GATE 2024

ENGINEERING SCIENCES

Exam held on
10/02/2024
(Afternoon Session)

Memory based
Questions & Solutions

Corporate Office : 44-A/1, Kalu Sarai, New Delhi - 110016
MADE EASY Centres : • Delhi • Hyderabad • Bhopal • Jaipur • Pune • Kolkata
9021300500
www.madeeasy.in

FLUID MECHANICS

Q.1 Velocity profile of a fluid flow is given as $u = \frac{a}{(b-x)^2}$

At a = 8 m^3/s, b = 4 m, x = 2 m, the magnitude of acceleration is ______ m/s^2.

Ans. (4)

Velocity in x-direction, $u = \frac{a}{(b-x)^2}$

Acceleration is given as

$$a = u\frac{\partial u}{\partial x} + \cancel{v\frac{\partial u}{\partial y}}^{0} + \cancel{w\frac{\partial u}{\partial z}}^{0} + \cancel{\frac{\partial u}{\partial t}}^{0}$$

$$\Rightarrow \quad a = \frac{a}{(b-x)^2}\cdot\frac{\partial}{\partial x}\left[\frac{a}{(b-x)^2}\right]$$

$$= \frac{a}{(b-x)^2}\cdot a(-2)(b-x)^{-3}(-1) = \frac{2a^2}{(b-x)^5}$$

At x = 2 m, b = 4 m and a = 8 m^3/s

$$a = \frac{2\times(8)^2}{(4-2)^5} = \frac{2\times(8)^2}{2^5} = 4 \text{ m/s}^2$$

$\Rightarrow$ Magnitude of acceleration is 4 m/s^2.

End of Solution

Q.2 What is the correct relation between Darcy's friction factor and fanning friction factor?

Ans. (*)

Darcy's friction factor = 4 × Fanning friction factor

$$\Rightarrow \quad F = 4f'$$

End of Solution

Q.3 The velocity potential function of a flow field is given as $\phi = -(axy + bx^2 - by^2)$ where constants $a = 2\text{s}^{-1}$ and $b = 0.5^{-1}$. The magnitude of velocity at the point x = 2 m, y = 1 m is ________ m/s.

Ans. (5)

$$\phi = -(axy + bx^2 - by^2)$$

The velocity component in x-direction is given as;

$$u = -\frac{\partial \phi}{\partial x} = -\frac{\partial}{\partial x}\left[-\left(axy + bx^2 - by^2\right)\right]$$

$$= \frac{\partial}{\partial x}\left(axy + bx^2 - by^2\right)$$

$$= ay + 2bx$$

The velocity component in y-direction is given as;

$$v = -\frac{\partial \phi}{\partial y} = -\frac{\partial}{\partial y}\left[-\left(axy + bx^2 - by^2\right)\right]$$

$$= \frac{\partial}{\partial y}\left(axy + bx^2 - by^2\right)$$

$$= ax - 2by$$

Hence, $$\vec{V} = u\hat{i} + v\hat{j}$$

$$= (ay + 2bx)\hat{i} + (ax - 2by)\hat{j}$$

At a = 2 per second, b = 0.5 per second, x = 2 m and y = 1 m

$$\vec{V} = (2 \times 1 + 2 \times 0.5 \times 2)\hat{i} + (2 \times 2 - 2 \times 0.5 \times 1)\hat{j}$$

$$= 4\hat{i} + 3\hat{j}$$

The magnitude of velocity,

$$\left|\vec{V}\right| = \sqrt{4^2 + 3^2} = 5 \text{ m/s}$$

End of Solution

Q.4 What is hydraulic diameter of a circular pipe of radius R?

Ans. (##)

The hydraulic diameter of a cross-section is given as;

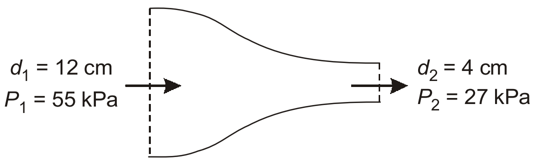
$$\text{Hydraulic diameter} = 4 \times \frac{\text{Area of cross-section}}{\text{Wetted perimeter}}$$

$$= 4 \times \frac{\pi R^2}{2\pi R} = 2R$$

Hence, the hydraulic diameter of a circular pipe of radius R is $2R$.

End of Solution

Q.5 Consider the flow through a converging section as shown below:

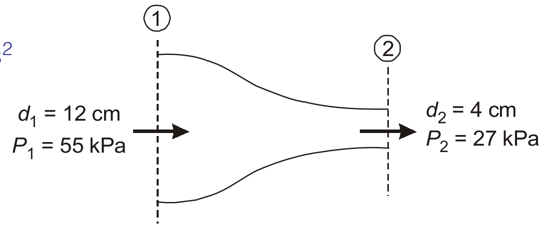


If specific weight of the fluid is 7 kN/m^3 and the acceleration due to gravity, g = 10 m/s^2, the mass flow rate will be _______.

Ans. (7.92)

