HSC 12TH STANDARD

MATHEMATICS

Model Question Paper - 1

Time : 2.30 Hours | Marks : 90

Part - I

All questions are compulsory **$20 \times 1 = 20$**

Choose the correct answer

1. Let A be a square matrix all of whose entries are integers. Then which one of the following is true ?

   a) If det (A) = ± 1, then A^{-1} exists but all its entries are not necessarily integers

   b) If det (A) $\neq \pm 1$, then A^{-1} exists and all its entries are non integers

   c) If det (A) = ± 1, then A^{-1} exists and all its entries are integers

   d) If det (A) = ± 1, then A^{-1} need not exist

2. If $A = \begin{bmatrix} 0 & 0 \\ 0 & 5 \end{bmatrix}$ then A is

   a) $\begin{bmatrix} 0 & 0 \\ 0 & 60 \end{bmatrix}$ b) $\begin{bmatrix} 0 & 0 \\ 0 & 5^{12} \end{bmatrix}$ c) $\begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$ d) $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

3. If $\vec{a}, \vec{b}, \vec{c}$ are vectors such that $\vec{a} + \vec{b} + \vec{c} = 0$ $|\vec{a}| = 7$, $|\vec{b}| = 5$, $|\vec{c}| = 3$ then angle between vectors $\vec{b}$ and $\vec{c}$ is

   a) 60^0 b) 30^0 c) 45^0 d) 90^0

4. If $\vec{a} \times (\overline{b} \times \overline{c}) + \vec{b} \times (\overline{c} \times \overline{a}) + \vec{c} \times (\overline{a} \times \overline{b}) = \vec{x} \times \vec{y}$ then

   a) $\overline{x} = 0$ b) $\overline{y} = 0$ c) $\overline{x}$ and $\overline{y}$ are parallel

   d) $\overline{x} = 0$ or $\overline{y} = 0$ or $\overline{x}$ and $\overline{y}$ are parallel

5. Let A and B denote the statements

   A : $\cos\alpha + \cos\beta + \cos\gamma = 0$

   B : $\sin\alpha + \sin\beta + \sin\gamma = 0$. If $\cos(\beta - \gamma) + \cos(\gamma - \alpha) + \cos(\alpha - \beta) = \frac{-3}{2}$ then

a) A is true and B is false   b) A is false and B is true

c) both A and B are true   d) both A and B are false

6. The conjugate of $i^{13} + i^{14} + i^{15} + i^{16}$ is

a) 1   b) -1 c) 0   d) $-i$

7. The eccentricity of an ellipse with its centre at the origin is $\frac{1}{2}$. If one of the directrices is $x = 4$, then the equation of the ellipse is

a) $3x^2 + 4y^2 = 1$ b) $3x^2 + 4y^2 = 12$ c) $4x^2 + 3y^2 = 12$

d) $4x^2 + 3y^2 = 1$

8. One of the foci of the rectangular hyperbola $xy = 18$ is

a) (6, 6)   b) (3, 3)   c) (4, 4)   d) (5, 5)

9. $\lim\limits_{x \to \infty} \left(\frac{x^2 + 5x + 3}{x^2 + x + 3} \right)$ is

a) e^4 b) e^2 c) e^3 d) 1

10. If $y = 6x - x^3$ and x increases at the rate of 5 units per second, the rate of change of slope when $x = 3$ is

c) 180 units/sec   d) -180 units/sec

11. The Rolles' constant for the function $y = x^2$ on $[-2, 2]$ is

a) $\frac{2\sqrt{3}}{3}$ b) 0   c) 2   d) -2

12. The point on the curve $x = at^2$, $y = 2at$, at which the tangent is at 45^0 to the x axis is

a) (2a, a)   b) (a, -2a)   c) $(2a, 2\sqrt{2}\,a)$ d) (a, 2a)

13. The area bounded by the parabola $x^2 = 4 - y$ and the lines $y = 0$ and $y = 3$ is

a) $\frac{14}{3}$ sq.units   b) $\frac{28}{3}$ sq. units   c) $4\sqrt{3}$ sq. units   d) $\frac{56}{3}$ sq.units

