Intermediate Examination Year 2023-2024

Mathematics Model Paper

Only Paper

Time – 3hrs.15min **M.M. - 100**

Note: First 15 minutes are allotted for the candidates to read the question paper.

Instructions:-

(i) There are nine questions in this question paper.

(ii) All questions are compulsory.

(iii) In the beginning of each question, the number of parts to be attempted are clearly mentioned.

(iv) Marks allotted to the questions are indicated against them.

(v) Start solving from the first question and proceed to solve till the last one. Do not waste your time over question which you cannot solve.

1. Write the correct alternative of each part in your answer book.

(A) If function F:R→R is defined as f(x)=3x write correct option. **01**

(i) f is one - one onto (ii) f is many one onto

(iii) f is one-one but not onto

(iv) Neither one-one nor onto

(B) Let R be the relation in the Set N given by **01**

R={(a, b):a = b-2, b>6}

Choose the correct answer

(i) (2,4) ∈ R (ii) (3,8) ∈R (iii) (6,8) ∈R (iv) (8,7) ∈R

(C) Value of $\int x\, e^x dx$ is **01**

(i) e^x (ii) $(x+1)e^x$ (iii) $(x-1)e^x$ (iv) $\frac{x^2}{2}e^x$

(D) The Order of differential equation **01**

$2x^2 \frac{d^2y}{dx^2} - 3\frac{dy}{dx} + y = 0$

(I) 2 (ii) 1 (iii) 0 (iv) not defined

(E) If vectors $2\hat{\imath} + \hat{\jmath} + \hat{k}$ and $\hat{\imath} - 4\hat{\jmath} + \lambda\hat{k}$ are mutually perpendicular to each other , then find out the value of λ . **01**

(i) 3 (ii) 2 (iii) 4 (iv) 0

2 Attempt all parts of the following —

(A) Find out principal value of $\cot^{-1}(-1/\sqrt{3})$ **01**

(B) Show that function f(x) = |x|, is continuous at x=0 **01**

(C) Find order and degree of differential equation **01**

$xy \frac{d^2y}{dx^2} + x(\frac{dy}{dx})^2 - y\frac{dy}{dx} = 0$

(D) Find the direction cosines of the line passing through the two points (-2, 4, -5) and (1, 2, 3) . **01**

(E) If P(A) = $\frac{7}{13}$, P(B) = $\frac{9}{13}$ and P(A∩ B) = $\frac{4}{13}$, find the value of P(A/B) . **01**

3. Attempt all parts of the following —

(A) If sets A ={1,2} and B ={3,4} then find out number of relation from A to B . **02**

(B) If y= Asinx + Bcosx, then prove that $\frac{d^2y}{dx^2}$ + y = 0 **02**

(C) Find out the angle between vectors $\hat{\imath}$ - 2$\hat{\jmath}$ + 3$\hat{k}$ and

3$\hat{\imath}$ - 2$\hat{\jmath}$ + $\hat{k}$. **02**

(D) If x$\begin{bmatrix}2\\3\end{bmatrix}$ + y$\begin{bmatrix}-1\\1\end{bmatrix}$ =$\begin{bmatrix}10\\5\end{bmatrix}$ Then find out the value of

x and y . **02**

4. Attempt all parts of the following —

(A) Show that f(x) = 7x -3 is an increasing function on R **02**

(B) Find a unit vector perpendicular to each of the vectors **02**

($\bar{a}$ + $\bar{b}$) and ($\bar{a}$ − $\bar{b}$), where $\bar{a}$ = $\hat{\imath}$ + $\hat{\jmath}$ + $\hat{k}$, $\bar{b}$ = $\hat{\imath}$ +2 $\hat{\jmath}$ +3 $\hat{k}$

(C) Find the area of the parallelogram whose adjacent **02**

sides are represented by the vectors $\bar{a}$ =3 $\hat{\imath}$ + $\hat{\jmath}$ +4 $\hat{k}$ and