Specific weight, $w = 7\ \text{kN/m}^3$

Acceleration due to gravity, $g = 10\ \text{m/s}^2$



Using continuity equation;

$$\rho A_1 v_1 = \rho A_2 v_2$$

$$\Rightarrow \quad A_1 v_1 = A_2 v_2$$

$$\Rightarrow \quad \frac{\pi}{4} d_1^2 v_1 = \frac{\pi}{4} d_2^2 v_2$$

$$\Rightarrow \quad d_1^2 v_1 = d_2^2 v_2$$

$$\Rightarrow \quad (12)^2 v_1 = (4)^2 v_2$$

$$\Rightarrow \quad 9 v_1 = v_2 \qquad \text{...(i)}$$

Using Bernoulli's equation

$$\frac{P_1}{w} + \frac{v_1^2}{2g} + z_1 = \frac{P_2}{w} + \frac{v_2^2}{2g} + z_2$$

$$\Rightarrow \quad \frac{P_1}{w} + \frac{v_1^2}{2g} = \frac{P_2}{w} + \frac{v_2^2}{2g} \qquad [\because\ z_1 = z_2]$$

$$\Rightarrow \quad \frac{55}{7} + \frac{v_1^2}{2\times 10} = \frac{27}{7} + \frac{v_2^2}{2\times 10}$$

$$\Rightarrow \quad v_2^2 - v_1^2 = 80 \qquad \text{...(ii)}$$

Using equation (i) and (ii),

$$v_1 = 1\ \text{m/s and } v_2 = 9\ \text{m/s}$$

$$\text{Mass flow rate, } \dot{m} = \rho A_1 v_1$$

$$= \frac{w}{g} \times A_1 v_1$$

$$= \frac{7\times 10^3}{10} \times \left(\frac{\pi}{4} \times (0.12)^2\right) \times 1$$

$$= 7.9168\ \text{kg/s} \simeq 7.92\ \text{kg/s}$$

End of Solution

Q.6 What is the vorticity component in y-z plane?

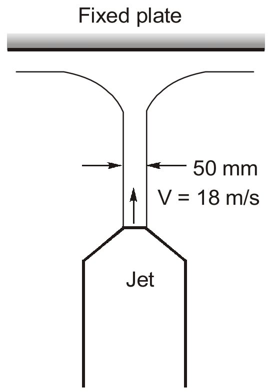
Ans. (##)

Vorticity component in *y-z* plane is given as:

$$\Omega_{\text{y-z plane}} = \Omega_x = \left(\frac{\partial w}{\partial y} - \frac{\partial v}{\partial z}\right)$$

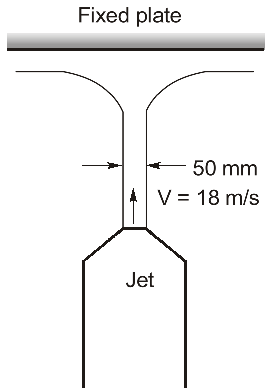
End of Solution

Q.7 A jet of diameter 50 mm is discharging water on a fixed plate with a velocity of 18 m/s. What is the force applied by a jet on a fixed plate? [Neglect the effect of change in potential energy]



Ans. (636.17)

Given : d = 50 mm; V = 18 m/s, ρ = 1000 kg/m^3



Force applied on the plate:

$$F = \rho A V^2$$

$$= 1000 \times \left(\frac{\pi}{4} \times (0.050)^2\right) \times (18)^2$$

$$= 636.1725 \text{ N} \simeq 636.17 \text{ N}$$

End of Solution

Q.8 What is the dimension of pressure?

(a) $[M\ L\ T^{-1}]$

(b) $[M\ L^{-1}\ T^{-2}]$

(c) $[M\ L^{-2}\ T^{-1}]$

(d) $[M\ L\ T^{-2}]$

Ans. (b)

Dimension of pressure can be obtained as;

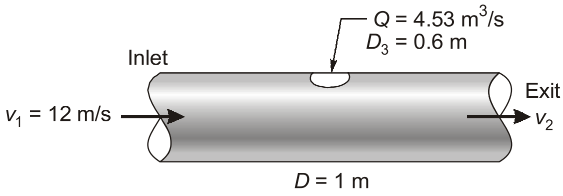
$$\text{Dimension of pressure} = \frac{\text{Dimension of force}}{\text{Dimension of area}}$$

$$= \frac{(\text{Dimension of mass}) \times (\text{Dimension of acceleration})}{\text{Dimension of area}}$$

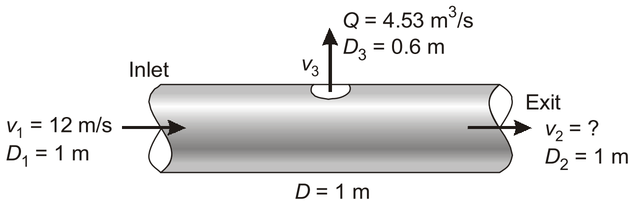
$$= \frac{[M] \times [LT^{-2}]}{L^2} = [ML^{-1}T^{-2}]$$

End of Solution

Q.9 The velocity of water at the inlet is 12 m/s. What is the velocity of water at exit, if there is leakage from the pipe which is of 0.6 m of diameter?



Ans. (6.23)



Using continuity equation;

$$\rho Q_1 = \rho Q_3 + \rho Q_2$$

$$\Rightarrow \quad Q_1 = Q_3 + Q_2$$

$$\Rightarrow \quad A_1 v_1 = Q_3 + A_2 v_2$$

$$\Rightarrow \quad \frac{\pi}{4}(1)^2 \times 12 = 4.53 + \frac{\pi}{4}(1)^2 \times v_2$$

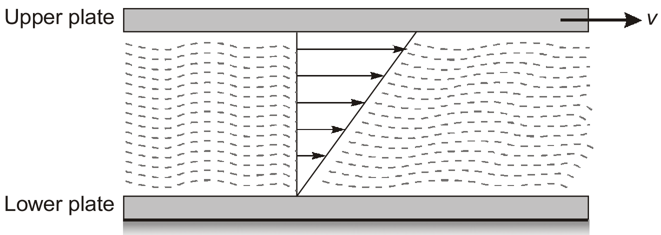
$$\Rightarrow \quad v_2 = 6.2322 \text{ m/s} \simeq 6.23 \text{ m/s}$$

Hence, velocity of water at exit is 6.23 m/s.

End of Solution

Q.10 In a simple Couette Flow, the lower plate is sationary and upper plate is moving with a speed 1 m/s. The distance between the plates is 1 cm. The viscosity is 10^{-3} Pa-s. Find the shear stress required?

