

MECHANICAL ENGINEERING

EXAM HELD ON

3rd FEBRUARY 2024

EVENING SESSION

DETAILED SOLUTION BY TEAM



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Targeted Batch For GATE 2025 Aspirants

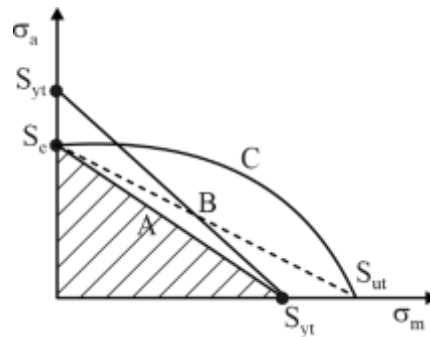
SECTION-1	Applied Mechanics and Design	Engineering Mechanics
		Strength of Material
		Theory of Machine
		Vibration
		Machine Design

[MCQ-1]

Q.1. Which one of the following failure theories in the most conservative design approach against fatigue failure

- (a) Soderberg line
- (b) Yield line
- (c) Gerber line
- (d) Modified Goodman line

Sol. (a)



Line A represents Soderberg line, $\frac{\sigma_a}{S_e} + \frac{\sigma_m}{S_{yt}} = 1$

Line B represents Yield line $\frac{\sigma_a}{S_{yt}} + \frac{\sigma_m}{S_{yt}} = 1$

Line C represents Gerber Parabola $\frac{\sigma_a}{S_e} + \left(\frac{\sigma_m}{S_{ut}}\right)^2 = 1$

The dashed line joining S_e and S_{ut} represents Goodman line $\frac{\sigma_a}{S_e} + \frac{\sigma_m}{S_{ut}} = 1$

The safe region as per Modified Goodman criteria is the safe region out of Goodman line and Yield line.

Out of all above criteria, Soderberg line is most conservative approach because its region of safety is smaller.

[MCQ-1]

Q.2. For a ball bearing, the fatigue life in million revolutions is given by $L = \left(\frac{C}{P}\right)^n$ where

P is the constant applied load and C is the basic dynamic load rating. Which one of the following statements is true

- (a) $n = 3$ assuming that the inner race is fixed and outer race is revolving
- (b) $n = 1/3$ assuming that the outer race is fixed and inner race is revolving
- (c) $n = 1/3$ assuming that the inner race is fixed and outer race is revolving



(d) $n = 3$ assuming that the outer race is fixed and inner race is revolving

Sol. (d)

For Rolling contact bearing, the fatigue life in million revolutions is given by

$$L = \left(\frac{C}{P}\right)^n$$

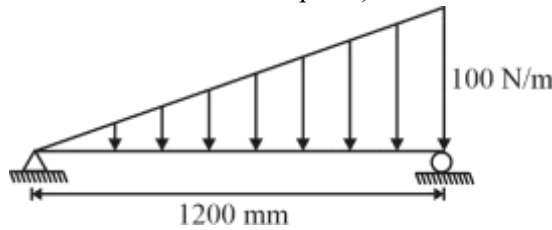
If the outer race is fixed and inner race is revolving, then

For Ball bearing $n = 3$

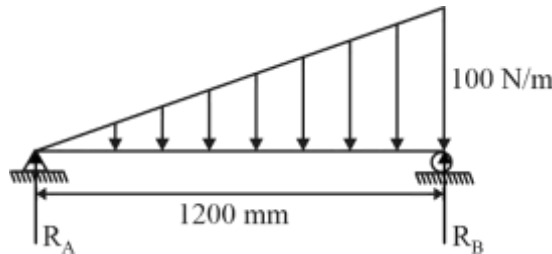
For Roller bearing $n = 10/3$

[NAT-2]

Q.3. A horizontal beam of length 1200 mm is pinned at the left end and is resting on the roller at the other end as shown in the figure. A linearly varying distributed load is applied on the beam. The magnitude of maximum bending moment acting on the beam is _____ Nm (Round off to one decimal place)



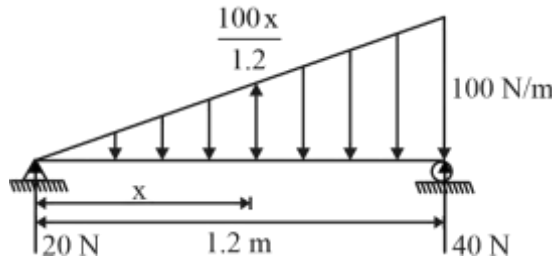
Sol. (9.237)



$$\Sigma M_B = 0$$

$$\Rightarrow R_A \times 1.2 = \left(\frac{1}{2} \times 1.2 \times 100\right) \times \frac{1.2}{3}$$

$$\Rightarrow R_A = 20 \text{ kN}$$



Bending moment will be maximum where the shear force changes its sign.

Location of zero shear force.

$$20 - \frac{1}{2} \times x \times \frac{100x}{1.2} = 0$$



$$\Rightarrow x = 0.693 \text{ m}$$

At $x = 0.693 \text{ m}$

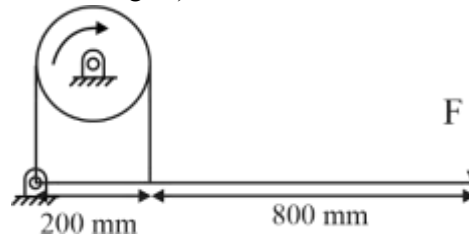
Maximum bending moment

$$B.M_{\max} = 20 \times 0.693 - \frac{1}{2} \times 0.693 \times \frac{100 \times 0.693}{1.2} \times \frac{0.693}{3}$$

$$\Rightarrow \boxed{B.M_{\max} = 9.237 \text{ N-m}}$$

[NAT-2]

Q.4. A band brake shown in the figure has a coefficient of friction of 0.3. The band can take a maximum force of 1.5 kN. The maximum braking force (F) that can be safely applied is _____ N. (Answer in integers)



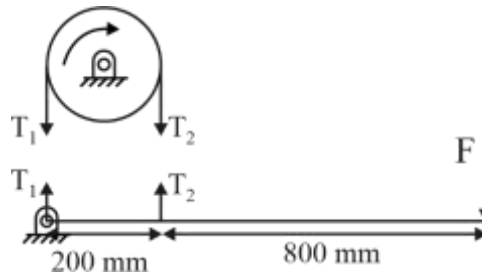
Sol. (116.9)

$$\mu = 0.3$$

$$T_1 = 1.5 \text{ kN} = 1500 \text{ N}$$

$$\theta = \pi \text{ rad}$$

$$F = ?$$



In FBD of lever

$$\Sigma M_O = 0$$

$$\Rightarrow F \times 1000 - T_2 \times 200 = 0$$

$$\Rightarrow F = \frac{T_2}{5} \quad \dots(1)$$

$$\frac{T_1}{T_2} = e^{\mu\theta}$$

$$\Rightarrow \frac{1500}{T_2} = e^{0.3 \times \pi}$$

$$\Rightarrow T_2 = 584.492 \text{ N}$$

From (i)

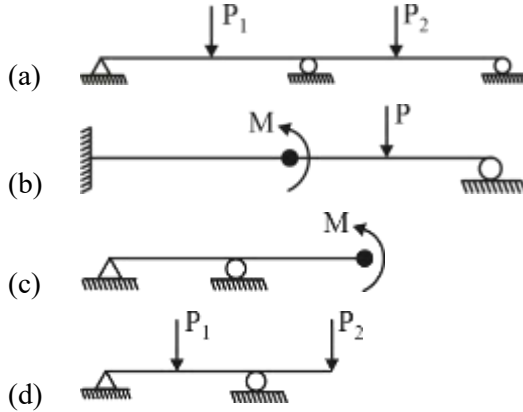


$$F = \frac{584.492}{5}$$

$$\Rightarrow \boxed{F = 116.9 \text{ N}}$$

[MSQ-2]

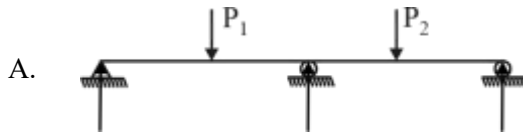
Q.5. Which of the following beams is/are statically indeterminate?



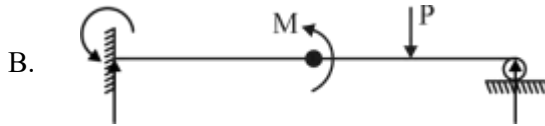
Sol. (a, b)

For statically indeterminate beam

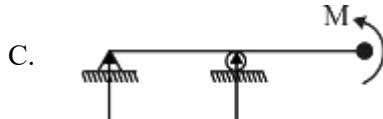
Number of reactions > Number of useful static equilibrium equation



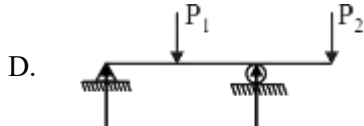
Three reactions > two useful static equilibrium equation
Therefore this beam is statically indeterminate beam



Three reactions > two useful static equilibrium equation
Therefore this beam is statically indeterminate beam



Two reactions = two useful static equilibrium equation
Therefore this beam is statically determinate beam



Two reactions = two useful static equilibrium equation
Therefore this beam is statically determinate beam

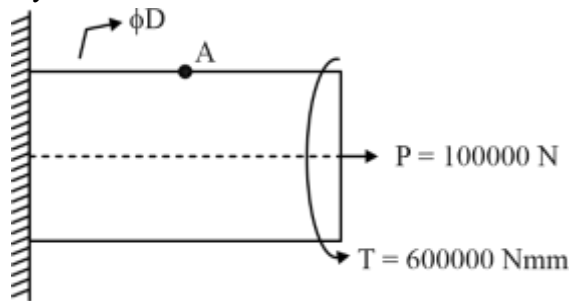


[NAT-2]

Q.6. A solid massless cylindrical member of 50 mm diameter is rigidly attached at one end, and is subjected to an axial force, $P = 100 \text{ kN}$ and torque $T = 600 \text{ Nm}$ at the another end as shown. Assume that the axis of the cylinder is normal is to the support. considering distortion energy theory with allowable yield stress as 300 MPa. The factor of safety in the design is ____ (Round of one decimal place)

Sol. (4.53)

$D = 50 \text{ mm}$
 $P = 100 \text{ kN} = 100000 \text{ N}$
 $T = 600 \text{ Nm} = 600000 \text{ Nmm}$
 $\sigma_y = 300 \text{ MPa}$
 FOS = ?
 Von-mises Theory of failure



Stress at critical point:

Axial stress due to P (σ_a)