$\bar{b}$ = $\hat{\imath}$ - $\hat{\jmath}$ + $\hat{k}$

(D) Let A and B be events such that P(A) = $\frac{1}{2}$, P(A∪ B)= $\frac{3}{5}$ and P(B)=p, Find the value of p such that A and B are mutually exclusive. **02**

5. Attempt all parts of the following —

(A) On the set Z of all integers consider the relation R={(a, b) : (a-b) is divisible by 2}. Show that R is an equivalence relation on Z . **05**

(B) If A =$\begin{bmatrix} 2 & 3 \\ 1 & -4 \end{bmatrix}$, B = $\begin{bmatrix} 1 & -2 \\ -1 & 3 \end{bmatrix}$ .Then prove that $(AB)^{-1} = B^{-1}A^{-1}$. **05**

(C) Differentiate $(sinx)^{cosx}$ with respect to x . **05**

(D) Evaluate : $\int_{-\pi/4}^{\pi/4} sin^2\, x\, dx$ **05**

(E) Find the shortest distance between the lines whose vector equation are **05**

$\bar{r}$ = $\hat{\imath}$ +2 $\hat{\jmath}$ -4 $\hat{k}$ + λ(2 $\hat{\imath}$ +3 $\hat{\jmath}$ +6 $\hat{k}$), and

$\bar{r}$ = 3$\hat{\imath}$ +3 $\hat{\jmath}$ -5 $\hat{k}$ + μ(2 $\hat{\imath}$ +3 $\hat{\jmath}$ +6 $\hat{k}$)

6. Attempt all parts of the following —

(A) Show that **05**

$$f(x) = \begin{cases} \frac{|x|}{x} \text{ , } if\ x \neq 0 \\ 0,\ if\ x = 0 \end{cases}$$

is discontinuous at x=0

(B) Find the area bounded by the curve **05**

y= cosx between x=0, and x= 2 π

(C) Find the value of P so that lines **05**

$\frac{1-x}{3} = \frac{7y-14}{2P} = \frac{z-3}{2}$ and

$\frac{7-7x}{3P} = \frac{y-5}{1} = \frac{6-z}{5}$ are at right angle

(D) Minimize Z=3x+2y, Subject to the constraints **05**

$x+y\geq 8$, $3x+5y\leq 15$, $x\geq 0$, $y\geq 0$.

(E) In a hostel , 60% of the students read Hindi news paper, 40% read English news paper and 20% read both Hindi and English news papers. A Student is selected at random.

(i) Find the probability that she reads neither Hindi nor English news paper. **$2\frac{1}{2}$**

(ii) If she reads Hindi news paper, find the probability that she reads English newspaper too. **$2\frac{1}{2}$**

7. Attempt any one part of the following —

(A) If $A^{-1}=\begin{bmatrix} 3 & -1 & 1 \\ -15 & 6 & -5 \\ 5 & -2 & 2 \end{bmatrix}$ and $B=\begin{bmatrix} 1 & 2 & -2 \\ -1 & 3 & 0 \\ 0 & -2 & 1 \end{bmatrix}$ then find out the value of $(AB)^{-1}$. **08**

(B) Solve the following system of equations by matrix method. **08**

$3x-2y+3z = 8$

$2x+y-z = 1$

$4x-3y+2z = 4$

8. Attempt any one part of the following —

(A) Prove that the volume of the largest cone that can be inscribed in a sphere is $\frac{8}{27}$ of the volume of the sphere. **08**

(B) Find the general solution of the differential equation $\frac{dy}{dx} - y = \cos x$. **08**

9. Attempt any one part of the following —

(A) Evaluate :- $\int_0^{\pi/2} log\ Sinxdx.$ **08**

(B) Evaluate:- $\int_0^{\pi} \frac{xdx}{a^2cos^2x + b^2sin^2\,x}$ **08**

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