Ans. (0.1)



$$\mu = 10^{-3} \text{ Pa.s}, \ v = 1 \text{ m/s}, \ h = 1 \text{ cm}$$

For simple Couette flow;

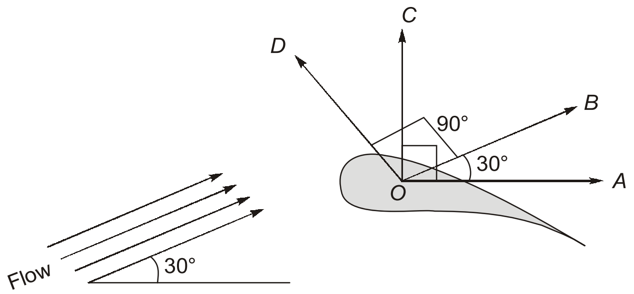
$$\frac{\partial P}{\partial x} \simeq 0$$

$$\text{Shear stress, } \tau = \mu \frac{v}{h}$$

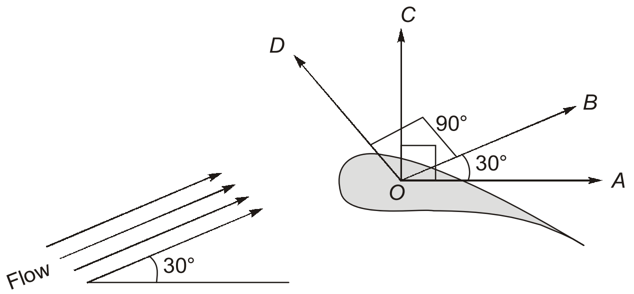
$$\Rightarrow \quad \tau = 10^{-3} \times \frac{1}{(0.01)} = 0.1 \text{ Pa}$$

End of Solution

Q.11 An aerofoil is kept in the flow as shown in figure. The correct direction of drag and lift is given by



Ans. (##)



The direction of drag force on aerofoil is always along the direction of flow i.e. OB direction.

The direction of lift force on aerofoil is always perpendicular to the direction of flow i.e. OD direction.

End of Solution

Q.12 In a drag force test of $\frac{1}{8}$ model-prototype, the actual velocity of car is 16 m/s. The velocity of the model car is _______ m/s.

Ans. (128)

$$v_p = 16 \text{ m/s}, \ \frac{L_m}{L_p} = \frac{1}{8}$$

In drag force testing, the viscous force are dominating.

Hence, $(Re)_{model} = (Re)_{prototype}$

$$\left(\frac{\rho vL}{\mu}\right)_m = \left(\frac{\rho vL}{\mu}\right)_p$$

$$\Rightarrow \quad \frac{\rho v_m L_m}{\mu} = \frac{\rho v_p L_p}{\mu}$$

$$\Rightarrow \quad v_m L_m = v_p L_p$$

$$\Rightarrow \quad v_m = v_p \times \left(\frac{L_p}{L_m}\right) = 16 \times \frac{1}{(1/8)} = 16 \times 8$$

$$= 128 \text{ m/s}$$

Q.13 At certain place atmospheric pressure is 700 mm of Hg and the absolute pressure is 400 mm of Hg. What is the vacuum pressure _______ mm of Hg.

Ans. (300)

$$P_{\text{local atm}} = 700 \text{ mm of Hg}, \; P_{abs} = 400 \text{ mm of Hg}$$

$$P_{vacuum} = P_{\text{local atm}} - P_{abs}$$

$$= 700 - 400 = 300 \text{ mm of Hg}$$

End of Solution

Q.14 Incompressible fluid flowing over a flat plate in x-direction. What is the pressure gradient along the flow direction?

(a) Positive
(b) Negative
(c) Constant
(d) Zero

Ans. (d)

In Prandtl boundary layer condition the pressure gradient is neglected $\left(\frac{\partial P}{\partial x} \simeq 0\right)$ and boundary layer increases in the flow direction.

But if the pressure gradient is favourable $\left(\frac{\partial P}{\partial x} = \text{Negative}\right)$, then the boundary layer decreases in the flow direction.

End of Solution

Q.15 For laminar boundary layer over a flat plate is given by $\frac{u}{U_\infty} = \frac{3y}{2\delta} - \frac{y^3}{2\delta^3}$. The free stream velocity is 1 m/s; $\nu = 10^{-6}$ m²/s and $\delta = \frac{4.64x}{\sqrt{Re_x}}$. Find the shear stress at a distance of 1 m from leading edge?

Ans. (##)

$U_\infty = 1$ m/s; $v = 10^{-6}$ m^2/s; $x = 1$ m

Reynolds number, $Re_x = \dfrac{U_\infty x}{\nu} = \dfrac{1\times 1}{10^{-6}} = 10^6$

Boundary layer thickness,

$$\delta = \frac{4.64x}{\sqrt{Re_x}} = \frac{4.64\times 1}{\sqrt{10^6}} = 4.64\times 10^{-3}\text{m}$$

Velocity profile, $\dfrac{u}{U_\infty} = \dfrac{3y}{2\delta} - \dfrac{y^3}{2\delta^3}$

$$\Rightarrow \quad \frac{du}{dy} = U_\infty\left(\frac{3}{2\delta} - \frac{3y^2}{2\delta^3}\right)$$

