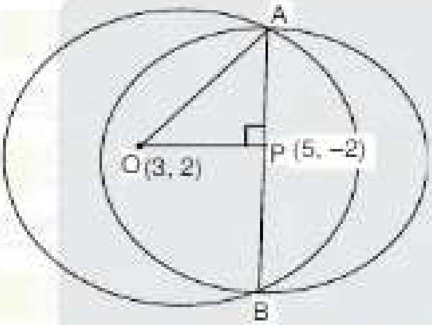
Sol. Point of intersection of lines

$2x + 3y = 12$

& $3x - 2y = 5$

$x = 3$

and $y = 2$



centre of required circle is (3, 2) and radius of given circle is = 4

and centre of given circle is (5, –2)

Now from fig.

$r = \sqrt{AP^2 + OP^2}$

$\Rightarrow r = \sqrt{16 + \left(\sqrt{4+16}\right)^2} = 6$

20. For any curve $y = y(x)$, $\frac{dy}{dx} = \frac{\tan x + y}{\sin x(\sec x - \sin x \tan x)}$ and $y\left(\frac{\pi}{4}\right) = 1$ then $y\left(\frac{\pi}{3}\right)$ is equal to

(1) $\frac{\sqrt{3}}{2}\left(1 - \ell n\sqrt{3}\right)$ (2) $\frac{\sqrt{3}}{2}\left(\ell n\sqrt{3} - 1\right)$ (3) $\frac{1}{2}\left(\ell n\sqrt{3} - 1\right)$ (4) $\sqrt{3}\left(\ell n\sqrt{3} + 1\right)$

Ans. (4)

Sol. $\frac{dy}{dx} = \frac{\tan x}{\sin x(\sec x - \sin x \tan x)} + \frac{y}{\sin x(\sec x - \sin x \tan x)}$

$\frac{dy}{dx} = \frac{1}{\cos x(\sec x - \sin x \tan x)} + \frac{y}{\sin x(\sec x - \sin^2 x \sec x)}$

$\frac{dy}{dx} = \frac{1}{(1 - \sin^2 x)} + \frac{y}{\sin x \sec x(1 - \sin^2 x)}$

$\frac{dy}{dx} = \frac{1}{\cos^2 x} + \frac{y}{\sin x \cos x}$

$\frac{dy}{dx} - y(\sec x \operatorname{cosec} x) = \sec^2 x$ linear differential equation

I.f. $= e^{\int -\frac{1}{\sin x \cos x}dx} = e^{2\int \operatorname{cosec} 2x dx}$

$= e^{\ell n \tan x} = \frac{1}{\tan x} = \cot x$

Hence solution of differential equation is

$y.\cot x = \int \sec^2 x.\cot x dx + c$

$y.\cot x = \int \frac{\sec^2 x}{\tan x} dx + c$

$y.\cot x = \int 2\text{cosec}2x + c$

$y.\cot x = \ln\tan x + c$

now $y\left(\frac{\pi}{4}\right) = 1$

$1 = 0 + c$

$y.\cot x = \ln\tan x + 1$

$y\left(\frac{\pi}{3}\right) = \left(\ln\sqrt{3} + 1\right)\sqrt{3}$

21. 3 rotten apples are mixed with 15 good apples. Two apples are drawn from basket at random. Let x be the variable representing number of rotten apples drawn, then variance of probability distribution of number of rotten apples drawn is

(1) $\frac{1024}{2601}$ (2) $\frac{40}{153}$ (3) $\frac{70}{153}$ (4) $\frac{641}{2501}$

Ans. **(2)**

Sol.

X_i	0	1	2
P_i	$\frac{^{15}C_2}{^{18}C_2} = \frac{35}{51}$	$\frac{^{15}C_1 \times ^3C_1}{^{18}C_2} = \frac{5}{17}$	$\frac{^3C_2}{^{18}C_2} = \frac{1}{51}$

$\text{Var}(x) = \text{Var}(x) = \sum p_i x_i^2 - \left(\sum p_i x_i\right)^2$

$= \frac{35}{51} \times (0)^2 + \frac{5}{17}(1)^2 + \frac{1}{51}(2)^2 - \left(0 + \frac{5}{17} + \frac{2}{51}\right)^2$

$= \frac{19}{51} - \left(\frac{17}{51}\right)^2$

$= \frac{680}{2601} = \frac{40}{153}$

22. If A (α, β), B (1, 0), C(γ, δ) and D (1, 2) are vertices of a parallelogram ABCD. If line $2x - 3y + 1 = 0$ passes through vertices A and C then value of $2(\alpha + \beta + \gamma + \delta)$ is equal to

Ans. **(0008)**

Sol. ABCD is a parallelogram so mid point of A,C = mid point of B, D

$\Rightarrow \left(\frac{\alpha+\gamma}{2}, \frac{\beta+\delta}{2}\right) = \left(\frac{1+1}{2}, \frac{0+2}{2}\right)$

$\Rightarrow \left(\frac{\alpha+\gamma}{2}, \frac{\beta+\delta}{2}\right) = (1, 1)$

$\Rightarrow \alpha + \gamma = 2$ & $\beta + \delta = 2$

So $2(\alpha + \beta + \gamma + \delta) = 2(2 + 2) = 8$

23. Let $S = \left\{y^2 \le 4x, x < 4, \frac{xy(x-1)(x-2)}{(x-3)(x-4)} < 0, x \ne 3\right\}$ then area of region S is

(1) $\frac{32}{3}$ (2) $\frac{16}{2}$ (3) $\frac{64}{3}$ (4) $\frac{8}{3}$

Ans. (1)

Sol. Case-1: when $y \ge 0$

Then $\frac{xy(x-1)(x-2)}{(x-3)(x-4)} < 0$

$\frac{x(x-1)(x-2)}{(x-3)(x-4)} < 0$

$x \in (1, 2) \cup (3, 4)$

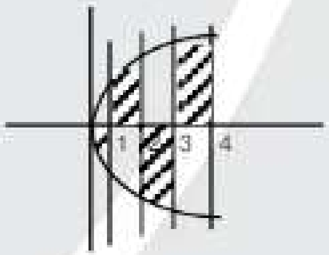
Case-2: $y < 0$

$\frac{xy(x-1)(x-2)}{(x-3)(x-4)} < 0$

$\frac{x(x-1)(x-2)}{(x-3)(x-4)} > 0$

, $x \in (0,1) \cup (2,3)$

Required region



So required area

$= \int_0^4 \sqrt{4x}\,dx = \left(2(x^{\frac{3}{2}})\frac{2}{3}\right)_0^4 = \frac{4}{3}(4 \times 2 - 0) = \frac{32}{3}$