NJR/KS/18/3105

Bachelor of Science (B.Sc.) Semester-IV (C.B.S.) Examination

MATHEMATICS (PARTIAL DIFFERENTIAL EQUATIONS & CALCULUS OF VARIATION)

Paper—I

Time : Three Hours] [Maximum Marks : 60

N.B. :— (1) Solve all the **FIVE** questions.

(2) All questions carry equal marks.

(3) Questions **1** to **4** have an alternative. Solve each question in full or its alternative in full.

UNIT—I

1. (A) Find the integral curves of the equations :

$$\frac{dx}{xz-y}=\frac{dy}{yz-x}=\frac{dz}{1-z^2}.$$ 6

(B) Solve the Pfaffian differential equation :

$$(x^2z - y^3)dx + 3xy^2dy + x^3dz = 0$$

first showing that it is integrable. 6

OR

(C) Solve the homogeneous Pfaffian differential equation :

$$yz(y + z)dx + xz(x + z)dy + xy(x + y)dz = 0.$$ 6

(D) Form the partial differential equation by eliminating arbitrary function f from the equation :

$$f(x^2 - y^2 - z^2,\ z^2 + 2xy) = 0.$$ 6

UNIT—II

2. (A) Prove that the general solution of the linear partial differential equation Pp + Qq = R is F(u, v) = 0 where F is an arbitrary function and $u(x, y, z) = c_1$ and $v(x, y, z) = c_2$ form a solution of the equations $\frac{dx}{P}=\frac{dy}{Q}=\frac{dz}{R}$. 6

(B) Find the integral surface of the equation

$$x(y^2 + z)p - y(x^2 + z)q = (x^2 - y^2)z$$

which contains straight line 2x + y = 0, z = 4. 6

OR

(C) Show that the equations :

$$xp = yq \text{ and } z(xp + yq) = 2xy$$

are compatible and hence find their solution. 6

(D) Find a complete integral of the equation :

$$p^2x + q^2y = z$$

by Jacobi's method. 6

UNIT—III

3. (A) Solve :

$$2\frac{\partial^2 z}{\partial x^2} - 7\frac{\partial^2 z}{\partial x \partial y} + 4\frac{\partial^2 z}{\partial y^2} = 7(y - x^2).$$ 6

(B) Solve $(2D^2 - D'^2 - 3)z = 5\,e^{x-y} + \cos(x + 2y)$, where $D = \frac{\partial}{\partial x}$ and $D' = \frac{\partial}{\partial y}$. 6

OR

(C) Solve $(D^2 + DD' + D'^2)z = 2\cos y - x \sin y$, where $D = \frac{\partial}{\partial x}$, $D' = \frac{\partial}{\partial y}$. 6

(D) Solve $x^2\frac{\partial^2 z}{\partial x^2} - y^2\frac{\partial^2 z}{\partial y^2} - y\frac{\partial z}{\partial y} + x\frac{\partial z}{\partial x} = x^3 y^2$

by using $x = e^u$ and $y = e^v$. 6

UNIT—IV

4. (A) Test for the extremum of the functional :

$$I[y(x)] = \int_0^1 (y'^2 + 7y' + 4)dx, \quad y(0) = 1, \ y(1) = 5$$

and show that the extremal is a straight line. 6

(B) From the Euler's equation $F_y - \frac{d}{dx}Fy' = 0$ for the extremals of the functional

$$I[y(x)] = \int_{x_0}^{x_1} F(x, y, y')dx,$$ deduce the following :

(i) If F depends only on y', then extremals are straight lines.

(ii) If F depends only on y and y' then extremals are given by $F_{y'}\, y' - F = \text{constant}$. 6

OR

(C) Find the extremal of the functional :

$$I[y(x)] = \int_0^{\pi} (y^2 - y''^2 + x^2)dx;$$

$y(0) = y(\pi) = 0$, $y'(0) = y'(\pi) = 1$. 6

(D) Find the Euler's equation of the functional :

$$I[y(x)] = \int_0^{\log 2} (e^{-x}\, y'^2 - e^x\, y^2)\, dx$$

and verify the invariance of Euler's equation under the co-ordinate transformation. 6

(Compulsory)

5. (A) Form a partial differential equation by eliminating arbitrary constants a and b from the equation :

$$x^2 - y^2 - (z - b)^2 = a^2.$$ 1½

(B) Find the integral curves of the equations :

$$\frac{dx}{1} = \frac{dy}{-2} = \frac{dz}{3x^2 \sin(y + 2x)}.$$ 1½

(C) Find the general integral of the linear partial differential equation $x^2p + y^2q = z$. 1½

(D) Using Charpit's method, solve the non-linear partial differential equation :

$$pq = 10, \quad p = \frac{\partial z}{\partial x}, q = \frac{\partial z}{\partial y}.$$ 1½

(E) Solve $t = \sin(xy)$, where $t = \frac{\partial^2 z}{\partial y^2}$ by integration. 1½

(F) Find P.I. of $(D^2 + D'^2 - 2DD' - 3D + 3D' + 2)z = e^{2x-y}$. 1½

(G) Define distance of order one between the functions $y = y_1(x)$ and $y = y_2(x)$ in the class $c^{(1)}\ [x_0, x_1]$. 1½

(H) Let $I[y(x)] = \int_0^1 [y(x)]^2\, dx$ be a functional. If $y(x) = e^x$, then obtain $I[y(x)]$. 1½