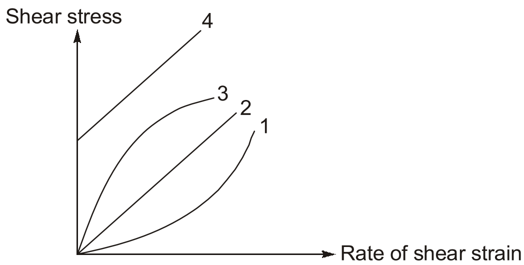
Wall shear stress, $\tau_w = \mu\left(\dfrac{du}{dy}\right)_{y=0}$

$$= \mu\times U_\infty\times\left(\frac{3}{2\delta} - \cancelto{0}{\frac{3\times 0}{2\delta^3}}\right) = \frac{3\mu U_\infty}{2\delta}$$

$$= \frac{3\times(10^{-6}\times 1)\times 1}{2\times(4.64\times 10^{-3})}$$

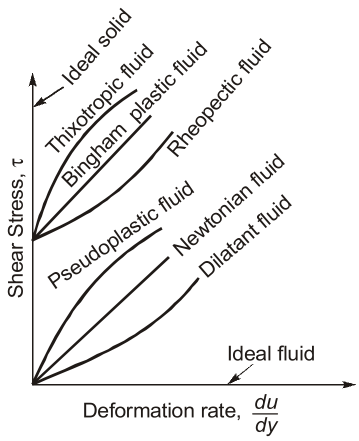
$= 3.23275 \times 10^{-4}$ Pa $\simeq 3.23 \times 10^{-4}$ Pa

Q.16 Which of the below fluid show the behaviour of Pseudoplastic fluid?



(a) 1
(b) 2
(c) 3
(d) 4

Ans. (##)



End of Solution

Q.17 In a fluid flow if fluid particles originating from a fixed origin. If we joint the location of different fluid particles at a given instant of time, is

(a) Streak line
(b) Path line
(c) Timeline
(d) Stream line

Ans. (a)

A streak line is the instantaneous picture of the positions of all the fluid particles that have passed through a fixed point in the flow field.

End of Solution

Q.18 Stability condition of fully submerged body is ____

Ans. (##)

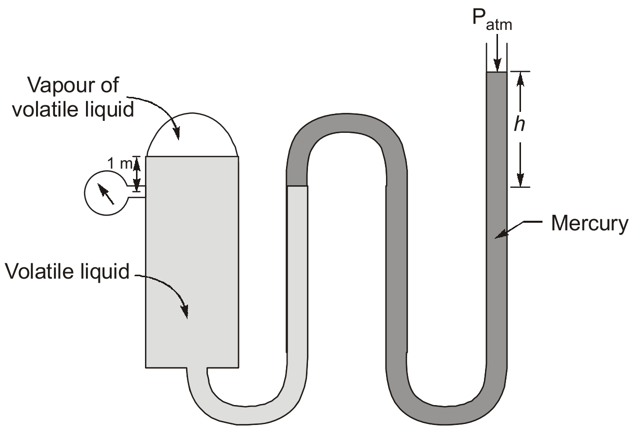
- For completely submerged body, the body is sad to be in stable equilibrium when the centre of buoyancy (B) should be above centre of gravity (G).
- For partially submerged body, the body is said to be in stable equilibrium when the metacentre (M) should be above centre of gravity (G).

End of Solution

Q.19 P_{atm} = 101 kPa, $P_{vapour\ pressure}$ = 107.6 kPa, $\rho_{volatile\ liquid}$ = 700 kg/m^3,

ρ_{Hg} = 13600 kg/m^3, g = 10 m/s^2

The value of h is ______ mm.



Ans. (100)

P_{atm} = 101 kPa, $P_{vapour\ pressure}$ = 107.6 kPa, x = 1 m ρ_L = 700 kg/m^3,

ρ_{Hg} = 13600 kg/m^3, g = 10 m/s^2

$P_{vapour\ pressure} + \rho_L g x - \rho_{Hg} g h = P_{atm}$

$(107.6 \times 10^3) + (700 \times 10 \times 1) - (13600 \times 10 \times h) = 101 \times 10^3$

$\Rightarrow$ 107600 + 7000 − 136000h = 101000

$\Rightarrow$ $h = 0.1$ m = 100 mm

End of Solution

Q.20 For laminar flow through circular pipe the velocity distribution is

$$u = \frac{1}{4\mu}\left(-\frac{\partial P}{\partial x}\right)R^2\left[1-\frac{r^2}{R^2}\right]$$

The average velocity is given by

$$v_{avg} = \frac{1}{k}\left[\frac{R^2}{\mu}\left(-\frac{\partial P}{\partial x}\right)\right]$$

What is the value of k?

(a) 2
(b) 4
(c) 8
(d) 16

Ans. (c)

Velocity distribution : $u = \frac{1}{4\mu}\left(\frac{-\partial P}{\partial x}\right)R^2\left(1-\frac{r^2}{R^2}\right)$

At $r = 0$, $u = u_{max} = \frac{1}{4\mu}\left(\frac{-\partial P}{\partial x}\right)R^2$

For Haigen-Poisuelle flow;

$$u_{avg} = \frac{u_{max}}{2}$$

$$= \frac{1}{8}\left[\frac{R^2}{\mu} - \left(\frac{-\partial P}{\partial x}\right)\right]$$

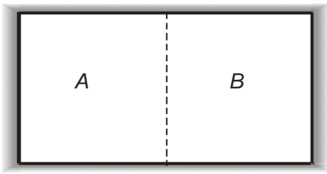
Hence, value of K is 8.

End of Solution

THERMODYNAMICS

Q.21 10 kg of water with constant specific heat 4.2 kJ/kgK at 300 K and and 10 kg of water with same specific heat at 350 K is mixed adiabatically. What is the change in entropy?

