NRT/KS/19/2075

Bachelor of Science (B.Sc.) Semester—III Examination

MATHEMATICS (Advanced Calculus, Sequence & Series)

Optional Paper—I

Time : Three Hours] [Maximum Marks : 60

Note :— (1) Solve all **FIVE** questions.

(2) All questions carry equal marks.

(3) Question Nos. **1** to **4** have an alternative. Solve each question in full or its alternative in full.

UNIT—I

1. (A) If the functions f(x) and F(x) are derivable in (a, b) and $F'(x) \neq 0$ for any x in [a, b], then prove that there exists at least one value 'C' in (a, b) such that $\frac{f(b)-f(a)}{F(b)-F(a)} = \frac{f'(c)}{F'(c)}$. 6

(B) By using $\in$-δ technique, show that $\lim_{(x,y)\to(2,3)} xy = 6$. 6

OR

(C) Show that the function $f(x, y) = \begin{cases} x \sin\frac{1}{y} & \text{when } (x, y) \neq (0, 0) \\ 0 & \text{when } (x, y) = (0, 0) \end{cases}$ is continuous at origin (0, 0). 6

(D) Expand $e^x \sin y$ in powers of x and y upto third degree terms. 6

UNIT—II

2. (A) Find the envelope of the family of straight lines $x \cos^3\alpha + y \sin^3\alpha = c$, where α is the parameter. 6

(B) Discuss the maximum or minimum values of $u = x^3y^2(1-x-y)$. 6

OR

(C) Find the envelope of the straight line $\frac{x}{a}+\frac{y}{b}=1$ where $ab = c^2$ and c is a constant and a and b are parameters. 6

(D) Find the minimum value of $x^2 + y^2 + z^2$ when $yz + zx + xy = 3a^2$ using Lagrange's multiplier method. 6

UNIT—III

3. (A) Prove that the sequence where n^{th} term is $x_n = \frac{1}{1.2} + \frac{1}{2.3} + ... + \frac{1}{n(n+1)}$ is monotonic and bounded. 6

(B) If sequences $<x_n>$ and $<z_n>$ each converge to ℓ and if $x_n < y_n < z_n$, $\forall\, n \in N$, then prove that sequence $<y_n>$ also coverges to ℓ. 6

OR

(C) Prove that the sequence $<x_n>$, where $x_n = \frac{3n+4}{2n+1}$ is bounded monotonic decreasing and tends to limit $\frac{3}{2}$. 6

(D) Prove that a sequence is convergent iff it is a Cauchy sequence. 6

UNIT—IV

4. (A) Test the convergence of the series $\sum\left[(n^3+1)^{1/3}-n\right]$ using comparison test. 6

(B) Test for convergence of the series $\sum_{n=2}^{\infty}\frac{1}{n(\log n)^p}$, $p \geq 0$ using Cauchy's integral test. 6

OR

(C) Show that the series $\sum\frac{(n+\sqrt{n})^n}{2^n n^{n+1}}$ is covergent using Cauchy's Root test. 6

(D) Test the absolute convergence and conditional convergence of the series $1-\frac{1}{3}+\frac{1}{5}-\frac{1}{7}+......$ 6

UNIT—V

5. (A) Verify Lagranges Mean value theorem for $f(x) = 2x^2 - 7x - 10$ in [2, 5]. 1½

(B) Verify whether the iterated limits are equal for $f(x, y) = \frac{x^2+y^2}{x+y}$. 1½

(C) Find the envelope of $y = t^2(x - t)$ where t is the parameter. 1½

(D) Find the stationary points of $f(x, y) = x^3 + y^3 - 3xy$. 1½

(E) Find $n_0 \in N$ such that $\left|\frac{n}{n+3}-1\right| < \frac{1}{5}$, $\forall\ n \geq n_0$. 1½

(F) Find the limit of the sequence $\left\langle\frac{n^2+1}{5n^2-1}\right\rangle$. 1½

(G) State the D'Alembert's Ratio test for convergence of a series. 1½

(H) Show that the alternating series $1-\frac{1}{2}+\frac{1}{2^2}-\frac{1}{2^3}+.......$ is convergent. 1½