NKT/KS/17/5078

Bachelor of Science (B.Sc.) Semester—II (C.B.S.) Examination

MATHEMATICS

Compulsory Paper—1

(M_3 Geometry, Differential and Difference Equations)

Time : Three Hours] [Maximum Marks : 60

N.B. :— (1) Solve all the **FIVE** questions.

(2) All questions carry equal marks.

(3) Question Nos. **1** to **4** have an alternative. Solve each question in full or its alternative in full.

UNIT—I

1. (A) Find the equation of the sphere which passes through the points (1, 0, 0), (0, 1, 0), (0, 0, 1) and has its radius as small as possible. 6

(B) Show that the plane $lx + my + nz = p$ will touch the sphere $x^2 + y^2 + z^2 + 2ux + 2vy + 2wz + d = 0$, if $(ul + vm + wn + p)^2 = (l^2 + m^2 + n^2)(u^2 + v^2 + w^2 - d)$. 6

OR

(C) Find the equation of the right circular cone which passes through the point (1, 1, 2) and has its vertex at the origin and axis the line $\frac{x}{2} = \frac{-y}{4} = \frac{z}{3}$. 6

(D) Find the equation of the right circular cylinder of radius 2 and whose axis is the line $\frac{x-1}{2} = \frac{y-2}{1} = \frac{z-3}{2}$. 6

UNIT—II

2. (A) Prove that the general solution of the linear differential equation $\frac{dy}{dx} + Py = Q$, where P and Q are functions of x or constants, is given by $ye^{\int Pdx} = \int Qe^{\int Pdx}dx + c$ and hence solve $\frac{dy}{dx} + y \cdot \tan x = \sec x$. 6

(B) Solve $(x^2 + y^2)\,dx + xy\,dy = 0$ by finding integrating factor. 6

OR

(C) Solve $p - \frac{1}{p} = \frac{x}{y} - \frac{y}{x}$, where $p = \frac{dy}{dx}$. 6

(D) Solve $y = 2px + y^2 p^3$, using method of solvable for x, where $p = \frac{dy}{dx}$. 6

UNIT—III

3\. (A) Solve $(D^2 + 3D - 4)y = x\, e^{-2x}$, where $D = \frac{d}{dx}$. 6

(B) Solve $x^2 \frac{d^2y}{dx^2} - x\frac{dy}{dx} + y = \log x$. 6

OR

(C) Solve $xy^{(2)} - (2x - 1)y^{(1)} + (x - 1)y = 0$ for which $y = e^x$ is an integral. 6

(D) Solve $y^{(2)} + 4y = \operatorname{cosec} 2x$ by using method of variation of parameters. 6

UNIT—IV

4\. (A) From the relation $u_x = c_1\, 3^x + c_2 (-1)^x$, derive the difference equation by eliminating the arbitrary constants c_1 and c_2. 6

(B) Solve $u_{x+2} - 3u_{x+1} + 2u_x = 4^x$, given that $u_0 = 0$, $u_1 = 1$. 6

OR

(C) Solve $u_{x+2} - 7u_{x+1} + 10\, u_x = 12.4^x$. 6

(D) Solve $u_{x+2} + u_x = \sin(x/2)$. 6

UNIT —V

5\. (A) Find the equation of the sphere through the circle $x^2 + y^2 + z^2 = 9$, $2x + 3y + 4z = 5$ and the point (1, 2, 3). 1½

(B) Prove that the semivertical angle of a right circular cone admitting sets of three mutually perpendicular generator is $\tan^{-1}\sqrt{2}$. 1½

(C) Reduce the equation $\frac{dy}{dx} - \frac{1}{x}\tan y = x^2 \sec y$ to the linear form. 1½

(D) Solve $p = \sin(y - xp)$, where $p = \frac{dy}{dx}$. 1½

(E) Find the particular integral of $(D^2 - 4D + 3)y = e^{3x}$. 1½

(F) Solve $(D^3 - D^2 - 12D)\, y = 0$. 1½

(G) Solve $u_{x+3} - 3u_{x+1} - 2u_x = 0$. 1½

(H) Write the difference equation $(\Delta^2 + 2\Delta + 5)u_x = 0$ in E-form. 1½