Ans. (0.25)



Given : $m_A = m_B = 10$ kg; $c_A = c_B = 4.2$ kJ/kgK; $T_A = 300$ K; $T_B = 350$ K

By first law of thermodynamics,

$$\delta Q = dU + \delta W$$

$\Rightarrow$ $dU = 0$ [since there is no heat and work transfer]

$m_A c_A (T_f - 300) + m_B c_B (T_f - 350) = 0$

where, T_f is the final temperature after mixing.

$$\Rightarrow 10 \times 4.2(T_f - 300) + 10 \times 4.2(T_f - 350) = 0$$

$$2T_f - 650 = 0$$

$$T_f = 325 \text{ K}$$

Thus, entropy change for system A,

$$\Delta s_A = m_A c_A \ln\frac{T_f}{T_A}$$

$$= 10 \times 4.2 \ln\left(\frac{325}{300}\right) = 3.362 \text{ kJ/K}$$

and entropy change for system B,

$$\Delta s_B = m_B c_B \ln\frac{T_f}{T_B}$$

$$= 10 \times 4.2 \ln\left(\frac{325}{350}\right) = -3.113 \text{ kJ/K}$$

∴ Total entropy change,

$$\Delta s_{total} = \Delta s_A + \Delta s_B + \Delta s_{surr.}$$
$$= 3.362 - 3.113 \qquad (\because \ \Delta s_{surr.} = 0)$$
$$\simeq 0.25 \text{ kJ/K}$$

End of Solution

Q.22 Efficiency of carnot cycle will increase most when
(a) the sink temperature is increased
(b) the sink temperature is decreased
(c) the source temperature is increased
(d) the source temperature is decreased

Ans. (b)

We know, for carnot cycle,

$$\eta = 1 - \frac{T_L}{T_H}$$

To increase efficiency of heat engine, decreasing the lower temperature (T_L) is more effective than increasing the higher temperature (T_H). This can be easily proved as shown below:

Increasing T_H by ΔT,

$$\eta_1 = \frac{(T_H + \Delta T) - T_L}{T_H + \Delta T} = \frac{(T_H - T_L) + \Delta T}{T_H + \Delta T} \quad \text{...(i)}$$

Decreasing T_L by ΔT,

$$\eta_2 = \frac{T_H - (T_L - \Delta T)}{T_H} = \frac{(T_H - T_L) + \Delta T}{T_H} \quad \text{...(ii)}$$

From (i) and (ii)

∴ $\eta_1 < \eta_2$

End of Solution

Q.23 Two rigid impermeable containers A and B. Only heat transfer takes place between the containers. There is no interaction between the containers and surrounding.

P = Pressure, V = Volume, N = Number of moles and T = Temperature

Which of the following relations are valid?

(a) $\frac{P_A V_A}{N_A} = \frac{P_B V_B}{N_B}$

(b) $T_A = T_B$

(c) $P_A = P_B$

(d) $\frac{P_A}{V_A} = \frac{P_B}{V_B}$

Ans. (a, b)

If T_f is the final temperature

For A, $$P_A V_A = N_A \bar{R} T_f$$

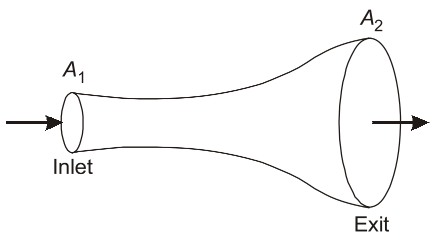
and for B, $$P_B V_B = N_B \bar{R} T_f$$

$\therefore$ $$\frac{P_A V_A}{N_A} = \frac{P_B V_B}{N_B}$$

End of Solution

Q.24 Air enters a nozzle with a mass flow rate of 2.5 kg/s. At inlet pressure, velocity and temperature 350 kPa, 3 m/s and 350 K respectively. At exit pressure and temperature are 101.5 kPa and 305 K, respectively, with Mach number is $\frac{9}{7}$ at exit. The ratio of inlet area to outlet area is ______.

Ans. (50)



Given : $\dot{m} = 2.5$ kg/s

At inlet, $T_1 = 350$ K, $P_1 = 350$ kPa, $c_1 = 3$ m/s

At exit, $T_2 = 305$ K, $P_2 = 101.5$ kPa, $M_2 = \frac{9}{7}$

Assuming air to be an ideal gas,

$$P_1 \dot{V}_1 = \dot{m} R T_1$$

$$\dot{V}_1 = \frac{\dot{m} R T_1}{P_1} = \frac{2.5 \times 0.287 \times 350}{350}$$

$$\dot{V}_1 = 0.7175 \text{ m}^3/\text{s}$$

Also, $$\dot{V}_1 = A_1 c_1$$

$$A_1 = \frac{0.7175}{3} = 0.2392 \text{ m}^2$$

Similarly, at exit,

$$\dot{V}_2 = \frac{\dot{m} R T_2}{P_2} = \frac{2.5 \times 0.287 \times 305}{101.5}$$

$$\dot{V}_2 = 2.15603 \text{ m}^3/\text{s}$$

Now, $$M_2 = \frac{c_2}{\bar{c}_2} = \frac{c_2}{\sqrt{\gamma R T_2}}$$ ($\bar{c}_2$ is velocity of sound at exit)

$\Rightarrow$ $$c_2 = M_2 \sqrt{\gamma R T_2}$$

$$c_2 = \frac{9}{7}\sqrt{1.4 \times 287 \times 305} = 450.09 \text{ m/s}$$

Again, $\dot{V}_2 = A_2 c_2$

$\Rightarrow$ $$A_2 = \frac{\dot{V}_2}{c_2} = \frac{2.15603}{450.09} = 0.00478 \text{ m}^2$$

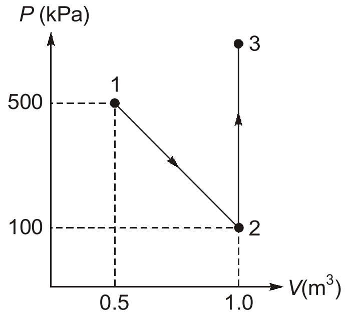
Thus, the ratio of inlet area to outlet area is

$$\frac{A_1}{A_2} = \frac{0.2392}{0.00478} \simeq 50$$

End of Solution

Q.25 Consider a thermodynamic process 1 - 2 - 3. The process 1 - 2 follows the equation $P + 800V = 900$. The process 2 - 3 is isochoric with $V_2 = V_3 = 1 \text{ m}^3$ and $\frac{P_3}{P_2} = 4$. If $V_1 = 0.5 \text{ m}^3$, the total work done during the process is ______.

