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NJR/KS/18/3091

B.Sc. Statistics Semester-III (C.B.S.) Examination

STATISTICS (Statistical Methods)

Compulsory Paper—1

Time : Three Hours] [Maximum Marks : 50

N.B. :— ALL questions are compulsory and carry equal marks.

1. (A) Define (i) joint p.m.f. of two discrete random variables X and Y, (ii) Marginal p.m.f. of X, (iii) Conditional p.m.f. of Y given x.

Let the joint probability function of discrete random variables X and Y be :

$$f(x, y) = \begin{cases} \dfrac{x+2y}{27}, & x = 0, 1, 2 \\ & y = 0, 1, 2 \\ 0 & , \text{elsewhere.} \end{cases}$$

(i) Present the joint distribution of X and Y in a bivariate table.

(ii) Find marginal distribution of X and Y.

(iii) Find E(X).

(iv) Derive conditional distribution of Y given X = 2. 10

OR

(E) Define :

(i) Bivariate p.d.f. of random variables X and Y.

(ii) Conditional p.d.f. of X given Y = y.

(iii) Conditional mean of X given Y = y.

(iv) Conditional variance.

(v) Stochastic independence of r-vs X and Y.

If the random variables X and Y have joint p.d.f.

$$f(x, y) = \begin{cases} 12xy(1-y), & 0 < x < 1 \\ & 0 < y < 1 \\ 0 & , \text{elsewhere.} \end{cases}$$

Check whether X and Y are stochastically independent. 10

2. (A) State the p.m.f. of trinomial distribution. Find the marginal p.m.f. of X. A manufactured item is classified as "good", a "second" or "defective" with probabilities 6/10, 3/10 and 1/10 respectively. Fifteen such items are selected at random from the production line. Let X denote the number of good items. Y denote the number of seconds and 15-X-Y the number of defectives.

(a) Give the joint p.m.f. of X and Y.

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(b) Find P(X = 10, Y = 4).

(c) Find P(X ≤ 1). 10

OR

(E) Define Bivariate normal distribution of a pair of random variables (X, Y). Find marginal p.d.f. of X. Also find m.g.f. of bivariate normal distribution. Show that X and Y are independent iff ρ, the correlation coefficient between X and Y is zero. 10

3. (A) Let X_1 and X_2 have joint p.d.f.

$$f(x_1, x_2) = \begin{cases} 2, & 0 < x_1 < x_2 < 1 \\ 0, & \text{elsewhere.} \end{cases}$$

Find :

(i) Joint p.d.f. of $Y_1 = \frac{X_1}{X_2}$ and $Y_2 = X_2$.

(ii) Marginal p.d.f. of Y_1 and Y_2.

(iii) Are Y_1 and Y_2 independent ?

(B) If X and Y are independent chi-square variables with n_1 and n_2 degrees of freedom respectively then show that probability distribution of $\frac{X}{Y}$ is $\beta_2\left(\frac{n_1}{2}, \frac{n_2}{2}\right)$. 5+5

OR

(E) Suppose that random variables X and Y have joint density function given by :

$$f(x, y) = \begin{cases} c(2x + y), & 2 < x < 6, 0 < y < 5 \\ 0, & \text{elsewhere.} \end{cases}$$

Find :

(i) c

(ii) Joint p.d.f. of U = 2X and V = 2Y.

(iii) Marginal p.d.f. of U and V.

(iv) Are U and V independent ? 10

4. (A) Define F-Statistic. Derive its p.d.f. Find r^{th} raw moment about origin and hence find mean and variance of F-distribution. 10

OR

(E) Define chi-square statistic. State its p.d.f. Find its m.g.f. and hence find mean and variance. Also prove additive property of chi-square distribution. 10

5. Solve any **10** of the following questions :

(A) Define Fisher's t.

(B) Show that student's t follows Fisher's t with (n – 1) d.f.

(C) What is mode of t distribution ?

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(D) State conditional mean of X given Y = y if (X, Y) follows bivariate normal distribution.

(E) State m.g.f. of Trinomial distribution.

(F) State conditional variance of $Y \mid X = x$ of bivariate normal distribution.

(G) Define a random sample.

(H) If joint p.d.f. of two r.vs, f(x, y) is given by :

$$f(x, y) = h(x) \cdot g(y)$$

where h(x) and g(y) are non-negative functions of x alone and y alone respectively, then show that rvs X and Y are stochastically independent.

(I) Define m.g.f. of bivariate probability distribution.

(J) Define $(r, s)^{th}$ product moment about origin. How can it be obtained from the m.g.f. of bivariate probability distribution ?

(K) Let a r.v. $X \sim N(\mu, \sigma^2)$ then derive the distribution of $Z = \left(\frac{X - \mu}{\sigma}\right)$.

(L) Define a statistic. 1×10=10

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