Bachelor of Science (B.Sc.) Semester—II (C.B.S.) Examination

MATHEMATICS

(Vector Calculus and Improper Integrals)

Compulsory Paper—2

Time : Three Hours] [Maximum Marks : 60

N.B. :— (1) Solve all the **FIVE** questions.

(2) All questions carry equal marks.

(3) Question No. **1** to **4** have an alternative. Solve each question in full or its alternative in full.

UNIT—I

1. (A) Find the angle between the surfaces $x^2 - y^2 + z^2 = 9$ and $z = x^2 - y^2 - 3$ at the point (0, 0, –3). 6

(B) Show that $\vec{F} = xz^2y^2\,\vec{i} + yx^2z^2\,\vec{j} + zx^2y^2\,\vec{k}$ is conservative force field. Hence find scalar potential ϕ. 6

OR

(C) Let $\vec{F} = (2y+3)\,\vec{i} + xz\,\vec{j} + (yz - x)\,\vec{k}$ be a force field. Show that the work done by $\vec{F}$ is $\int_C \vec{F} \circ d\vec{r} = 10$ along the path C consists of straight lines from (0, 0, 0) to (0, 0, 1), then to (0, 1, 1) and then to (2, 1, 1). 6

(D) Evaluate $\vec{\nabla}^2 r^{\frac{7}{2}}$, where $\vec{r} = x\,\vec{i} + y\,\vec{j} + z\,\vec{k}$. 6

UNIT—II

2. (A) Evaluate $\iint_R \sqrt{4x^2 - y^2}\,dx\,dy$, where R is the region bounded by y = 0, y = x, x = 1. 6

(B) Evaluate $\int_0^1 \int_0^{\sqrt{1-x^2}} y^2\,dydx$ by changing the order of integration. 6

OR

(C) Using double integration, find area between the circle $x^2 + y^2 = a^2$ and the line x + y = a in the first quadrant. 6

(D) Evaluate $\int_0^1 \int_0^x \frac{x^3}{\sqrt{x^2 + y^2}}\,dxdy$ by changing to polar coordinates. 6

UNIT—III

3. (A) Evaluate $\oint_C \left[(x^2 - 2xy)\,dx + (x^2y + 3)\,dy\right]$ around the boundary of the region defined by $y^2 = 8x$ and x = 2 by using Green's theorem in the plane. 6

(B) Evaluate $\iint_S \vec{A} \circ \vec{n}\, dS$ over the entire surface S of the region bounded by the cylinder $x^2 + z^2 = 9$, $x = 0$, $y = 0$, $z = 0$ and $y = 8$ if $\vec{A} = 6z\,\vec{i} + (2x + y)\,\vec{j} - x\,\vec{k}$. 6

OR

(C) Evaluate $\iiint_V (2x - y)\, dV$, where V is the closed region bounded by the cylinder $z = 4 - x^2$ and the planes $x = 0$, $y = 0$, $y = 3$ and $z = 0$. 6

(D) Prove that :

$$\iiint_V \frac{dV}{r^2} = \iint_S \frac{\vec{r} \circ \vec{n}}{r^2}\, dS.$$ 6

UNIT—IV

4. (A) Test convergence of the integrals :

(i) $$\int_1^{\infty} \frac{x^2 dx}{3x^5 + 5x^3 + 3}$$

(ii) $$\int_0^{\infty} \frac{1 - \cos x}{x^2}\, dx.$$ 6

(B) Show that $\int_a^{\infty} \frac{dx}{x^p}$, where $a > 0$ is convergent when $p > 1$ and divergent when $p \le 1$. 6

OR

(C) Show that $\left|\overline{\frac{1}{2}}\right. = \sqrt{\pi}$, using definition of Gamma function. 6

(D) Show that :

(i) $$\int_0^a (a - x)^{m-1} \cdot x^{n-1}\, dx = a^{m+n-1} \cdot B(m, n)$$

(ii) $$\int_0^{\pi/2} \sqrt{\sin\theta}\, d\theta \times \int_0^{\pi/2} \frac{d\theta}{\sqrt{\sin\theta}} = \pi.$$ 6

QUESTION—V

5. (A) Find the unit tangent vector to the curve $x = 1 - t^2$, $y = 4t - 3$, $z = t^2 - 6$ at the point where $t = 2$. 1½

(B) Find the directional derivative of $\phi = x^2yz + 2xz - 4$ at the point (2, –1, 1) in the direction of Y-axis.
1½

(C) Evaluate $\int_{1}^{3}\int_{0}^{x} \frac{1}{x^2 + y^2}\,dx\,dy$. 1½

(D) Evaluate $\iiint_V dxdydz$ over the volume enclosed by three coordinate planes and the plane $x + y + z = 1$ in the first octant. 1½

(E) Show that the area enclosed by a simple closed curve C is given by $\frac{1}{2}\oint [x\,dy - y\,dx]$. 1½

(F) Show that $\iint_S (ax\,\vec{i} + by\,\vec{j} + cz\,\vec{k}) \circ \vec{n}\,dS = \frac{4}{3}\pi(a + b + c)$, where S is the surface of the sphere $x^2 + y^2 + z^2 = 1$. 1½

(G) Evaluate $\int_{0}^{\infty} x^5 e^{-2x} dx$ by reducing it into Gamma function. 1½

(H) Find the value of Beta function $B\left(\frac{1}{2}, \frac{7}{2}\right)$. 1½