Ans. (150)



Given, for process 1 - 2,

$$P + 800V = 900 \quad \text{(Linear function)}$$

$\therefore$ $$P_1 + 800V_1 = 900$$

$$P_1 + 800 \times 0.5 = 900$$

$\therefore$ $$P_1 = 500 \text{ kPa}$$

Also, $$P_2 + 800V_2 = 900$$

$$P_2 + 800 = 900$$

$$P_2 = 100 \text{ kPa}$$

Now, total work done is

$$W = W_{12} + W_{23}$$

$$W = W_{12} \quad [\because W_{23} = 0 \text{ as the process is isochoric}]$$

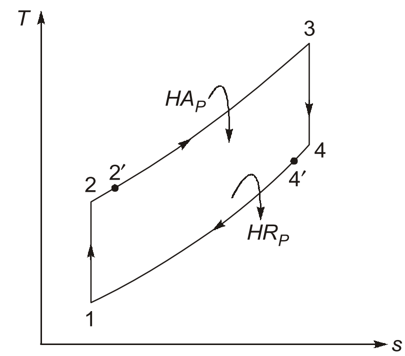
$$W = \frac{1}{2}(500 + 100) \times (1 - 0.5) \quad [\because \text{Work} = \text{Area under PV curve}]$$

$$W = 150 \text{ kJ}$$

End of Solution

Q.26 Consider a system undergoing Brayton cycle with following data:
Enthalpy at turbine inlet = 1400 kJ/kg
Enthalpy at turbine exit = 880 kJ/kg
Enthalpy at compressor exit = 600 kJ/kg
If the cycle is employed with a regenerator of effectiveness 0.8, the percentage change in heat input will be ______. [Answer in integer]

Ans. (28)



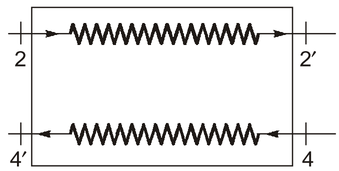
Given, $h_3 = 1400$ kJ/kg, $h_4 = 880$ kJ/kg
$h_2 = 600$ kJ/kg, $\varepsilon = 0.8$

Head addition in cycle without regeneration,

$$HA_1 = h_3 - h_2 = 1400 - 600 = 800 \text{ kJ/kg}$$

Now, with use of regenerator,

$$\varepsilon = \frac{\text{Actual heat gain}}{\text{Ideal heat gain}}$$



$$0.8 = \frac{h_2' - h_2}{h_4 - h_2}$$

$$0.8 = \frac{h_2' - 600}{880 - 600}$$

$$h_2' = 824 \text{ kJ/kg}$$

∴ Heat addition in cycle with regeneration,

$$HA_2 = h_3 - h_2'$$
$$= 1400 - 824 = 576 \text{ kJ/kg}$$

∴ Percentage change in heat input will be $= \dfrac{HA_1 - HA_2}{HA_1} \times 100$

$$= \frac{800 - 576}{800} \times 100 = 28\%$$

End of Solution

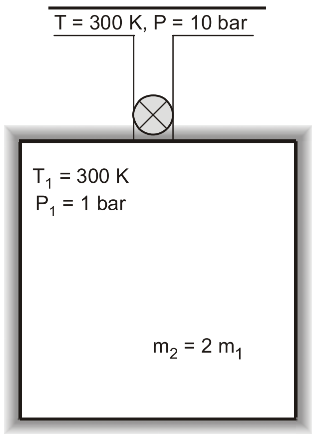
Q.27 $PV^{\gamma} = C$ is valid for

(a) Ideal gas and reversible process
(b) #
(c) #
(d) #

Ans. (a)

End of Solution

Q.28 Consider an insulated rigid tank with initial conditions $P = 100$ kPa, $T = 300$ K. The tank is connected to a pipe with $T = 300$ K by a valve. The valve is now opened such that the mass in the tank doubles. Find the temperature of the tank.



Ans. (360)

From conservation of mass,

$$m_2 - m_1 = m_i - m_e$$
$$m_i = m_2 - m_1 \qquad \ldots(i) \qquad (\because \; m_e = 0)$$

where, $i \rightarrow$ inlet; $e \rightarrow$ exit; $1 \rightarrow$ Initial condition in tank and $2 \rightarrow$ Final condition in tank

From conservation of energy,

$$U_2 - U_1 = m_i h_i + \cancelto{0}{Q} - \cancelto{0}{m_e h_e} - \cancelto{0}{W_{C.V.}}$$

$$m_2 C_V T_2 - m_1 C_V T_1 = (m_2 - m_1) C_P T_i \qquad \text{(By (i))}$$
$$C_V (m_2 T_2 - m_1 T_1) = (m_2 - m_1) C_P T_i$$

Assuming ideal gas behavior to be valid,

$$C_V \left(\frac{P_2 V}{R} - \frac{P_1 V}{R} \right) = \left(\frac{P_2 V}{R T_2} - \frac{P_1 V}{R T_1} \right) C_P \times T_i$$

$$P_2 - P_1 = \left(\frac{P_2}{T_2} - \frac{P_1}{T_1} \right) \times \gamma \times T_i \qquad \left(\because \; \frac{C_P}{C_V} = \gamma \right)$$

$$P_2 - 100 = \left[\frac{P_2}{T_2} - \frac{100}{300}\right] \times 1.4 \times 300 \quad \text{...(ii)}$$