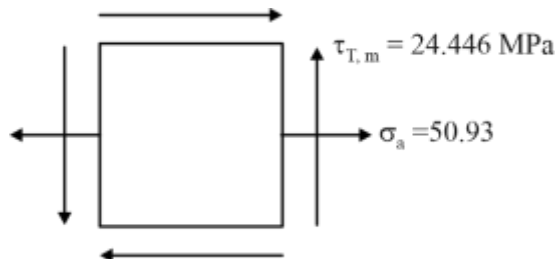
$$\sigma_a = \frac{P}{\frac{\pi}{4} D^2} = \frac{100000}{\frac{\pi}{4} \times 50^2}$$

$$\Rightarrow \sigma_a = 50.93 \text{ MPa}$$

Torsional shear stress due to ($\tau_{T,m}$)

$$\tau_{T,m} = \frac{16T}{\pi D^3} = \frac{16 \times 600000}{\pi \times 50^3}$$

$$\Rightarrow \tau_{T,m} = 24.446 \text{ MPa}$$



$$R = \sqrt{\left(\frac{\sigma_a}{2}\right)^2 + \tau_{T,m}^2}$$

$$\Rightarrow R = \sqrt{\left(\frac{50.93}{2}\right)^2 + 24.446^2}$$

$$\Rightarrow R = 35.3 \text{ MPa}$$

$$\sigma_{1,2} = \frac{\sigma_a}{2} \pm R \text{ and } \sigma_3 = 0$$

$$\Rightarrow \sigma_{1,2} = \frac{50.93}{2} \pm 35.3 \text{ and } \sigma_3 = 0$$

$$\Rightarrow \sigma_1 = 60.765 \text{ MPa}, \sigma_2 = -9.835 \text{ MPa}, \sigma_3 = 0$$

As per distortion energy theory of failure (when σ_3 is 0)

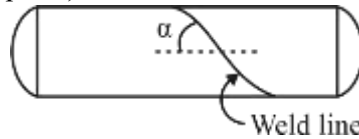
$$\text{FOS} = \frac{\sigma_y}{\sqrt{\sigma_1^2 + \sigma_2^2 - \sigma_1\sigma_2}}$$

$$\Rightarrow \text{FOS} = \frac{300}{\sqrt{60.765^2 + (-9.835)^2 - \{(60.765)(-9.835)\}}}$$

$$\Rightarrow \boxed{\text{FOS} = 4.53}$$

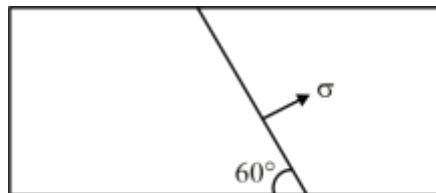
[NAT-2]

Q.7. The figure shows a thin cylindrical pressure vessel constructed by welding plates together along a line that makes an angle $\alpha = 60^\circ$ with the horizontal. The closed vessel has a wall thickness of 10 mm and diameter = 2m. When subjected to an internal pressure of 200 kPa, the magnitude of the normal stress along the weld is _____ MPa (Round of one decimal place)



Sol. (12.5)

$$P = 0.2 \text{ MPa}, d = 2 \text{ m}, t = 10 \text{ mm}$$



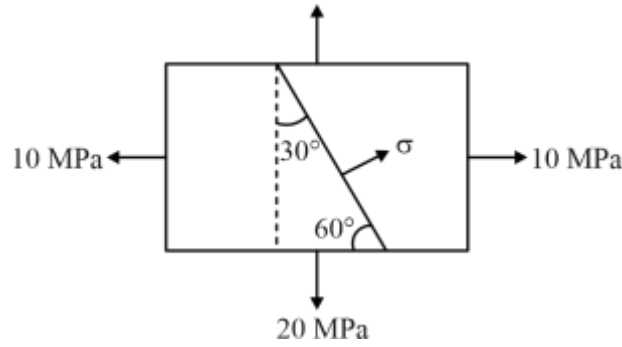
$$\sigma_x = \sigma_\ell = \frac{Pd}{4t}$$

$$\sigma_x = \frac{0.2 \times 2000}{4 \times 10}$$

$$\Rightarrow \sigma_x = 10 \text{ MPa}$$

$$\sigma_y = \sigma_c = 2\sigma_\ell$$

$$\Rightarrow \sigma_y = 20 \text{ MPa}$$



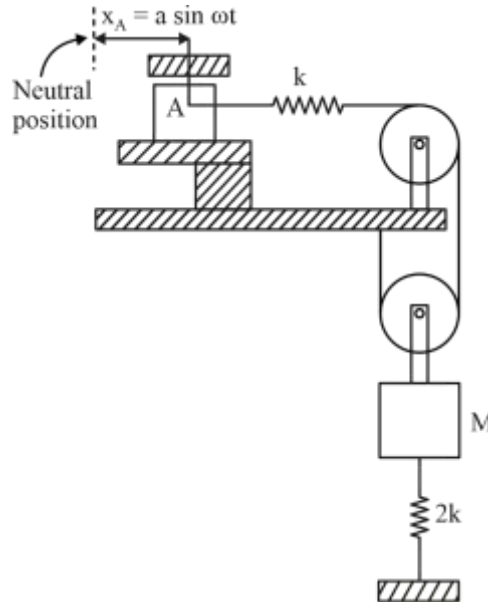
$$\sigma_{\theta} = \left(\frac{\sigma_x + \sigma_y}{2} \right) + \left(\frac{\sigma_x - \sigma_y}{2} \right) \cos 2\theta + \tau_{xy} \sin 2\theta$$

$$\Rightarrow \sigma_{\theta} = \left(\frac{10 + 20}{2} \right) + \left(\frac{10 - 20}{2} \right) \cos 60 + 0$$

$$\Rightarrow \sigma_{\theta} = 12.5 \text{ MPa}$$

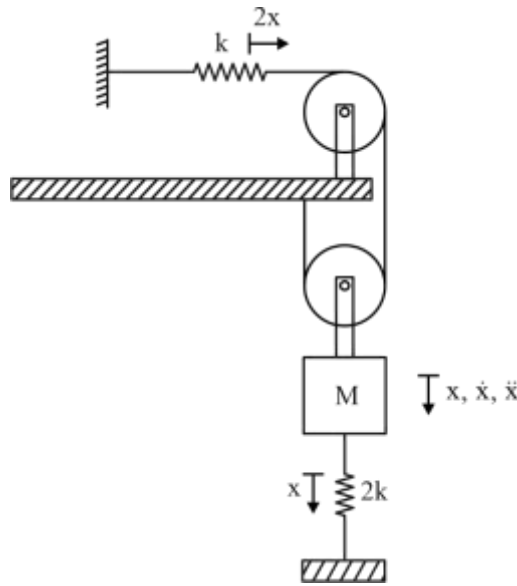
[NAT-2]

Q.8. A vibratory system consists of mass m , a vertical spring of stiffness $2k$ and a horizontal spring of stiffness k . The end A of the horizontal spring is given a horizontal motion $x_A = a \sin \omega t$. The other end of the spring is connected to an inextensible rope that passes over two massless pulleys as shown. Assume $m = 10 \text{ kg}$, $k = 1.5 \text{ kN/m}$ and neglect friction. The magnitude of critical driving frequency for which the oscillations of mass m tend to become excessively large is _____ rad./s (Answer in integer)



Sol. (30)





For excessively large oscillation of mass M , the driving frequency (ω) should be equal to natural undamped frequency (ω_n).

To calculate natural undamped frequency, we are assuming block A is fixed.

$$\text{Total kinetic energy } (E_k) = \frac{1}{2} M \dot{x}^2$$

$$\text{Total potential energy } (E_p) = \frac{1}{2} \times 2k \times x^2 + \frac{1}{2} kx(2x)^2$$

$$= \frac{1}{2} \times 6kx^2$$

As per energy method:

$$\frac{d}{dt}(E_k + E_p) = 0$$

$$\frac{d}{dt}\left(\frac{1}{2} M \dot{x}^2 + \frac{1}{2} \times 6kx^2\right) = 0$$

$$\frac{1}{2} M 2\dot{x}\ddot{x} + \frac{1}{2} \times 6k \times 2kx\dot{x} = 0$$

$$M\ddot{x} + 6kx = 0$$

$$\text{Natural frequency of system } \omega_n = \sqrt{\frac{6k}{M}} = \sqrt{\frac{6 \times 1500}{10}} = 30 \text{ rad/s}$$

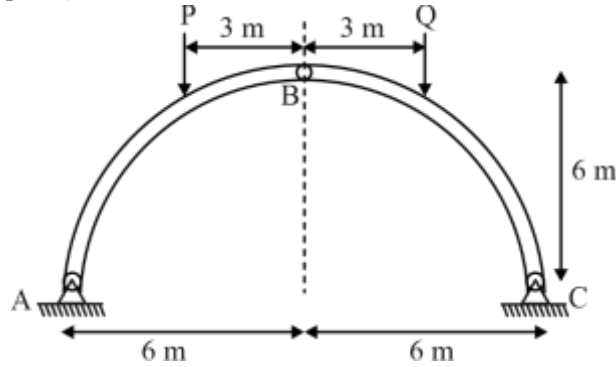
Since,

$$\omega = \omega_n = 30 \text{ rad/s}$$

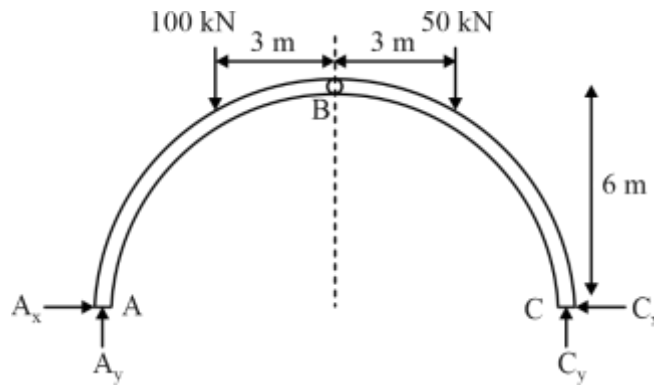


[NAT-2]

Q.9. A three-hinge arch ABC in the form of a semi-circle is shown in the figure. The arch is in static equilibrium under vertical loads $P = 100 \text{ kN}$ and $Q = 50 \text{ kN}$. Neglect friction at all the hinges. The magnitude of horizontal reaction at B is _____ kN (Round of one decimal place)

