Bachelor of Science (B.Sc.) Semester–I Examination

MATHEMATICS (ALGEBRA AND TRIGONOMETRY)

Optional Paper—1

Time : Three Hours] [Maximum Marks : 60

N.B. :— (1) Solve all the **FIVE** questions.

(2) All questions carry equal marks.

(3) Questions 1 to 4 have an alternative. Solve each question in full or its alternative in full.

UNIT—I

1. (A) Find non-singular matrices P and Q, so that PAQ is in the normal form :

where $A = \begin{bmatrix} 1 & -1 & -1 \\ 1 & 1 & 1 \end{bmatrix}$ 6

(B) Investigate the values of λ and μ so that the system of equations :

$2x + 3y + 5z = 9$

$7x + 3y - 2z = 8$

$3x + 4y - \lambda z = \mu$

has

(i) no solution,

(ii) a unique solution, and

(iii) an infinite number of solutions. 6

OR

(C) Find eigen values of the matrix :

$$\begin{bmatrix} 2 & 2 & 0 \\ 2 & 1 & 1 \\ -7 & 2 & -3 \end{bmatrix}$$

and find eigen vectors corresponding to the lowest eigen value. 6

(D) Verify Cayley-Hamilton theorem for the matrix $\begin{bmatrix} 2 & 3 \\ 1 & 5 \end{bmatrix}$ and hence find its inverse.

Further, express $7A^3 + 11A^2 - A - 10I$

as a linear polynomial in A. 6

UNIT—II

2. (A) Solve the equation :

$x^3 - 14x^2 + 56x - 64 = 0$

whose roots are in geometric progression. 6

(B) Solve the equation :

$x^4 + 20x^3 + 143x^2 + 430x + 462 = 0$

by removing its cubic term. 6

OR

(C) Solve the equation : 6

$x^3 - 18x = 35$ by Cardon's method.

(D) Solve the reciprocal equation : 6

$2x^5 - 15x^4 + 37x^3 - 37x^2 + 15x - 2 = 0.$

UNIT—III

3. (A) Prove that :

$$(1+i)^n + (1-i)^n = 2^{\frac{n+2}{2}} . \cos\left(\frac{np}{4}\right),$$

where n is any integer. 6

(B) Solve the equation $x^7 + 1 = 0$. 6

OR

(C) If cosh y = x then prove that : 6

$$y = \cosh^{-1}x = \log_e\left[x + \sqrt{x^2 - 1}\right]$$

(D) Prove that :

$$\text{Log}_e(x+iy) = \frac{1}{2}\log_e(x^2+y^2) + i\left[2np + \tan^{-1}\left(\frac{y}{x}\right)\right]$$

Hence find $\text{Log}_e(-7)$. 6

UNIT—IV

4. (A) Prove that the set G = {1, –1, i, –i} is an abelian multiplicative finite group of order 4. 6

(B) Let $G = \{a, a^2, a^3, a^4 = 1\}$ be a finite group of order 4 under multiplication and $H = \{1, a^2\}$ be a sub-group of G.

(i) Find all the cosets of H in G.

(ii) Hence, prove that G is a union of all these cosets.

(iii) Further, establish that any two cosets are either disjoint or indentical. 6

OR

(C) Show that the order of a sub-group of a finite group is a divisor of the order of the group. 6

(D) Show that the permutations :

$I = f_1$, $(a\ b) = f_2$, $(c\ d) = f_3$

$(a\ b) . (c\ d) = f_4$ on four symbols a, b, c, d form a finite abelian group with respect to the product of permutations. 6

UNIT—V

5. (A) Find the condition on x, so that rank of the matrix $A = \begin{bmatrix} -1 & 2 & 1 \\ 4 & 1 & 0 \\ x & 2 & 2 \end{bmatrix}$ is 3. 1½

(B) Solve the system of linear equations :

$$2x - 2y - 2z = 0, \; x - y = 0.$$ 1½

(C) Discuss the nature of roots of the equation :

$$x^7 + 3x^6 - 5x^3 + 2x^2 - 10x + 5 = 0$$

by using Descarte's rule of sign. 1½

(D) Find the condition that the product of two roots of the equation :

$x^3 + px^2 + qx - r = 0$ is 2. 1½

(E) Separate $\sinh(\alpha - i\beta)$ into real and imaginary parts. 1½

(F) Prove that $\cosh^2 x - \sinh^2 x = 1$. 1½

(G) Let $f = \begin{pmatrix} 1 & 2 & 3 \\ 1 & 3 & 2 \end{pmatrix}$ and $g = \begin{pmatrix} 1 & 2 & 3 \\ 2 & 3 & 1 \end{pmatrix}$ be two permutations of degree 3. Find fg and gf and write whether fg = gf. 1½

(H) Determine the identity element of a group (I, o) of all integers under the operation

$a \circ b = a + b - 1$. 1½