Also, $P_1V = m_1RT_1$ and $P_2V = m_2RT_2$

$\therefore$
$$\frac{P_1}{P_2} = \frac{m_1}{m_2} \times \frac{T_1}{T_2}$$

$$\frac{100}{P_2} = \frac{1}{2} \times \frac{300}{T_2}$$

$$\frac{P_2}{T_2} = \frac{200}{300} = \frac{2}{3}$$

Substituting this in equation (ii),

$$\frac{2}{3}T_2 - 100 = \left[\frac{2}{3} - \frac{1}{3}\right] \times 1.4 \times 300$$

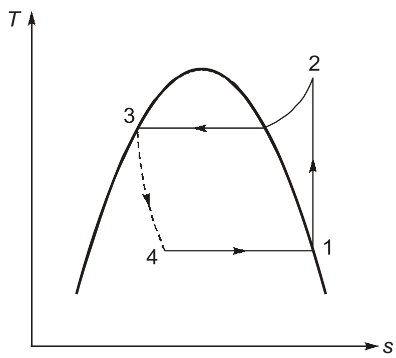
$$T_2 = \left(\frac{1}{3} \times 1.4 \times 300 + 100\right)\frac{3}{2}$$

$$T_2 = 360 \text{ K}$$

End of Solution

Q.29 Consider a system working on simple VCRS cycle. The enthalpies at the entry and exit of compressor is 246 kJ/kg and 286 kJ/kg respectively. If the refrigeration effect is 158 kJ/kg, the enthalpy at the exit of condenser will be _______ kJ/kg.

Ans. (88)



Given, $h_1 = 246$ kJ/kg; $h_2 = 286$ kJ/kg

RE = 158 kJ/kg

We know, RE = $h_1 - h_4$

$158 = 246 - h_4$

$h_4 = 246 - 158 = 88$ kJ/kg

Since process 3 - 4 is isenthalpic,

$\therefore$ $h_3 = h_4 = 88$ kJ/kg

Hence, enthalpy at the exit of condenser is 88 kJ/kg.

End of Solution

Q.30 For a gas obeying Van der Walls equation $\left(P+\frac{a}{v^2}\right)(v-b)=RT$

The value of $\left.\frac{\partial v}{\partial T}\right|_P \left.\frac{\partial P}{\partial v}\right|_T \left.\frac{\partial T}{\partial P}\right|_v =$ ______.

(a) 1
(b) –1
(c) 0
(d) None of these

Ans. **(–1)**

We know that for a function,

$$f(x, y, z) = c$$

$$\left(\frac{\partial x}{\partial y}\right)_z \left(\frac{\partial y}{\partial z}\right)_x \left(\frac{\partial z}{\partial x}\right)_y = -1$$

∴ For a gas obeying Van der Walls equation

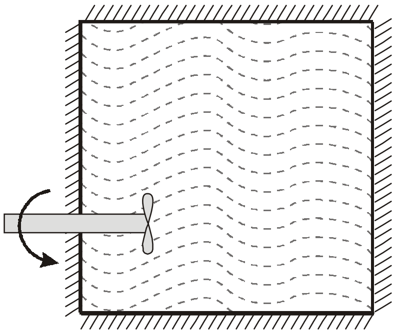
$$\left(P+\frac{a}{V^2}\right)(V-a) = RT$$

or $$f(V, P, T) = C$$

$$\Rightarrow \left(\frac{\partial V}{\partial T}\right)_P \left(\frac{\partial P}{\partial V}\right)_T \left(\frac{\partial T}{\partial P}\right)_V = -1$$

End of Solution

Q.31 An insulated tank contain 10 grams of water. If the temperature of water is to be raised by 5 K then what is the amount of heat to be supplied?



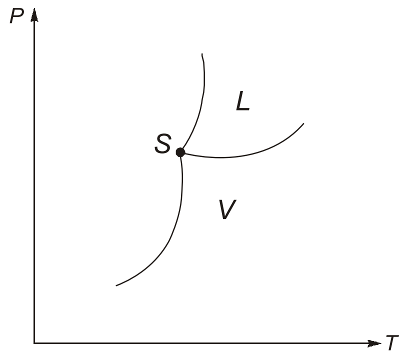
Ans. **(0)**

In case of stirrer, energy crosses the boundary in form of work and it is added to the fluid in form of work only. Thus, the amount of heat supplied is zero.

End of Solution

Q.32 At 273 K melting point and 1 bar density of solid is 900 kg/m^3. The solid liquified and density increases to 1000 kg/m^3 , the latent heat is 300 kJ/kg. What will be the melting point at 101 bar, assuming latent heat and density to be constant?

Ans. (271.9907)



$$\frac{dP}{dT} = \frac{\Delta s}{\Delta v} = \frac{LH}{T\Delta v}$$

$$\int_1^2 dP = \int_1^2 \frac{LH}{\Delta v} \times \frac{dT}{T}$$

$$P_2 - P_1 = \frac{LH}{V_l - V_s} \ln\left(\frac{T_2}{T_1}\right)$$

$$(101 - 1) \times 100 = \frac{300}{\left(\frac{1}{1000} - \frac{1}{900}\right)} \ln\left(\frac{T_2}{273}\right)$$

$$\ln\left(\frac{T_2}{273}\right) = \frac{100 \times 100}{300} \times \frac{(900 - 1000)}{1000 \times 900}$$

$$\ln\left(\frac{T_2}{273}\right) = -0.003704$$

$$T_2 = 0.996302 \times 273$$
$$T_2 = 271.9907 \text{ K}$$

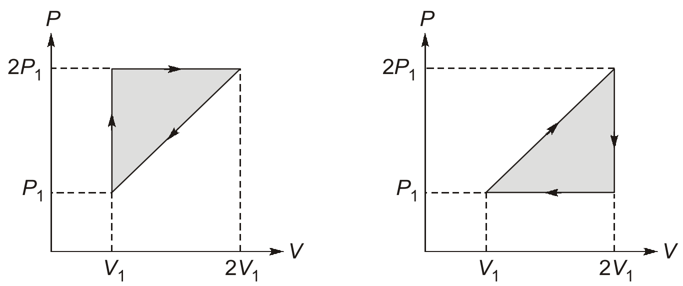
End of Solution

Q.33 In ideal rankine cycle if superheating is done then
(a) pump work increases
(b) pump work decreases
(c) pump work remain same, turbine work increases, efficiency increase and moisture content increases
(d) pump work remain same, turbine work increases, efficiency increase and moisture content decreases