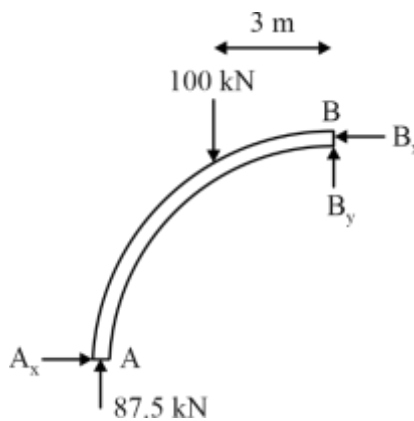


Sol. (37.5)



$$\begin{aligned} \sum M_C &= 0 \\ A_y \times 12 &= 100 \times 9 + 50 \times 3 \\ A_y &= 87.5 \text{ kN} \end{aligned}$$

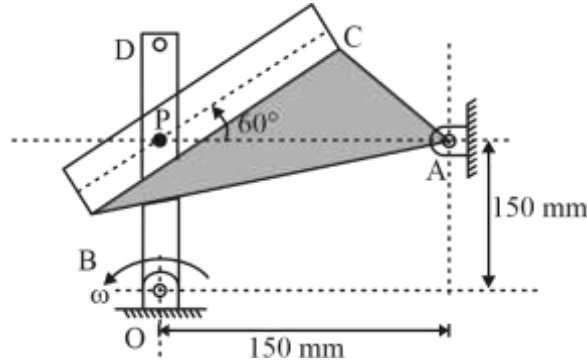
FBD of member AB



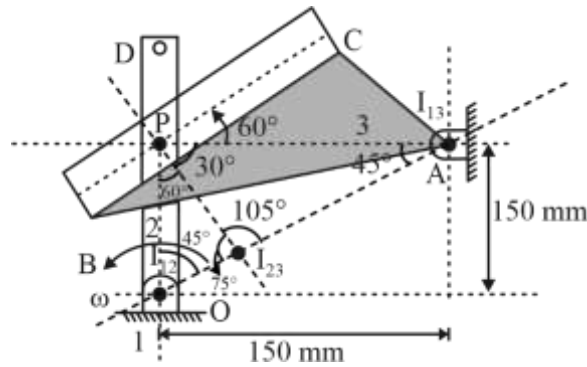
$$\begin{aligned} \sum M_B &= 0 \\ A_x \times 6 &= 87.5 \times 6 - 100 \times 3 \\ A_x &= 37.5 \text{ kN} \\ \sum F_x &= 0 \\ B_x &= A_x = 37.5 \text{ kN} \end{aligned}$$

[NAT-2]

Q.10. At the instant when OP is vertical and AP is horizontal, the link OD is rotating counter clockwise at a constant rate $\omega = 7$ rad/s. Pin P on link OD slides in the slots BC of link ABC which is hinged at A, and causes a clockwise rotation of the link ABC. The magnitude of angular velocity of link ABC for this instant is _____ rad/s (Round of two decimal place)



Sol. (12.124)



$$v_{I_{23}} = \omega_2(I_{12} I_{23}) = \omega_3(I_{13} I_{23}) \quad \dots(1)$$

In $\Delta I_{12} P I_{23}$

$$\frac{I_{12} I_{23}}{\sin 60^\circ} = \frac{I_{12} P}{\sin 75^\circ}$$

$$\frac{I_{12} I_{23}}{\sin 60^\circ} = \frac{150}{\sin 75^\circ} \Rightarrow I_{12} I_{23} = 134.49 \text{ mm}$$

In $\Delta I_{13} P I_{23}$

$$\frac{I_{13} I_{23}}{\sin 30^\circ} = \frac{I_{13} P}{\sin 105^\circ}$$

$$\frac{I_{13} I_{23}}{\sin 30^\circ} = \frac{150}{\sin 105^\circ} \Rightarrow I_{13} I_{23} = 77.65 \text{ mm}$$

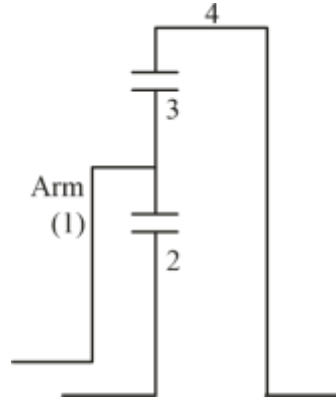
From equation (1)

$$\Rightarrow 7 \times 134.49 = \omega_3 \times 77.65$$

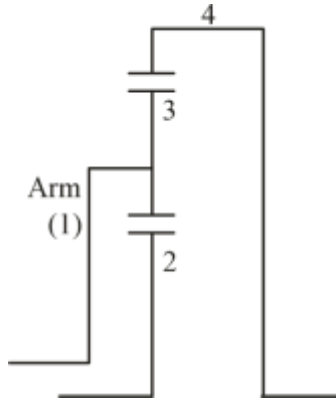
$$\Rightarrow \boxed{\omega_3 = 12.124 \text{ rad/s}}$$

[NAT-2]

Q.11. The Leval type-A train illustrated in the figure has gears with module $m = 8 \text{ mm/teeth}$. Gear 2 and 3 have 19 and 24 teeth respectively. Gear 2 is fixed and internal gear 4 rotates at 20 rev/min counter-clockwise. The magnitude of angular velocity of the arm is ____ rev/min (Round of to two decimal places)



Sol. (15.58)



$$m = 8 \text{ mm}$$

$$T_2 = 19, T_3 = 24, N_2 = 0, N_4 = 20 \text{ rpm CCW}$$

$$r_4 = r_2 + 2r_3$$

$$m \frac{T_4}{2} = \frac{mT_2}{2} + 2 \times \frac{mT_3}{2}$$

$$T_4 = T_2 + 2T_3 = 19 + 2 \times 24 = 67$$

By relative velocity method

$$\frac{N_2 - N_a}{N_4 - N_a} = \left(-\frac{T_3}{T_2} \right) \times \left(\frac{T_4}{T_3} \right) = -\frac{T_4}{T_2}$$

{Considering CCW as +ve}

$$\frac{0 - N_a}{20 - N_a} = -\frac{67}{19} = -3.526$$

$$\frac{N_a}{20 - N_a} = 3.526$$

$$N_a = 70.52 - 3.526 N_a$$



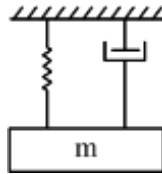
$$N_a = \frac{70.52}{4.526} = 15.58 \text{ rpm}$$

[MCQ-1]

Q.12. A linear spring mass-dashpot system with a mass of 2 kg is set in motion with viscous damping. If the natural frequency is 15 Hz, and the amplitude of two successive cycles measured are 7.75 mm and 7.20 mm, the coefficient of viscous damping in (N.s/m) is

- (a) 7.51
- (b) 4.41
- (c) 2.52
- (d) 6.11

Sol. (b)



$$m = 2 \text{ kg}$$

$$f_n = 15 \text{ Hz}$$

Amplitude of two successive cycles

$$x_n = 7.75 \text{ mm} \quad x_{n+1} = 7.20 \text{ mm}$$

$$\delta = \ln\left(\frac{x_n}{x_{n+1}}\right) = \ln\left(\frac{7.75}{7.20}\right) = 0.0736$$

As we know

$$\delta = \frac{2\pi\zeta}{\sqrt{1-\zeta^2}}$$

$$\Rightarrow 0.0736 = \frac{2\pi\zeta}{\sqrt{1-\zeta^2}}$$

$$\Rightarrow \zeta = 0.0117$$

Critical damping coefficient

$$C = \zeta \times 2m\omega_n = \zeta \times 2m \times 2f_n$$

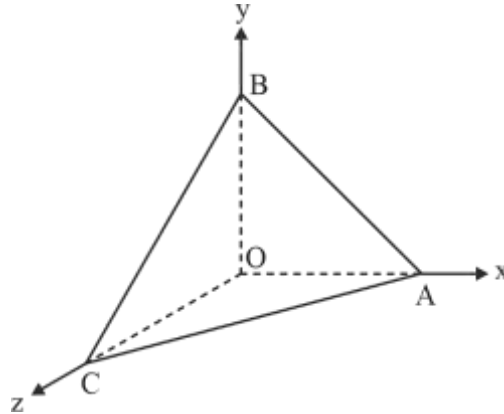
$$= 0.0117 \times 2 \times 2 \times 2 \times 15$$

$$= 4.41 \text{ N.s/m}$$



[MCQ-1]

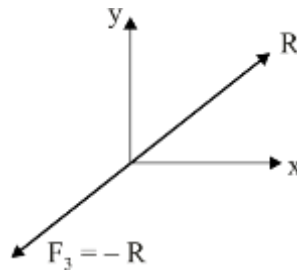
Q.13. A rigid massless tetrahedron is placed such that vertex O is at the origin and the other three vertices A, B and C lie on the co-ordinate axes as shown in the figure. The body is acted on by three point loads, of which one is acting at A along x-axis and other at point B along y-axis. for the body to be in equilibrium, the third point load acting at point O must be



- (a) in z-x plane but not along z or x axis
- (b) in x-y plane but not along x or y axis
- (c) in y-z plane but not along y or z axis
- (d) along z axis

Sol. (b)

Three forces may be in equilibrium only when the three forces are coplanar and either parallel or concurrent.



Here, the resultant of the given two forces is in the x-y plane, so the third force will be in the x-y plane but not along the x and y axes.



$$\Delta E = 300 \text{ J}$$

$$\omega_{\max} = 280 \text{ rad/s}, \omega_{\min} = 220 \text{ rad/s} \rightarrow \text{Without mounting flywheel}$$

$$K_s = 0.05$$

I_s = Inertia of shaft

I_f = Inertia of flywheel to be mounted to ensure $K_s = 0.05$

$I = I_s + I_f$ = total inertia after mounting flywheel

Before mounting flywheel

$$\Delta E = \frac{1}{2} I_s (\omega_{\max}^2 - \omega_{\min}^2)$$

$$\Rightarrow 300 = \frac{1}{2} I_s (280^2 - 220^2)$$

$$\Rightarrow I_s = 0.02 \text{ kgm}^2$$

After mounting flywheel

$$\Delta E = I \omega^2 K_s \quad \dots (i)$$

$$\omega = \frac{\omega_{\max} + \omega_{\min}}{2} = \frac{280 + 220}{2} = 250 \text{ rad/s}$$

From (i)

$$\Delta E = I \omega^2 K_s$$

$$\Rightarrow 300 = I \times 250^2 \times 0.05$$

$$\Rightarrow I_f + I_s = 0.096$$

$$\Rightarrow I_f + 0.02 = 0.096$$

$$\Rightarrow I_f = 0.076 \text{ kgm}^2$$

SECTION-2	Materials,	Manufacturing Engineering
	Manufacturing, and	Industrial Engineering
	Industrial Engineering	Material Science

[MCQ- 1]

- Q.16.** Allowance provided to a pattern for easy withdrawal from a sand mold
- | | |
|--------------------------|-------------------------|
| (a) Shrinkage allowance | (b) Shake allowance |
| (c) Distortion allowance | (d) Finishing allowance |

Sol. (b)

When the pattern is to be removed from a tightly packed or rammed sand mould, a slight shaking or rapping is required. This may result in a slight increase in the dimensions of the cavity. This results in an increase in the final dimensions of the casting. And to prevent this change in dimensions, the pattern is made smaller than the actual casting dimensions. This is shaking or rapping allowance in pattern.

