# BOARD QUESTION PAPER : FEBRUARY 2020

## MATHEMATICS AND STATISTICS

**Time: 3 Hours** **Max. Marks: 80**

***General Instructions:***
*The question paper is divided into **FOUR** sections.*
1. ***Section A:** Q. 1 contains **Eight** multiple choice type of questions carrying **Two** marks.*
   *Q. 2 contains **Four** sub-questions each carrying **One** mark each.*
2. ***Section B:** Q. 3 to Q. 14 each carries **Two** mark.(Attempt any **Eight**)*
3. ***Section C:** Q. 15 to Q. 26 carries **Three** marks.(Attempt any **Eight**)*
4. ***Section D:** Q. 27 to Q. 34 each carries **Four** marks.(Attempt any **Five**)*
5. *Use of log table is allowed. Use of calculator is not allowed.*
6. *Figures to the right indicate full marks.*
7. *Use of graph paper is not necessary. Only rough sketch of graph is expected.*
8. *For each MCQ, correct answer must be written along with its alphabet:*
   *e.g.(a)……….. / (b) ……….. / (c) ……….. / (d) ……….. etc.*
9. *Start answers to each section on a new page.*

### SECTION- A

**Q.1. Select and write the most appropriate answer from the given alternatives for each question:** **[16]**

i. In $\Delta ABC$, if a = 2, b = 3 and $\sin A = \frac{2}{3}$, then $\angle B$ = _______.

(A) $\frac{\pi}{4}$ (B) $\frac{\pi}{2}$ (C) $\frac{\pi}{3}$ (D) $\frac{\pi}{6}$ (2)

ii. If $\bar{a} = 3\hat{i} - \hat{j} + 4\hat{k}$, $\bar{b} = 2\hat{i} + 3\hat{j} - \hat{k}$ and $\bar{c} = -5\hat{i} + 2\hat{j} + 3\hat{k}$, then $\bar{a}\cdot(\bar{b}\times\bar{c})$ is _______.

(A) 100 (B) 110 (C) 109 (D) 108
(2)

iii. The cartesian equation of the line passing through the points A(4, 2, 1) and B(2, –1, 3) is _______.

(A) $\frac{x+4}{2} = \frac{y-2}{3} = \frac{z-1}{-2}$ (B) $\frac{x-4}{-2} = \frac{y-2}{-3} = \frac{z-1}{-2}$

(C) $\frac{x-4}{2} = \frac{y-2}{3} = \frac{z-1}{-2}$ (D) $\frac{x-4}{-2} = \frac{y-2}{3} = \frac{z-1}{-2}$ (2)

iv. If the line $\bar{r} = (\hat{i} - 2\hat{j} + 3\hat{k}) + \lambda(2\hat{i} + \hat{j} + 2\hat{k})$ is parallel to the plane $\bar{r}\cdot(3\hat{i} - 2\hat{j} + m\hat{k}) = 10$, then value of m is _______.

(A) – 2 (B) 2 (C) ± 2 (D) 0 (2)

v. If f($x$) = 1 – $x$, for $0 < x \le 1$ = k, for $x$ = 0 is continuous at $x$ = 0, then k = ______.

(A) 0 (B) –1 (C) 2 (D) 1 (2)

vi. The function f($x$) = $x^x$ is minimum at $x$ = _______.

(A) e (B) – e (C) $\frac{1}{e}$ (D) $-\frac{1}{e}$ (2)

vii. If $\int_0^k 4x^3 dx = 16$, then the value of k is _______.

(A) 1 (B) 2 (C) 3 (D) 4 (2)

viii. Order and degree of differential equation $\frac{d^4y}{dx^4}=\left[1+\left(\frac{dy}{dx}\right)^2\right]^3$ respectively are ________.

(A) Order: 1, Degree: 4 (B) Order: 4, Degree: 1
(C) Order: 6, Degree: 1 (D) Order: 1, Degree: 6 (2)

**Q.2. Answer the following questions:** **[4]**

i. Write the dual of $p \wedge \sim p \equiv F$ (1)

ii. Find the general solution of $\tan 2x = 0$ (1)

iii. Differentiate $\sin(x^2 + x)$ w.r.t. $x$ (1)

iv. If X ~ B(n, p) and n = 10, E(X) = 5, then find the value of p. (1)

## SECTION- B

**Attempt any EIGHT of the following questions:** **[16]**

**Q.3.** Using truth table verify that $\sim (p \vee q) \equiv \sim p \wedge \sim q$ (2)

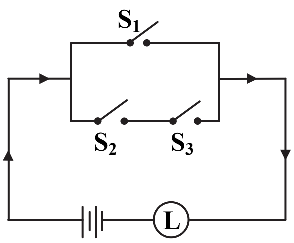
**Q.4.** Find the matrix of co-factors for the matrix $\begin{bmatrix} 1 & 3 \\ 4 & -1 \end{bmatrix}$ (2)

**Q.5.** Find the angle between the lines represented by $3x^2 + 4xy - 3y^2 = 0$ (2)

**Q.6.** $\bar{a}$ and $\bar{b}$ are non-collinear vectors. If $\bar{c} = (x - 2)\bar{a} + \bar{b}$ and $\bar{d} = (2x + 1)\bar{a} - \bar{b}$ are collinear, then find the value of $x$. (2)

