NJR/KS/18/3075

Bachelor of Science (B.Sc.) Semester—III (C.B.S.) Examination

MATHEMATICS (Advanced, Calculus, Sequence and Series)

Paper—I

Time : Three Hours] [Maximum Marks : 60

Note :— (1) Solve all the **five** questions.

(2) All questions carry equal marks.

(3) Question Nos. **1** to **4** have an alternative. Solve each question in full or its alternative in full.

UNIT—I

1. (a) If a function f(x) is continuous on [a, b] and derivable in (a,b), then prove that there exists a point $c \in (a, b)$ such that :

$$\frac{f(b)-f(a)}{b-a} = f'(c).$$ 6

(b) By using Lagrange's mean value theorem show that :

$$\frac{x}{1+x} < \log(1+x) < x, \quad x > 0$$

Hence show that :

$$0 < \frac{1}{\log(1+x)} - \frac{1}{x} < 1, \forall x > 0.$$ 6

OR

(c) If $\lim\limits_{(x,y)\to(x_0,y_0)} f(x,y) = A$ and $\lim\limits_{(x,y)\to(x_0,y_0)} g(x,y) = B$ then prove that

$$\lim_{(x,y)\to(x_0,y_0)} [f(x,y)\cdot g(x,y)] = A \cdot B.$$ 6

(d) Expand by Taylor's series :

$f(x, y) = x^2 + xy + y^2$ in powers of (x – 2) and (y – 3). 6

UNIT—II

2. (a) Find the envelope of the family of lines $ax \sec \alpha - by \operatorname{cosec} \alpha = a^2 - b^2$, where α being a parameter and a, b are constants. 6

(b) Discuss maxima or minima of

$u = x^3 + y^3 - 3axy.$ 6

OR

(c) Find the maxima or minima of u, when $u = 2xy - 3x^2y - y^3 + x^3y + xy^3$. 6

(d) Find the maximum and minimum values of $x^2 + y^2 + z^2$ subject to the conditions $x + y + z = 1$ and $xyz + 1 = 0$, by using Lagrange's multiplier method. 6

UNIT—III

3. (a) Prove that every convergent sequence has a unique limit. 6

(b) Show that the sequence whose n^{th} term is $\left\langle \frac{5n+4}{2n+1} \right\rangle$ is bounded, monotonic decreasing and tends to the limit 5/2. 6

OR

(c) If a sequence $< x_n >$ is a Cauchy sequence then prove that it is convergent. 6

(d) If $< x_n >$ is a sequence in R, where

$$x_n = 1 + \frac{1}{2} + \frac{1}{3} + ...+ \frac{1}{n}.$$

Evaluate $\lim_{n\to\infty} |x_{n+1} - x_n|$. Verify whether this sequence satisfies the Cauchy criterion. 6

UNIT—IV

4. (a) Test the convergence of the series :

(i) $\sum_{n=1}^{\infty} \left[\sqrt{n^4+1} - \sqrt{n^4-1}\right]$ by comparison test.

(ii) $\sum_{n=1}^{\infty} \frac{n}{n^2+1}$ by integral test. 6

(b) Test the convergence of the series $\sum_{n=1}^{\infty} \frac{n^3+a}{2^n+a}$ by ratio test. 6

OR

(c) Show that the alternating series $\frac{2}{1} - \frac{3}{2^2} + \frac{4}{3^2} - \frac{5}{4^2} + ...$ is conditionally convergent. 6

(d) Test the convergence of the series $1 + \frac{3}{2}x + \frac{5}{9}x^2 + \frac{7}{28}.x^3 + ... + \frac{2n+1}{n^3+1}x^n + ...$ by ratio test. 6

Question—V

5. (a) Verify Rolle's theorem for the function $f(x) = \frac{\sin x}{e^x}, x \in [0, \pi]$. 1½

(b) Examine whether the function f(x, y) is continuous at (1, 2), where

$$f(x,y) = x^2 + 4y \quad \text{when}(x,y) \neq (1,2)$$
$$= 0 \quad \text{when}(x,y) = (1,2)$$ 1½

(c) Find the envelope of

$y = mx + a\sqrt{1+m^2}$, where m is a parameter. 1½

(d) Define : Stationary point and Saddle point of the function f(x, y). 1½

(e) Find $n_0 \in N$ such that :

$$\left| \frac{2n}{n+3} - 2 \right| < \frac{1}{6}.$$ 1½

(f) Show that the sequence $< x_n >$ where $x_n = \frac{3n^2+1}{3n^2-4}$ converges to 1. 1½

(g) Show that the series $\sum_{n=2}^{\infty} \frac{1}{(\log n)^n}$ is divergent by using root test. 1½

(h) Show that an alternating series $\sum_{n=1}^{\infty} (-1)^{n-1} \frac{1}{n^2}$ is absolutely convergent. 1½