[MCQ-1]

- Q.17.** Preparatory function in CNC machining programming denoted by alphabet
- | | |
|-------|-------|
| (a) G | (b) O |
| (c) M | (d) P |

Sol. (a)

Preparatory functions are the G-codes that identify the type of activities the machine will execute. A program block may contain one or more G-codes.

[MCQ-1]

- Q.18.** Grinding wheel used to provided best surface finish.
- | | |
|-------------|-------------|
| (a) A60 L5V | (b) A80 L5V |
| (c) A54 L5V | (d) A36 L5V |

Sol. (b)

As the number increased in second place grain become finer

(10 to 24 toughening)

(70 to 180- finishing)

(220 to 600 super finishing)

[MCQ-1]

- Q.19.** Earing phenomenon in metal forming
- | | |
|---------------|------------------|
| (a) Extrusion | (b) Deep drawing |
|---------------|------------------|



(c) Forging

(d) Rolling

Sol. (b)
[NAT-2]

Q.20. For machining single point cutting tool has 60 min tool life with 60 m/min cutting speed, for the same tool and same material it provide a tool life = 10 min, cutting speed is 100 m/min. for the same cutting condition determine the tool life (min) of cutting tool having 80 m/min cutting speed.

Sol. (21.86)

$$V_1 T_1^n = V_2 T_2^n = V_3 T_3^n$$

$$(60)(60)^n = (100)(10)^n$$

$$\Rightarrow n = 0.285$$

$$V_1 T_1^n = V_3 T_3^n$$

$$(80)(T)^{0.285} = (60)(60)^{0.285}$$

$$T = 21.86 \text{ mint}$$

[NAT-2]

Q.21. A cubical casting size of 20 mm × 20 mm × 20 mm is cast under identical condition a another sphere casting has the diameter of 20 mm find the ratio of solidification time of cube to sphere casting. (in integer)

Sol. (1)

$$\frac{\tau_{\text{cube}}}{\tau_{\text{sphere}}} = \frac{\left(\frac{A}{6}\right)^2}{\left(\frac{D}{6}\right)^2} = \frac{\left(\frac{20}{6}\right)^2}{\left(\frac{20}{6}\right)^2} = 1$$

[NAT-2]
Q.22.

Supply

11	16	19	13	300
5	10	7	8	300
12	14	17	11	300
8	15	11	9	300

Demand 300 300 300 300

Manager wishes to minimize the total cost of transportation. The optimal cost of satisfying the total demand.

Sol. (12300)

Supply

11	16	19	13	300
5	10	7	8	300
12	14	17	11	300
8	15	11	9	300

Demand 300 300 300 300

6	6	12	5	=	1	1	7	0	=	0	0	6	0
0	0	0	0		0	0	0	0		0	0	0	1
7	4	10	3		4	1	7	0		3	0	6	0
3	5	4	1		2	4	3	0		1	3	2	0

Total cost $(11 + 7 + 14 + 9) \times 300 = \text{Rs. } 12300$

[MCQ-1]

Q.23.

	U	V	W	X	Y	Z
W51	5	7	3	4	6	8
W52	4	6	6	8	5	7

Sequence total make span of production is to be minimized.

- (a) W X Z V Y U
- (b) Y X Z V W U
- (c) W X Z V U Y
- (d) W V Y Z V U

Sol. (a)

Sequence is

W X Z V Y U

[MCQ-1]

Q.24. Arrival rate of a job 5 jobs/hr follow the Poisson distribution and service rate follow the exponentially distributed with a mean of 6 min determined the probability that the server is not busy at any point in time.

- (a) 0.17
- (b) 0.83
- (c) 0.5
- (d) 0.2

Sol. (c)

$\lambda = 5/\text{hr}$

$\mu = 10/\text{hr (6 min)}$

$\rho = \frac{\lambda}{\mu} = \frac{5}{10} = 0.5$



If server is not busy $P_o = 1 - 0.5 = 0.5$

[MCQ-1]

- Q.25.** Phase present in pearlite
- Ferrite and Cementite
 - Ferrite and austenite
 - Martensite and Ferrite
 - Martensite and Cementite

Sol. (a)

Pearlite has a mixer of Ferrite and Cementite

[MCQ-2]

- Q.26.** Demand = 8000 gear/year, Ordering cost (C_o) = ₹ 300, Holding cost (C_h) = ₹ 12/month/gear
The company uses an order size that is 25% more than the optimal order quantity determined by EOQ model. Find the percentage change of total cost
- 5.02%
 - 2.50%
 - 7%
 - 12%

Sol. (b)

$$Q = \sqrt{\frac{2DC_o}{C_h}} = \sqrt{\frac{2 \times 8000 \times 300}{12 \times 12}} = 182.5$$

$$\begin{aligned} \text{Total cost for } Q &= \sqrt{2DC_oC_h} = \sqrt{2 \times 8000 \times 300 \times 12 \times 12} \\ &= 26290.06 \end{aligned}$$

Now the optimal order quantity is increased by 1.25 times

$$Q_1 = 1.25 \times Q = 1.25 \times 182 = 228.21 \text{ order}$$

$$\text{Total cost for } Q_1 = \frac{D}{Q_1} C_o + \frac{Q_1}{2} C_h$$

$$= \frac{8000}{228.21} \times 300 + \frac{228.21}{2} \times 12 \times 12$$

$$= 29947.74$$

$$\text{Percentage change of total cost} = \frac{49947.74 - 26290.6}{26290.6} = 0.0249 = 2.5\%$$

[NAT-2]

- Q.27.** In arc welding voltage 30V and current 200A is supplied. The weld bead cross sectional area is 20 mm² and welding speed is 5 mm/s, heat rate to melt is 20 J/s (the unit of heat



rate to melt is wrong, we used J/mm^3) determine percentage of heat loss to surrounding. (Correct upto two decimal places)

Sol. (66.67)

$$\text{Power} = V \times I = 30 \times 200 = 6000 \text{ W}$$

$$\text{Heat required for melting} = A \times v \times \text{heat rate to melt} = 20 \times 5 \times 20 = 2000 \text{ W}$$

$$\text{Percentage of heat generate} = \frac{\text{heat required for melting}}{\text{Power}} = \frac{2000}{6000} \times 100 = 33.33\%$$

$$\text{Percentage of heat lost to surrounding} = 100 - 33.33 = 66.67\%$$

[NAT-2] Data is Insufficient

Q.28. Bfs v_0 ($v_0 \in R^5$)

LPP

$$\text{Minimize } Z = -x_1 - 2x_2$$

$$\text{Statement } x_3 = 2 + 2x_1 - x_2$$

$$x_4 = 7 + x_1 - 2x_2$$

$$x_5 = 3 - x_1$$

$$x_1, x_2, x_3, x_4, x_5 \geq 0$$

Then the objective function is written as a function of non-basic variables. If the function move the one step (bfs) v_1 ($v_1 \in R^5$) then the optimum linear programming function

(a) $Z = -6 - 5x_1 + 2x_3$

(b) $Z = -4 - 5x_1 + 2x_4$

(c) $Z = -4 - 5x_1 + 2x_3$

(d) $Z = -3 + x_5 - 2x_2$

Sol. (Data is Insufficient)

[NAT-2]

Q.29. Blanking operation C20 steel sheet, $D = 20$ mm, $t = 2$ mm, allowance = 0.04 is provided. Punch size _____ mm.

Sol. (Data is Insufficient)

$$\text{For blanking operation punch size} = \text{die size} - 2 \text{ clearance} - \text{allowance}$$

$$= 20 - 2C - 0.04$$

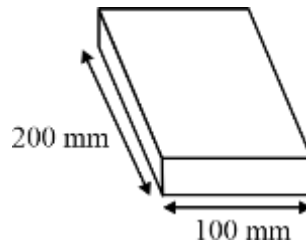
Since clearance is not given will not get exact answer.

[NAT-2]

Q.30. A work piece has length = 100 mm and width = 200 mm is produced by HSS slab mill cut has the teeth on the cutter = 8 and diameter of cutter $D = 100$ mm, width of cutter $W = 200$ mm, Feed per tooth = 0.1 mm, cutting speed $v = 20$ m/min, depth of cut $d = 2$ mm, the machining time required to remove entire stock _____ min. (Correct upto two decimal places)

Sol. (2.23)





$$Z = 8, D = 100 \text{ mm}, W = 100 \text{ mm}, f_z = 0.1 \text{ mm/tooth}$$

$$A = \sqrt{d(D-d)} = \sqrt{2(100-2)} = 14 \text{ mm}$$

$$V = \frac{\pi DN}{1000} = 20 \text{ m/min}$$

$$\frac{\pi(100)N}{1000} = 20 \Rightarrow N = 63.661 \text{ rpm}$$

$$F = f_z z N \frac{\text{mm}}{\text{min}}$$

$$= (0.1) (8) (63.661) \text{ mm/min} = 50.929 \text{ mm/min}$$

$$T_m = \frac{100 + A}{F} = \frac{100 + 14}{50.929} = 2.238 \text{ min}$$

[MCQ-1]

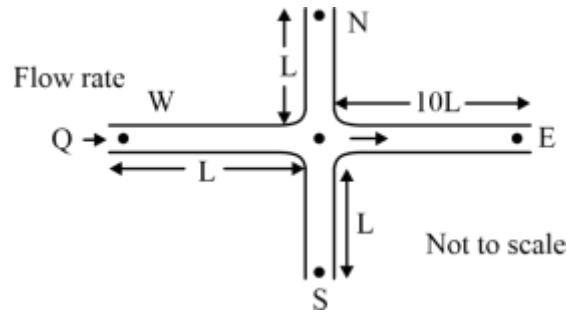
Q.31. Most suitable electrode material used for joining low alloy steel using GMAW.

- (a) Tungsten
- (b) Copper
- (c) Low alloy steel
- (d) Cadmium

Sol. (c)

We used electrode same as parent material in gas metal arc welding because electrode is consumed.