**Q.7.** If a line makes angles 90°, 135°, 45° with X, Y and Z axes respectively, then find its direction cosines. (2)

**Q.8.** Express the following circuit in symbolic form:




(2)

**Q.9.** Differentiate log (sec $x$ + tan $x$) w.r.t. $x$. (2)

**Q.10.** Evaluate: $\int \frac{dx}{x^2 + 4x + 8}$ (2)

**Q.11.** Evaluate: $\int_0^{\frac{\pi}{2}} \cos^2 x \, dx$ (2)

**Q.12.** Solve the differential equation $\frac{dy}{dx} = x^2y + y$ (2)

**Q.13.** Find expected value of the random variable X whose probability mass function is: (2)

| X = $x$ | 1 | 2 | 3 |
|---|---|---|---|
| P(X = $x$) | $\frac{1}{5}$ | $\frac{2}{5}$ | $\frac{2}{5}$ |

**Q.14.** If $y = x \log x$, then find $\frac{d^2y}{dx^2}$. (2)

## SECTION- C

**Attempt any EIGHT of the following questions:** **[24]**

**Q.15.** State the converse, inverse and contrapositive of the conditional statement:
'If a sequence is bounded, then it is convergent'. (3)

**Q.16.** Show that: $\sin^{-1}\left(\frac{8}{17}\right) + \sin^{-1}\left(\frac{3}{5}\right) = \sin^{-1}\left(\frac{77}{85}\right)$. (3)

**Q.17.** Show that the points A(2, 1, –1), B(0, –1, 0), C(4, 0, 4) and D(2, 0, 1) are coplanar. (3)

**Q.18.** If $\Delta$ABC is right angled at B, where A(5, 6, 4), B(4, 4, 1) and C(8, 2, $x$), then find the value of $x$. (3)

**Q.19.** Find the equation of the line passing through the point (3, 1, 2) and perpendicular to the lines $\frac{x-1}{1} = \frac{y-2}{2} = \frac{z-3}{3}$ and $\frac{x}{-3} = \frac{y}{2} = \frac{z}{5}$. (3)

**Q.20.** Find the distance of the point $\hat{i} + 2\hat{j} - \hat{k}$ from the plane $\bar{r}\cdot\left(\hat{i} - 2\hat{j} + 4\hat{k}\right) = 10$ (3)

**Q.21.** If $e^x + e^y = e^{x+y}$, show that $\frac{dy}{dx} = -e^{y-x}$ (3)

**Q.22.** The surface area of a spherical balloon is increasing at the rate of 2 $cm^2$/sec. At what rate the volume of the balloon is increasing when the radius of the balloon is 6 cm? (3)

**Q.23.** Find the approximate value of $e^{1.005}$; given e = 2.7183. (3)

**Q.24.** Evaluate: $\int \frac{x^2 \cdot \tan^{-1}\left(x^3\right)}{1+x^6}\,dx$ (3)

**Q.25.** Solve the differential equation $\frac{dy}{dx} + y = e^{-x}$ (3)

**Q.26.** If f($x$) = k$x$, $0 < x < 2$
= 0, otherwise,
is a probability density function of a random variable X, then find:
i. Value of k.
ii. P(1 < X < 2) (3)

## SECTION- D

**Attempt any FIVE of the following questions:** **[20]**

**Q.27.** Prove that a homogeneous equation of degree two in $x$ and $y$ i.e. $ax^2 + 2hxy + by^2 = 0$ represents a pair of lines passing through the origin, if $h^2 - ab \geq 0$. (4)

**Q.28.** Solve the following linear programming problem:
Maximise: $z = 150x + 250y$
Subject to; $4x + y \leq 40$
$3x + 2y \leq 60$
$x \geq 0$
$y \geq 0$ (4)

**Q.29.** Solve the following equations by the method of reduction:
$x + 3y + 3z = 12$
$x + 4y + 4z = 15$
$x + 3y + 4z = 13$ (4)

**Q.30.** In $\Delta$ABC, if a + b + c = 2s, then prove that $\sin\left(\frac{A}{2}\right) = \sqrt{\frac{(s-b)(s-c)}{bc}}$, with usual notations. (4)

**Q.31.** Function f($x$) is continuous on its domain [–2, 2], where

$$f(x) = \frac{\sin ax}{x} + 2, \text{ for } -2 \le x < 0$$
$$= 3x + 5, \text{ for } 0 \le x \le 1$$
$$= \sqrt{x^2 + 8} - b, \text{ for } 1 < x \le 2$$

Find the value of a + b + 2. (4)

**Q.32.** Prove that: $\int \sqrt{x^2 + a^2} \cdot dx = \frac{x}{2}\sqrt{x^2 + a^2} + \frac{a^2}{2} \log\left|x + \sqrt{x^2 + a^2}\right| + c$ (4)

**Q.33.** A fair coin is tossed 8 times. Find the probability that:

i. is shows no head

ii. it shows head at least once. (4)

**Q.34.** Prove that:

$$\int_0^{2a} f(x)\,dx = \int_0^{a} f(x)\,dx + \int_0^{a} f(2a - x)\,dx$$ (4)