JEE MAIN 1 FEBRUARY 2024 SHIFT 1 QUESTION PAPER

MATHEMATICS

1. $\int_0^{\pi/4} \frac{x}{\sin^4(2x)+\cos^4(2x)} dx = ?$

2. If 3, a, b, c are in A.P. and 3, (a - 1), (b + 1) are in G.P., then find the arithmetic mean of a, b and c.

3. If $(t + 1)dx = (2x + (t + 1)^3)dt$ and x(0) = 2, then x(1) = ?

4. Five people are distributed in four identical rooms. A room can also contain zero people. Find the number of ways to distribute them.

5. If the hyperbola $x^2 - y^2\csc^2\theta = 5$ and ellipse $x^2\csc^2\theta + y^2 = 5$ has eccentricity e_H and e_E respectively and $e_H = \sqrt{7}\, e_E$, then find the value of θ.

6. If $A = \begin{bmatrix} \sqrt{2} & 1 \\ -1 & \sqrt{2} \end{bmatrix}, B = \begin{bmatrix} 1 & 0 \\ 1 & 1 \end{bmatrix}, C = ABA^T$ and $X = AC^2A^T$, then find $|X|$.

7. Let S = {1,2,3,..., 20}
    $R_1 = \{(a, b): a \text{ divide } b\}$,
    $R_2 = \{(a, b): a \text{ is integral multiple of } b\}$ and $a, b \in S$.
    $n(R_1 - R_2) = ?$

8. If $AP_1 = 3, 7, 11,..., 403$ and $AP_2 = 2, 5, 8,..., 401$. Find the sum of common terms of AP_1 and AP_2.

9. $\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \frac{8\sqrt{2}\cos x}{(1+e^{\sin x})(1+\sin^4 x)} dx = a\pi + b\, log\left(3 + 2\sqrt{2}\right)$, then find $a + b$.

10. If $5f(x) + 4f\left(\frac{1}{x}\right) = x^2 - 4$ and $y = 9f(x) * x^2$

    If y is strictly increasing, then find the interval of x.