Which one of the following options is correct?

- (a) Both statements I & II are true
- (b) Both statements I & II are false
- (c) Statement I is true & II is false
- (d) Statement I is false & II is true

Sol.

(d)
 $Q_1 + Q_2 + Q_3 = Q$ (i)

$Q_1 = Q_2$ (ii)

From equation (i) and (ii)

$2Q_1 + Q_3 = Q$ (A)

$P_J - P_N = P_J - P_E$

Now $\frac{P_J - P_N}{\rho g} = \frac{P_J - P_E}{\rho g}$

$(h_L)_{JN} = (h_L)_{JE}$

$\frac{fL_{JN}Q_1^2}{12.1 D^5} = \frac{fL_{JE}Q_3^2}{12.1 D^5}$

$LQ_1^2 = 10LQ_3^2$

$Q_1 = \sqrt{10}Q_3$

From equation (A)

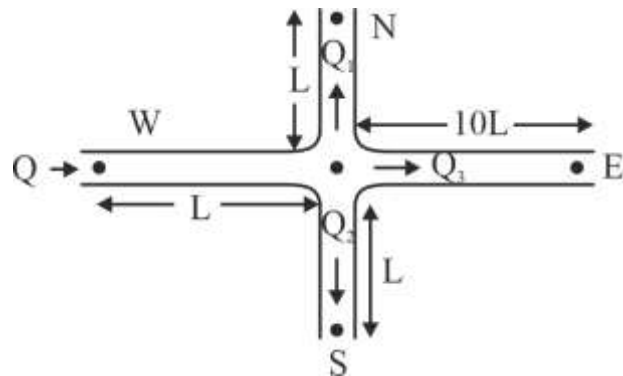
$2Q_1 + Q_3 = Q$

$2\sqrt{10}Q_3 + Q_3 = Q$

$Q_3 = \frac{Q}{1 + 2\sqrt{10}}$

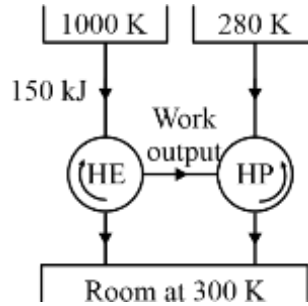
$Q_3 = 0.136 Q$

So statement 1 is wrong and statement 2 is correct.



[MCQ-2]

Q.34. A Heat Pump is driven by the work output of the heat engine (H.E) as shown in the figure. The heat engine extracts 150 kJ of heat from the source of 1000 K. The heat pump absorbs heat from the ambient at 280 K and delivers heat to the room which is maintained at 300K. Considering the combined system to be ideal, the total amount of heat delivered to the room together by the heat engine and heat pump is _____ in kJ
[Put Answer in integer]



Sol. (1620)

$$Q_1 = 150 \text{ kJ}$$

$$Q_2 + Q_4 = ?$$

For Heat engine

$$\eta = 1 - \frac{T_L}{T_H}$$

$$\eta = 1 - \frac{300}{1000} = 1 - \frac{Q_2}{Q_1}$$

$$Q_2 = 0.3Q_1$$

$$Q_2 = 45 \text{ kJ}$$

$$W_{\text{net}} = Q_1 - Q_2$$

$$W_{\text{net}} = (150 - 45) \text{ kJ} = 105 \text{ kJ}$$

COP of heat pump is given by

$$(\text{COP})_{\text{HP}} = \frac{Q_4}{W_{\text{net}}} = \frac{T_H}{T_H - T_L} = \frac{300}{300 - 280}$$

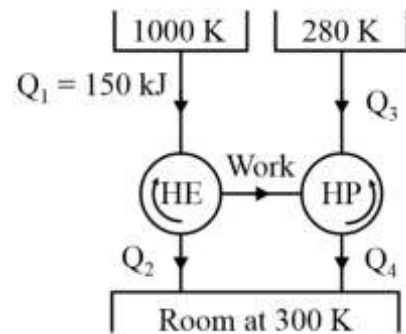
$$\frac{Q_4}{105} = 15$$

$$Q_4 = 1575 \text{ kJ}$$

Now we can write total heat provided into the room

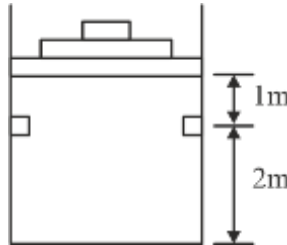
$$Q_2 + Q_4 = (45 + 1575) \text{ kJ}$$

$$\boxed{Q_2 + Q_4 = 1620 \text{ kJ}}$$



[NAT-2]

Q.35. A piston–cylinder arrangement as shown in the figure has stopper located 2m above the base. The cylinder initially contains air at 140 kPa and 350°C and the piston is resting in equilibrium at a position which is 1 m above the stops. The system is now cooled to the ambient temperature of 25°C. Consider air to be an ideal gas with a value of gas constant $R = 0.287 \text{ kJ/kg.K}$. The absolute value of specific work done duration process as ____ kJ/kg [Round off to the one decimal places]



Sol. (59.60)

$$P_1 = 140 \text{ kPa}$$

$$T_1 = 350^\circ\text{C}$$

$$T_3 = 25^\circ\text{C}$$

Total work done

$$W_{1-3} = W_{1-2} + W_{2-3}$$

For constant pressure process (1–2)

$$W_{1-2} = P_1 (V_2 - V_1)$$

Or

$$W_{1-2} = P_1 V_2 - P_1 V_1$$

$$\therefore P_1 = P_2$$

$$W_{1-2} = P_2 V_2 - P_1 V_1$$

or

$$W_{1-2} = mR (T_2 - T_1)$$

For constant pressure process

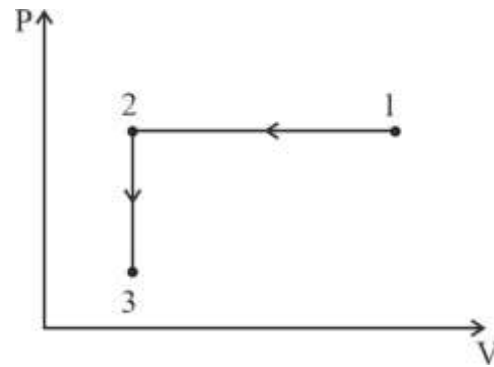
$$\frac{T_2}{T_1} = \frac{V_2}{V_1}$$

or

$$\frac{T_2}{T_1} = \frac{A \cdot L_2}{A \cdot L_1} \quad (A = \text{cross sectional area})$$

$$\frac{T_2}{(350 + 273)} = \frac{A \cdot (2)}{A \cdot (3)}$$

$$T_2 = 415.33 \text{ K}$$



Specific work done

$$W_{1-2} = 1 \times 0.287 (415.33 - 623)$$

$$W_{1-2} = -59.60 \text{ kJ}$$

For process 2-3

$$W_{2-3} = 0 \quad (\text{constant volume process})$$

Total work done

$$\boxed{W_{1-3} = -59.60 \text{ kJ}}$$

[NAT-2]

Q.36. A liquid fills a horizontal capillary tube whose one end is dipped in a large pool of the liquid. Experiment shows that the distance L travelled by the liquid meniscus inside the capillary in time t is given by

$$L = k.\gamma^a .R^b \mu^c .\sqrt{t}$$

Where γ is the surface tension. R is the inner radius of the capillary and μ is the dynamic viscosity of the liquid. If k is a dimensionless constant, the exponent a is ____ *[Round off to the one decimal places]*

Sol (0.5)

Given

$$L = k.\gamma^a .R^b \mu^c .\sqrt{t}$$

$$\Rightarrow [L] = [MT^{-2}]^a .[L]^b .[ML^{-1}T^{-1}]^c .[T]^{1/2}$$

$$\Rightarrow [L] = [M^{a+c}L^{b-c}T^{-2a-c+\frac{1}{2}}]$$

$$\Rightarrow a + c = 0 \Rightarrow c = -a$$

$$b - c = 0 \Rightarrow c = b$$

$$-2a - c + \frac{1}{2} = 0 \Rightarrow -2a - (-a) + \frac{1}{2} = 0$$

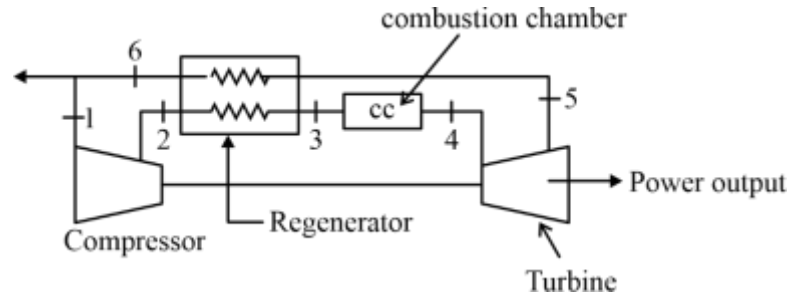
$$\Rightarrow -a + \frac{1}{2} = 0$$

$$\boxed{\Rightarrow a = 0.5}$$



[NAT-2]

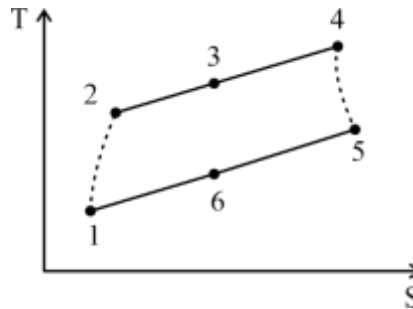
Q.37. Consider an air standard Brayton cycle with adiabatic compressor and turbine and a regenerator as shown in figure. Air enters the compressor at 100 kPa and 300 K and exit the compressor of 600 kPa and 550 K. The air exits and combustion chamber at 1250 K and exits the adiabatic turbine at 100 kPa and 800 k. The exhaust air from the turbine is used to preheat the air in the regenerator. The exhaust air exits the regenerator (state 6) at 600 K. There is no pressure drop across the generator and the combustion chamber. Also, there is no heat loss from the regenerator to the surroundings. The ratio of specific heats at constant pressure and volume is 1.4 . The thermal efficiency of the cycle is _____% [Put answer in integer]



Sol. (40)

Given

$$T_1 = 300 \text{ K}; T_2 = 550 \text{ K}; T_4 = 1250 \text{ K}; T_5 = 800 \text{ K}; T_6 = 600 \text{ K}$$



$$\eta_{th} = \frac{w_{net}}{w_{net} + q_R} = \frac{c_p(T_4 - T_5) - c_p(T_2 - T_1)}{c_p(T_4 - T_5) - c_p(T_2 - T_1) + c_p(T_6 - T_1)}$$

$$= \frac{(450 - 250)}{(450 - 250 + 300)} = \frac{2}{5}$$

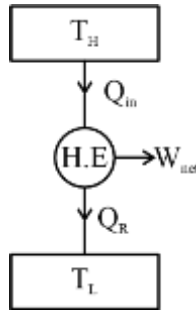
$$\therefore \eta_{th} = 0.4 = 40\%$$

[MCQ-1]

Q.38. A furnace can supply heat steadily at 1200 K at a rate of 24000 kJ/min. The maximum amount of power (in kW) that can be produced by using the heat supplied by this furnace in an environment at 300 K is

- (a) 300
- (b) 18000
- (c) 150
- (d) 0

Sol. (a)



Given

$$T_H = 1200 \text{ K}$$

$$T_L = 300 \text{ K}$$

$$Q_{in} = 24000 \text{ kJ/min or } 400 \text{ kW}$$

For Carnot heat engine we can write

$$\eta = 1 - \frac{T_L}{T_H} = \frac{W_{net}}{Q_{in}}$$

$$1 - \frac{300}{1200} = \frac{W_{net}}{400}$$

$$W_{net} = 300 \text{ kW}$$

[MCQ-1]

Q.39. Which one of the following statements regarding Rankine cycle is false?