14. The value of $\int_0^1 x\,(1 - x)^4\, dx$ is

a) $\frac{1}{12}$ b) $\frac{1}{30}$ c) $\frac{1}{24}$ d) $\frac{1}{20}$

15. The degree of the differential equation $\frac{\left[\left(1 + \frac{dy}{dx}\right)^3\right]^{2/3}}{\frac{d^3y}{dx^3}} = C$ where C is a constant is

a) 1   b) 3   c) -2 d) 2

16. The particular integral of $\frac{d^2y}{dx^2} - 6\frac{dy}{dx} + 9y = e^{3x}$

a) $x\, e^{3x}$ b) e^{3x} c) $\frac{x^2}{2} e^{3x}$ d) ∞

17. The order of an element a of a group is 10. (ie) 0(a) = 10 Then the order of $(a^2)^{-1}$ is

a) 10 b) 5 c) 2 d) 1

18. The value of $[3] +_{11} ([5] +_{11} [6])$ is

a) [0] b) [1] c) [2] d) [3]

19. When two dice are thrown the probability of getting one five is

a) $\frac{25}{36}$ b) $\frac{5}{36}$ c) $\frac{1}{36}$ d) $\frac{5}{18}$

20. If in a poission distribution P (X = 0) = K, then the variance is

a) $\log\frac{1}{K}$ b) log K c) e^{λ} d) $\frac{1}{K}$

Part - II

Answer any Seven questions. Question 30 is Compulsory **$7 \times 2 = 14$**

21. Consider the system of linear equation x + 2x + x = 3, 2x + 3x + x = 3, 3x + 5x + 2x = 1 find the solution if exists.

22. For any vector a , prove that the value of $\left(\vec{a} \times \vec{i} \quad + \vec{a} \times \vec{j} \quad + \vec{a} \times \vec{k} \quad = \quad .\right.$

23. If the cube roots of unity are $1, \omega, \omega^2$ then find the roots of the equation $(x - 1)^3 + 8 = 0$.

24. Find the condition that y = mx + c may be a tangent to the conics parabola $y^2 = 4ax$.

25. Prove that the function $f(x) = x^2 - x + 1$ is neither increasing nor decreasing in [0, 1].

26. Find $\frac{\partial w}{\partial t}$, if $w = x^2y - 10y^3z^3 + 43x - 7\tan(4y)$ where $x = t$, $y = t^2$, $z = t^3$

27. Find the value of $\int_{-1}^{2} |x|\,dx$.

28. Solve $e^x\sqrt{1-y^2}\,dx + \frac{y}{x}\,dy = 0$.

29. Prove that the set of all 4th roots of unity forms an abelian group under multiplication.

30. For the probability density function $f(x) = \begin{cases} 2e^{-2x}, & x > 0 \\ 0, & x \le 0 \end{cases}$ find F(2).

Part - III

Answer any Seven questions. Question No.40 is compulsory. **$7 \times 3 = 21$**

31. Solve by matrix inversion method x + y = 3, 2x + 3y = 8

32. What is the radius of the circle in which the sphere $x^2 + y^2 + z^2 + 2x - 2y - 4z - 19 = 0$ is

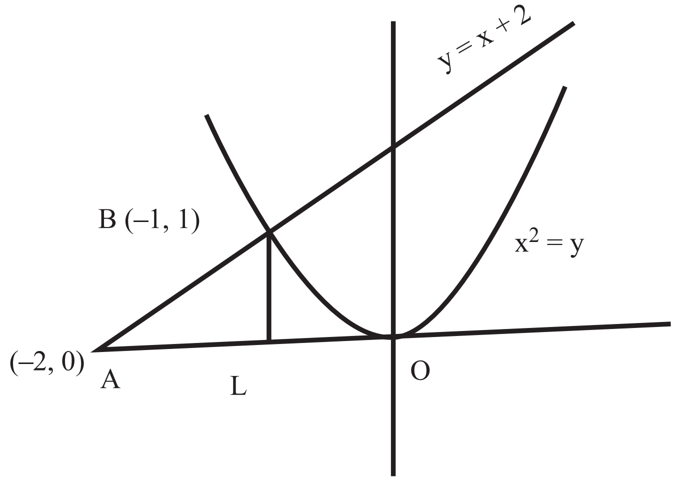
cut by the plane x + 2y + 2z + 7 = 0.