Ans. (d)

End of Solution

Q.34



(a) Heat addition in both is same
(b) Heat rejection in both is same
(c) Both having same efficiency
(d) Net heat transfer for both is same

Ans. (d)

From first law of thermodynamics, for a cycle

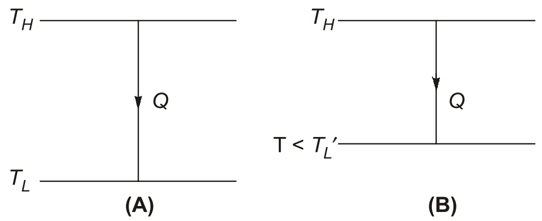
$$\oint \delta Q = \oint \delta W$$

Now, on PV diagram, net work in cycle is given by area enclosed.

Also, the area in both PV diagrams is same and hence net work and heat transfer both will be same.

End of Solution

Q.35



Which one of the following is more reversible?

(a) A is more reversible than B and entropy generation is positive.

(b) B is more reversible than A and entropy generation is positive.

(c) A is more reversible than B and entropy generation is zero.

(d) A and B have same reversibility and entropy generation in both is positive.

Ans. (b)

Change in entropy for case A,

$$(\Delta s_{univ})_A = \frac{-Q}{T_H} + \frac{Q}{T_L}$$

and for case B,

$$(\Delta s_{univ})_B = \frac{-Q}{T_H} + \frac{Q}{T_L'}$$

Since $T_{L'} > T_L$, we can say that

$$(\Delta s_{univ})_B < (\Delta s_{univ})_A$$

$\therefore$ Case B is more reversible than A and entropy generation is positive.

End of Solution

Q.36 Mixture of ideal gases, contains 5 moles of O_2, 4 moles of N_2, 3 moles of H_2. total pressure is 100 kPa. If 2 moles of O_2 is removed what will be the partial pressure of oxygen, if temperature does not change?

Ans. (0.25)

Given : $(n_{O_2})_1$ = 5 moles, (n_{N_2}) = 4 moles, (n_{H_2}) = 3 moles, (p_{t_1}) = 100 kPa

Now, $$p_{t_1}V = n_1\bar{R}T_1$$

$$100 \times V = 12 \times \bar{R} \times T_1 \qquad \text{...(i)}$$

After removing 2 moles of O_2,

$$(n_{O_2})_2 = 3 \text{ moles}$$

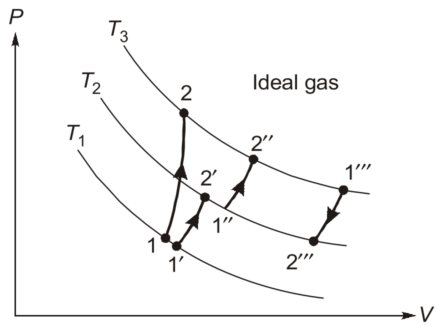
Also,
$$p_{t_2}V = n_2\bar{R}T_2$$
$$p_{t_2}V = 10\times\bar{R}\times T_1 \quad \text{...(ii)} \quad [\because \; T_1 = T_2]$$

By equations (i) and (ii),
$$\frac{100}{p_{t_2}} = \frac{12}{10}$$

$\therefore$
$$p_{t_2} = 83.33 \text{ kPa}$$

Partial pressure of O_2,
$$p_{O_2} = x_{O_2}\times p_{t_2}$$
$$= \frac{3}{10}\times 83.33 = 24.999 \text{ kPa} \simeq 0.25 \text{ bar}$$

End of Solution

Q.37 

What is the maximum change in internal energy?

(a) $1 \rightarrow 2$
(b) $1' \rightarrow 2'$
(c) $1'' \rightarrow 2''$
(d) $1''' \rightarrow 2'''$

Ans. (a)

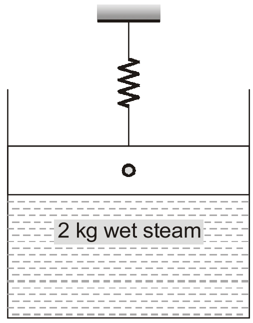
For an ideal gas, the internal energy is proportional to temperature.

$\therefore$ Maximum change in internal energy will be for the process which is undergone in between the two extreme temperatures i.e. process 1 - 2.

End of Solution

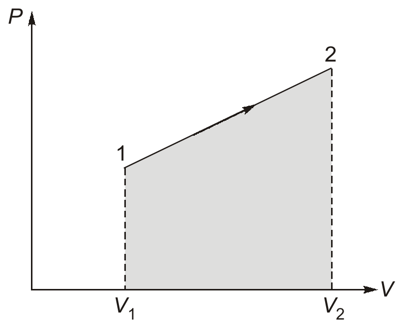
Q.38 Given: mass of wet steam is 2 kg. Heat is added till system recahes state 2 by steam. What is the amount of work done?

S.No.				v_f	v_g
1	t_1(°C)	P_{s1} =	x = 0.1	v_f	v_g
2	t_2(°C)	P_{s2} =	v_2 (given)		



Ans. (?)

The pressure variation w.r.t. volume will be linear in this case.



Amount of work done will be given by the shaded area.

$$W = \frac{1}{2}(P_1 + P_2) \times (V_2 - V_1)$$

$$= \frac{1}{2}(P_1 + P_2) \times m(v_2 - v_1)$$

End of Solution