- (a) Superheating the steam in the boiler increases the cycle efficiency
- (b) Cycle efficiency increases as boiler pressure decreases
- (c) Cycle efficiency increases as condenser pressure decreases
- (d) The pressure at the turbine outlet depends on the condenser temperature

Sol. (b)

a – super heating increases the mean temperature of heat addition → correct statement

b – cycle efficiency increases as pressure increases → wrong statement

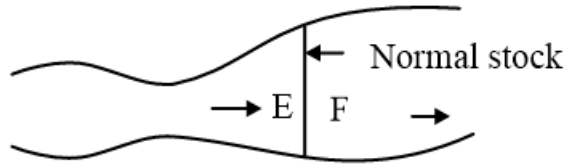
c – efficiency increases as condenser temperature decreases as mean temperature of heat rejection decreases → correct statement

d – pressure of condenser is function of condenser temperature → correct statement

[MSQ-2]

Q.40. Steady compressible flow of air takes place through an adiabatic converging–diverging nozzle as shown in the figure. For a particular value of pressure differences across the nozzle, a stationary normal shockwave forms in the diverging section of the nozzle. If E & F denote the flow conditions just upstream & downstream of the normal shock respectively, which of the following statements is/are True?





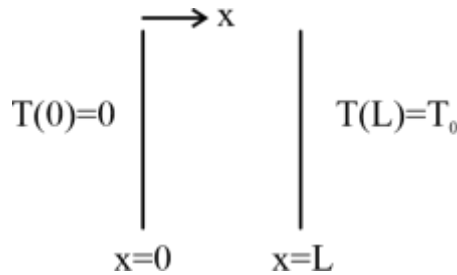
- (a) Density of E is lower than the density at F
- (b) Static pressure at E is lower than its static processes at F
- (c) Mach number at E is lower than the Mack number at F
- (d) specific gravity at E is lower than specific gravity at F.

Sol. (a, b, d)

- a) $\rho_E < \rho_F \rightarrow$ correct statement
- b) $P_E > P_F \rightarrow$ correct statement
- c) $M_E < M_F \rightarrow$ wrong as velocity decreases across shock wave \rightarrow wrong statement
- d) Specific entropy increases across shock wave \rightarrow correct statement as shock wave is irreversible phenomena and entropy increases across it.

[MCQ-1]

Q.41. A plane, solid slab of thickness L , shown in figure, has thermal conductivity k that varies with the spatial coordinate x as $k = A + Bx$ where A and B are positive constant ($A > 0, B > 0$). The slab walls are maintained at fixed temperature of $T(x = 0) = 0$ and $T(x = L) = T_0 > 0$. The slab has no internal heat sources. Consider 1-D heat transfer, which one of the following plots qualitatively depicts the steady state temperature distribution within slab



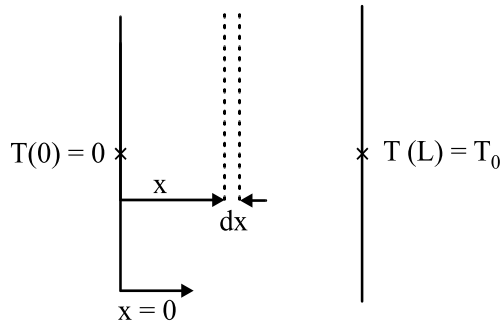
- (a)

(b)

(c)

(d)

Sol. (c)



$$T(0) > T(L)$$

$$k = A + Bx; (A, B > 0)$$

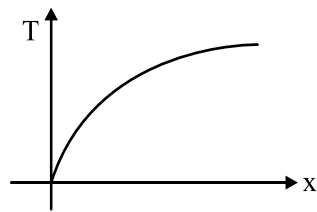
From Fourier's law of heat conduction,

$$\dot{Q} = -kA \frac{dT}{dx}$$

$$\frac{\dot{Q}}{A} = -k \frac{dT}{dx}$$

$$k \cdot \frac{dT}{dx} = \text{constant}$$

$$\text{As } x \uparrow, k \uparrow \frac{dT}{dx} \downarrow$$



[MCQ-1]

Q.42. Consider incompressible laminar flow over of flat plate with free stream velocity of U_∞ . The Nusselt number corresponding to this flow velocity is Nu_1 . If the free stream velocity is doubled, the Nusselt number changes to Nu_2 . Choose correct option for Nu_2/Nu_1

- (a) 1
- (b) 1.26
- (c) $\sqrt{2}$
- (d) 2

Sol. (c)

For laminar flow over flat plate

$$Nu = C Re_L^{1/2} Pr^{1/3}$$

$$Nu \propto U_\infty^{1/2}$$

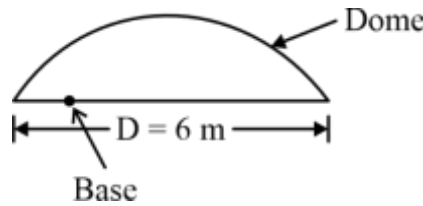
$$\Rightarrow \frac{Nu_1}{Nu_2} = \left(\frac{u_{\infty,1}}{2U_{\infty,1}} \right)^{\frac{1}{2}}$$

$$\Rightarrow \frac{Nu_2}{Nu_1} = \sqrt{2}$$

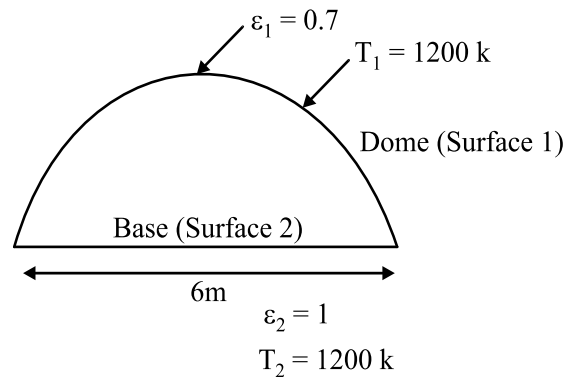
[NAT-2]

Q.43. Consider a hemispherical furnace of diameter $D = 6\text{ m}$ with flats base. The dome of the furnace has an emissivity of 0.7 and flat base is a blackbody. The base and the dome are maintained at uniform temperature of 300 k and 1200 k respectively under steady state condition. The rate of radiation heat transfer from the dome to the base is _____ kW [Round off to the nearest integer]

Use Stefan Boltzmann constant $= 5.67 \times 10^{-8}\text{ W/m}^2\text{K}^4$



Sol. (2727)



Hemispherical furnace

figure

Surface

$$F_{22} = 0, F_{21} = 1$$

From reciprocity Theorem.

$$A_1 F_{12} = A_2 F_{21}$$

$$F_{12} = \frac{A_2}{A_1} = \frac{\pi R^2}{2\pi R^2} = 0.5$$

$$\dot{Q}_{Net} = A_1 (Fg)_{12} \sigma_b [T_1^4 - T_2^4]$$

$$(Fg)_{12} = \frac{1}{\frac{1-\epsilon_1}{\epsilon_1} + \frac{1}{F_{12}} + \frac{1-\epsilon_2}{\epsilon_2} \frac{A_1}{A_2}}$$

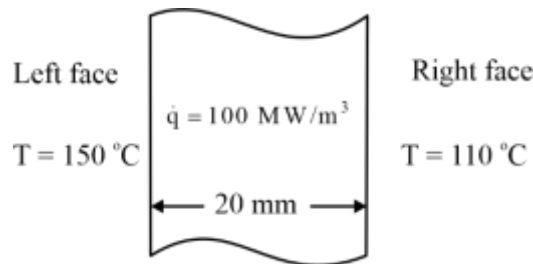
$$(Fg)_{12} = \frac{1}{\frac{1-0.7}{0.7} + \frac{1}{0.5}}$$

$$(Fg)_{12} = 0.41176$$

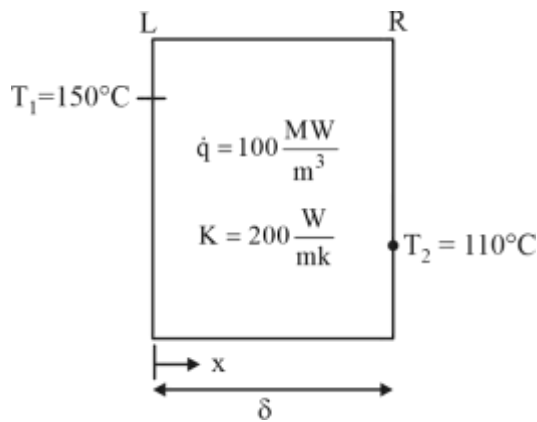
$$\begin{aligned} \dot{Q}_{Net} &= 2\pi(3)^2 \times 0.41176 \times 5.67 \times 10^{-8} \times [1200^4 - 300^4] \\ &= 2726.935 \text{ kW} \end{aligned}$$

[NAT-2]

Q.44. Consider a slab of 20 mm thickness. There is a uniform heat generation of $\dot{q} = 100 \text{ MW/m}^3$ inside the slab. The left and Right face of the slab are maintained at 150°C and 110°C respectively. The plates have constant thermal conductivity (k) of 200 W/m.K . Considering one dimensional steady state heat conduction. The location of the maximum temperature from left face (in mm) *[Put Answer in integer]*