33. Find the real and imaginary parts of the complex number $Z = \dfrac{3i^{20} - i^{19}}{2i - 1}$

34. The tangent at any point of the rectangular hyperbola $xy = c^2$ makes intercepts a, b and the normal at the point makes intercepts p, q on the axes. Prove that ap + bq = 0.

35. Find the point of inflection to the curve $y = \sin^2 x$ where $\dfrac{-\pi}{2} < x < \dfrac{\pi}{2}$.

36. Compute the area of the figure enclosed by the curves $x^2 = y$, $y = x + 2$ and x axis.



38. Find the order of each element of the group (z , +).

39. In a binomial distribution the arithmetic mean and variance are respectively 4 and 3. If the random variable X denotes the number of successes in the corresponding experiment then find P(x = 2) / P (x =3) .

40. Verify Euler's theorem for $f(x, y) = \dfrac{1}{\sqrt{x^2 + y^2}}$

Part - IV

Answer all the questions **7 × 5 = 35**

41. a) Examine the consistency of the following system of equations. If it is consistent then solve using rank method.

$4x + 3y + 6z = 25$, $x + 5y + 7z = 13$, $2x + 9y + z = 1$

or

b) Find the vector and cartesian equations to the plane through the point (–1, 3, 2) and perpendicular to the plane x + 2y + 2z = 5 and 3x + y + 2z = 8.

42. a) $\overline{a}, \overline{b}, \overline{c}$ are three non-zero vectors of magnitudes a, b, c respectively. Also $[\overline{a}\ \overline{b}\ \overline{c}]$ = abc. Then prove that $\overline{a} \cdot \overline{b} = \overline{b} \cdot \overline{c} = \overline{c} \cdot \overline{a} = 0$.

or

b) If α and β are the roots of $x^2 - 2x + 2 = 0$ and $\cot\theta = y + 1$ show that

$$\frac{(y+\alpha)^n - (y+\beta)^n}{\alpha - \beta} = \frac{\text{Sin } n\,\theta}{\text{Sin}^n\,\theta}$$

43. a) Find directrix, latus rectum of the ellipse $6x^2 + 9y^2 + 12x - 36y - 12 = 0$ also draw the diagram.

or

b) The path of a ship can be described by a hyperbolic model centered at the origin, relative to two stations on the shore 168 miles apart that are located at the foci. If the ship is 40 miles south of the centre of the hyperbola, find the equation of the hyperbola.

44. a) Find the values of x, y whose product xy = 64 and such that $4x + 27y^3$ is maximum.

or

b) Prove that the sum of the intercepts on the co-ordinate axes of any tangent to the curve

45. a) $u = \tan^{-1}\left(\frac{x}{y}\right)$ Verify $\frac{\partial^2 u}{x\,\partial y} = \frac{\partial^2 u}{\partial y\,\partial x}$

or

b) The plane region bounded by the curve $y = \sqrt{\cos x}$, $0 \le \theta \le \frac{\pi}{2}$ and the lines x = 0, y = 0 is rotated about x axis. Find the volume of the solid.

46. a) Derive the formula for the volume of a right circular cone with radius 'r' and height 'h'. using integration.

or

b) A Bank pays interest by continuous compounding that is by treating the interest rate as the instantaneous rate of change of principal. Suppose in an account interest accures at 8% per year compounded continuously. Calculate the percentage increase in such an account over one year.

47. a) Show that the set of all matrices of the form $\begin{bmatrix} a & 0 \\ 0 & 0 \end{bmatrix}$, $a \in R - \{0\}$ is an abelian group under matrix multiplication.

or

b) Solve : $x\frac{dy}{dx} - y = (x - 1)\,ex$