Sol. (6 mm)



From GHCE

$$\frac{d^2T}{dx^2} + \frac{\dot{q}_g}{k} = 0$$

$$T(x) = T_1 + (T_2 - T_1) \frac{x}{\delta} + \frac{\dot{q}_g}{2k} [\delta x - x^2]$$

To get location of maximum temperature

$$\frac{dT}{dx} = 0$$

$$(T_2 - T_1) \frac{1}{\delta} + \frac{\dot{q}_g}{2k} [\delta - 2x] = 0$$



$$(110 - 150) \times \frac{1}{0.02} + \frac{100 \times 10^6}{2 \times 200} [0.02 - 2x] = 0$$

$$-2000 + \frac{100 \times 10^6}{400} [0.02 - 2x] = 0$$

$$x = 6 \text{ mm}$$

[NAT-2]

Q.45. A condenser is used as a Heat Exchanger in a large steam power plant in which steam is converted to liquid water. The condenser is a shell and tube heat exchanger which consists of 1 shell and 20,000 tubes. Water flows through each of the tubes at a rate of 1 kg/s with an inlet temperature of 30°C. The steam in the condenser shell condenses at the rate of 430 kg/s at a temperature of 50°C. If the heat of vaporization is 2.326 MJ/kg and the specific heat of water is 4 kJ/kg.K, the effectiveness of the heat exchanger is _____. (Round off to 3 decimal places)

Sol. (0.625)

Given data:

Shell and tube Heat exchanger (1 shell + 20000 tubes)

$$\dot{m}_w / \text{tube} = 1 \text{ kg/s},$$

$$\text{Total } \dot{m}_w = 20000 \text{ kg/s}$$

$$\dot{m}_s = 430 \text{ kg/s (mass of steam condensed)}$$

$$\text{Latent Heat (LH)} = 2.326 \text{ MJ/kg}$$

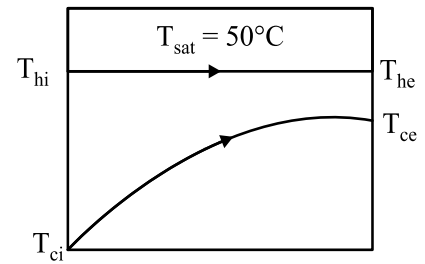
Effectiveness (ϵ)

$$\epsilon = \frac{\dot{Q}_{\text{act}}}{\dot{Q}_{\text{max}}}$$

$$\epsilon = \frac{\dot{m}_s \times \text{LH}}{(mc)_{\text{min}} [T_{\text{hi}} - T_{\text{ci}}]}$$

$$\epsilon = \frac{430 \times 2.326 \times 10^3}{20000 \times 1 \times 4 \times [50 - 30]}$$

$$\boxed{\epsilon = 0.625}$$



SECTION-4	General Aptitude and Engineering Mathematics	General Aptitude
		Engineering Mathematics

[MCQ-2]

Q.46. The matrix $\begin{bmatrix} 1 & \alpha \\ 8 & 3 \end{bmatrix}$ where $(\alpha > 0)$ has a negative eigen value if α is greater than

- | | |
|-------------------|-------------------|
| (a) $\frac{1}{8}$ | (b) $\frac{1}{4}$ |
| (c) $\frac{3}{8}$ | (d) $\frac{1}{5}$ |

Sol. (c)

Since trace of the given Matrix is positive, both the eigen values can not be negative.

For one of the eigen value to be negative, $\lambda_1 \cdot \lambda_2 < 0$

$$\text{Product of eigen values} = \text{determinant of matrix} \Rightarrow 3 - 8\alpha < 0 \Rightarrow \boxed{\alpha > \frac{3}{8}}$$

[NAT-2]

Q.47. Let x be a continuous random variable defined on $[0, 1]$ such that its probability density function $f(x) = 1$ for $0 \leq x \leq 1$ and 0 otherwise. Let $y = \log_e(x + 1)$, then the expected value y is ____ (Round of to two decimal places)

Sol. (0.39)

$$\text{For } y = \log_e(x+1)$$

$$\Rightarrow e^y = x+1 \Rightarrow x = 1 - e^y$$

Considering cumulative function for y .

$$P(Y \leq y) = P(Y \leq \log_e(x + 1))$$

$$= P(x \geq 1 - e^y) = 1 - P(x \leq 1 - e^y) = 1 - (1 - e^y) = e^y \text{ (since, 'x' is uniformly distributed)}$$

$$\therefore P(y) = \frac{d}{dy}(P(Y \leq y)) = e^y$$

$$\therefore \text{Mean of 'y' is } \int_0^{\ln 2} y \cdot e^y dy$$

$$= y \cdot e^y - e^y \Big|_0^{\ln 2}$$

$$= e^y (y - 1) \Big|_0^{\ln 2}$$

$$= 2(\ln 2 - 1) - (-1) = \ln 4 - 1 = 0.386$$



[NAT-2]

Q.48. If the value of double integral

$$x = \int_{x=3}^4 \int_{y=1}^2 \frac{dydx}{(x+y)^2} \text{ is } \log_e \left(\frac{a}{24} \right), \text{ then } a \text{ is } \underline{\hspace{2cm}} \text{ (Answer in integer)}$$

Sol. (25)

$$\begin{aligned} &= \int_{x=3}^{x=4} \left(\frac{-1}{x+y} \Big|_{y=1}^{y=2} \right) dx \\ &= \int_{x=3}^{x=4} \left(\frac{-1}{x+2} - \left(\frac{-1}{x+1} \right) \right) dx \\ &= \left\{ -\ln(x+2) + \ln(x+1) \right\} \Big|_3^4 \\ &= -\ln 6 + \ln 5 - \{ -\ln 5 + \ln 4 \} \\ &= \ln 25 - \ln 24 \\ &= \ln \left(\frac{25}{24} \right) \end{aligned}$$

Now comparing with $\ln \left(\frac{a}{24} \right)$

We get $\boxed{a = 25}$

[NAT-2]

Q.49. If $x(t)$ satisfies the differential equation $t \frac{dx}{dt} + (t-x) = 0$.

Subject to the condition $x(1) = 0$, then the value of $x(2)$ is _____ (Round of to two decimal places)

Sol. (-1.39)

Given

$$t \frac{dx}{dt} + (t-x) = 0 \qquad x(1) = 0 \qquad x(2) = ?$$

We can write

$$\Rightarrow t \cdot dx = (x-t)dt$$

$$\Rightarrow t \cdot dx - x \cdot dt = -t \cdot dt$$

Now dividing by t^2 on both sides

$$\frac{t \cdot dx - x \cdot dt}{t^2} = \frac{-t}{t^2} dt \Rightarrow d \left(\frac{x}{t} \right) = -\frac{1}{t} dt$$

Integrating both sides



$$\int d\left(\frac{x}{t}\right) = -\int \frac{1}{t} dt \Rightarrow \frac{x}{t} = -\ln t + C$$

Now putting the boundary condition

$$\frac{0}{1} = \ln 1 + C \Rightarrow C = 0$$

$$\therefore x = -t \cdot \ln t$$

$$\text{at } t = 2, x = -2 \cdot \ln 2$$

$$\Rightarrow \boxed{x = -\ln 4 = -1.386}$$

[MCQ-1]

Q.50. Let $f(\cdot)$ be a twice differentiable function from $\mathbb{R}^2 \rightarrow \mathbb{R}$. If $p, x_0 \in \mathbb{R}^2$.

Where $\|p\|$ is sufficiently small (here $\|\cdot\|$ is the Euclidean norm or distance function,

$$\text{then } f(x_0 + p) = f(x_0) + \nabla f(x_0)^T p + \frac{1}{2} p^T \nabla^2 f(\psi) p.$$

Where $\psi \in \mathbb{R}^2$ is a point on the line segment joining x_0 and $x_0 + p$.

If x_0 is a strict local minimum of $f(x)$, then which one of the following statements is true

- (a) $\nabla f(x_0)^T p > 0$ and $p^T \nabla^2 f(\psi) p = 0$
- (b) $\nabla f(x_0)^T p = 0$ and $p^T \nabla^2 f(\psi) p = 0$
- (c) $\nabla f(x_0)^T p = 0$ and $p^T \nabla^2 f(\psi) p > 0$
- (d) $\nabla f(x_0)^T p = 0$ and $p^T \nabla^2 f(\psi) p < 0$

Sol. (c)

For local minimum first derivative should be equal to 0 and second derivative should be greater than 0.

In given equation first derivative is should be zero and second derivative is positive.

Thus,

$$\nabla f(x_0)^T p = 0 \text{ and } p^T \nabla^2 f(\psi) p > 0$$

[MCQ-1]

Q.51. Let $f(z)$ be an analytic function, where $z = x + iy$. If the real part of $f(z)$ is $\cosh x \cos y$, and the imaginary part of $f(z)$ is zero for $y = 0$, then $f(z)$ is

- (a) $\cosh z$
- (b) $\cosh x \exp(-iy)$
- (c) $\cosh z \exp z$
- (d) $\cosh z \cos y$

Sol. (a)

Given

$$z = x + iy$$



$$\operatorname{Re}(f(z)) = \cosh x \cdot \cos y$$

$$\operatorname{Im}(f(z)) = 0 \text{ for } y = 0$$

$$\text{Assuming } f(z) = u + iv$$

where,

$$u = u(x, y) = \cosh x \cdot \cos y$$

$$v = v(x, y) = ?$$

Taking partial differentiation with respect to x

$$f'(z) = \frac{\partial u}{\partial x} + i \left(\frac{\partial v}{\partial x} \right)$$

$$\text{since for analytic function, } \frac{\partial v}{\partial x} = -\frac{\partial u}{\partial y}$$

$$f'(z) = \frac{\partial u}{\partial x} + i \left(-\frac{\partial u}{\partial y} \right)$$

$$f'(z) = \sinh x \cdot \cos y + i \cosh x \cdot \sin y$$

from Milne Thomson method, replace x by z and y by 0.

$$f'(z) = \sinh z$$

taking integration on both side we get

$$\int f'(z) \cdot dz = \int \sinh z \cdot dz$$

$$\boxed{f(z) = \cosh z}$$

[MCQ-1]

Q.52. The value of surface integral $\oiint_s z dx dy$ where s is the external surface of the sphere $x^2 + y^2 + z^2 = R^2$ is

- | | |
|---------------|--------------------------|
| (a) 0 | (b) $\frac{4\pi}{3} R^3$ |
| (c) πR^3 | (d) $4\pi R^3$ |

Sol. (b)

$$S: x^2 + y^2 + z^2 = R^2$$

$$\oiint_s z dx dy = \iiint_v dz \cdot dx \cdot dy$$

$$\text{Volume of sphere} = \frac{4}{3} \pi R^3$$



[MCQ]

Q.53. In order to numerically solve the ordinary differential equation $\frac{dy}{dt} = -y$ for $t > 0$ with an initial condition $y(0) = 1$, the following scheme is employed $\frac{y_{n+1} - y_n}{\Delta t} = -\frac{1}{2}(y_{n+1} + y_n)$, here Δt is the time step and $y_n = y(n\Delta t)$ for $n = 0, 1, 2, \dots$. This numerical scheme will yield a solution with non-physical oscillation for $\Delta t > h$. The value of h is

- | | |
|-------|-------------------|
| (a) 2 | (b) $\frac{3}{2}$ |
| (c) 1 | (d) $\frac{1}{2}$ |

Sol. (a)

$$\frac{dy}{dt} = -y \text{ and } y(0) = 1$$

$$\text{Given scheme is } \frac{y_{n+1} - y_n}{\Delta t} = -\frac{1}{2}(y_{n+1} + y_n)$$

$$\Rightarrow 2y_{n+1} - 2y_n = -\Delta t y_{n+1} - \Delta t y_n$$

$$\Rightarrow (2 + \Delta t)y_{n+1} = (2 - \Delta t)y_n$$

$$\Rightarrow y_{n+1} = \left(\frac{2 - \Delta t}{2 + \Delta t}\right)y_n$$

$$\text{For the scheme to yield physical oscillations } \left|\frac{2 - \Delta t}{2 + \Delta t}\right| < 1$$

$$\Rightarrow |2 - \Delta t| > |2 + \Delta t|$$

$$\text{On solving } |\Delta t| < 2$$

$$\therefore \text{ scheme yield no physical solution for } |\Delta t| > 2$$

[MCQ-1]

Q.54. $x + 2y + z = 5$

$$2x + ay + 4z = 12$$

$$2x + 4y + 6z = b$$

Values of a and b such that there exists a non-trivial null space and system admits infinite solution.

- | | |
|---------------------|---------------------|
| (a) $a = 4, b = 12$ | (b) $a = 8, b = 14$ |
| (c) $a = 4, b = 14$ | (d) $a = 8, b = 12$ |

Sol. (c)

Given equations are

$$x + 2y + z = 5$$



$$2x + ay + 4z = 12$$

$$2x + 4y + 6z = b$$

$$\Rightarrow \begin{bmatrix} 1 & 2 & 1 \\ 2 & a & 4 \\ 2 & 4 & 6 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 12 \\ b \\ b \end{bmatrix} \Rightarrow Ax = B$$

For the system to have non trivial null space

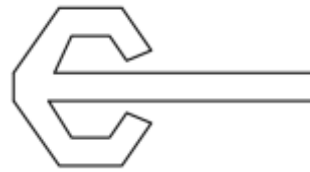
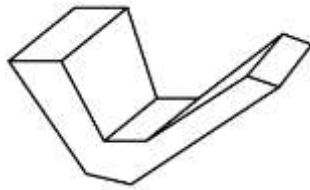
$$\rho(A) = \rho(A/B) < 3$$

$$\Rightarrow \begin{vmatrix} 1 & 2 & 1 \\ 2 & a & 4 \\ 2 & 4 & 6 \end{vmatrix} = 0 \& \begin{vmatrix} 1 & 1 & 5 \\ 2 & 4 & 12 \\ 2 & 6 & b \end{vmatrix} = 0$$

$$\Rightarrow a = 4 \text{ and } b = 14$$

[MCQ-1]

Q.55. A plan rectangular paper has two v-shaped pieces attracted as shown below. This piece of paper is folded to make the following closed. 3-D-object. The number of folds required to form the above object.



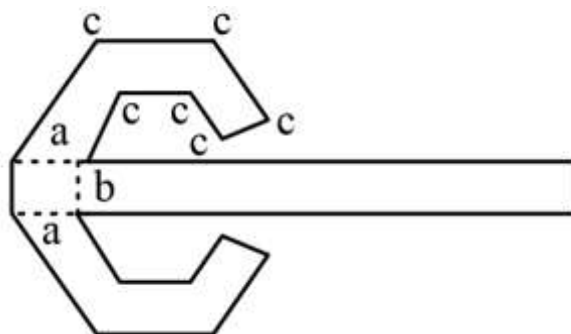
- (a) 11
- (c) 7

- (b) 9
- (d) 8

Sol. (b)

The plane rectangular paper when folded to form the 3-D shape the number of folding required is = 2 + 1 + 3 + 3 = 9

$$(a) + (b) + (c) + (d)$$



[MCQ-1]

Q.56. In the following series identify the number that need to be changed to form the Fibonacci series

1, 1, 2, 3, 6, 8, 13, 21

- (a) 6
- (b) 21
- (c) 8
- (d) 13

Sol. (a)

In the following series follow sum of first two terms gives the next term

$$1 + 1 = 2$$

$$1 + 2 = 3$$

$$3 + 2 = (6) \rightarrow 5$$

$$5 + 8 = 13$$

$$8 + 13 = 21$$

Here 6 is to be changed to form the Fibonacci series.

[MCQ-1]

Q.57. How many combinations of non-null sets A B C are possible from the subsets of (2, 3, & 6 satisfying (i) A is a subset of B and (ii) B is a subset of C.

- (a) 27
- (b) 28
- (c) 19
- (d) 18

Sol. (*)

Question can be challenged.

[MCQ-2]

Q.58. In the given text blanks are numbered (i – iv) select the best match for all the blanks.

Prof. P (i) merely a man with narrated fanny stores (ii) in his blackest moments. He was capable of self-deprecating humour.

Prof. Q (iii) a man who hardly narrated funny stories (iv) in his blackest moments was he able to find humour.

- (a) wasn't only was even
- (b) wasn't even was only
- (c) was even wasn't only
- (d) was only wasn't even

Sol. (a)

[MCQ-2]

Q.59. The real variables x, y, z and the real constant p, q, r is satisfies.

$$\frac{x}{pq - r^2} = \frac{y}{qr - p^2} = \frac{z}{rp - q^2}$$

Given the denominator are non-zero. The value of $px + qr + rz =$

- (a) pqr
- (b) 0



- (c) $p^2 + q^2 + r^2$ (d) 1

Sol. (b)

$$\frac{x}{pq - r^2} = \frac{y}{qr - p^2} = \frac{z}{rp - q^2} = k$$

$$\Rightarrow x = k(pq - r^2); y = k(qr - p^2); z = k(rp - q^2)$$

$$\therefore Px + qy + rz = k\{p(pq - r^2) + q(qr - p^2) + r(rp - q^2)\}$$

$$= k\{p^2q - pr^2 + q^2r - qp^2 + q^2p - rq^2\}$$

$$= k(0) = 0$$

$$\therefore px + qy + rz = 0$$

[MCQ-1]

Q.60. Find the odd one out in the set.

(19, 37, 21, 17, 23, 29, 31, 11)

- (a) 23 (b) 21
(c) 29 (d) 37

Sol. (b)

19, 37, 21, 17, 23, 29, 31, 11 are Prime Number

21 is Non-Prime or Composite.

[MCQ-1]

Q.61. If \rightarrow denotes increasing order of intensity, the meaning of two words.

[Smile \rightarrow giggles \rightarrow laugh] is analogous to [disapprove \rightarrow _____ \rightarrow chide] which one of the given options is appropriate to fill the blank.

- (a) response (b) grieve
(c) reprove (d) praise

Sol. (c)

Smile \rightarrow giggles \rightarrow laughs similarly disapprove \rightarrow reprove \rightarrow chide.

[MCQ-2]

Q.62. Take Two long disc (rectangular parallel piped) each having four rectangular faces

labelled as 2, 3, 5, 7. If thrown the long dice cannot land on the square face and has $\frac{1}{4}$

probability of landing on any of four rectangular faces. The label on the top face of the dices is the score of the throw.



If thrown together, what is the probabilities of getting the sum of the two long dice scores greater than.

- (a) $\frac{1}{16}$ (b) $\frac{3}{16}$
 (c) $\frac{1}{8}$ (d) $\frac{3}{8}$

Sol. (b)

The combination of numbers on the dices to get sum of more than 11 are

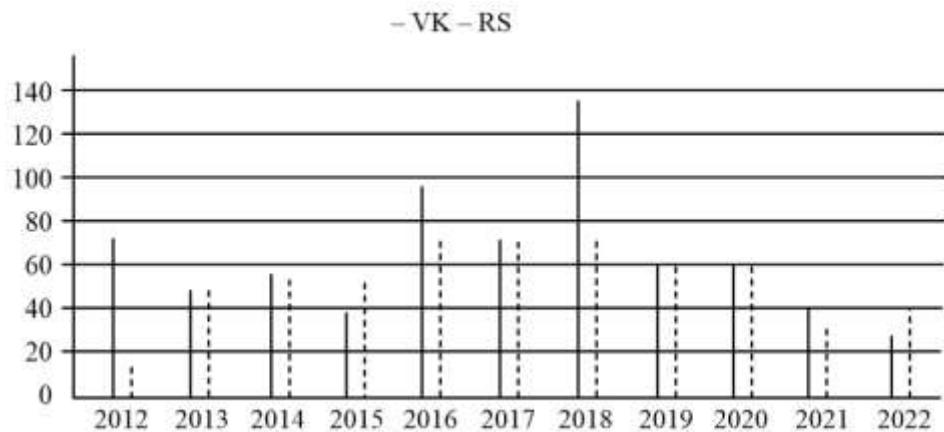
(5, 7), (7, 5) and (7, 7)

$$\begin{aligned} \text{So, the probability} &= \frac{1}{4} \times \frac{1}{4} + \frac{1}{4} \times \frac{1}{4} + \frac{1}{4} \times \frac{1}{4} \\ &= \frac{3}{16} \end{aligned}$$

[MCQ-2]

Q.63. The bar chart gives the batting overseas of UK & Rs for 11 calendar years from 2012 to 2022. Considering that 2015 and 2019 are world cup years. When one of the following options is true?

- (a) Rs has a higher yearly batting average than that of the in every W.C. year
 (b) UK has a higher yearly batting average than that of Rs in average
 (c) UK is yearly batting average is considering has of Rs between the two world cup years
 (d) Rs yearly batting average is constants higher than that of UK in the last three years



Sol. (b)

From the graph it is clear that between two world cups years (i.e. between 2015 & 2019), UK has higher yearly batting average than of Rs.





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