NRT/KS/19/2105

Bachelor of Science (B.Sc.) Semester—IV Examination

MATHEMATICS

(Partial Differential Equations and Calculus of Variation)

Optional Paper—I

Time : Three Hours] [Maximum Marks : 60

N.B. :— (1) Solve all the **FIVE** questions.

(2) All questions carry equal marks.

(3) Questions **1** to **4** have an alternative. Solve each question in full (A & B) or its alternative in full (C & D).

UNIT—I

1. (A) Show that integral curves of the equation $\dfrac{dx}{5y-4z}=\dfrac{dy}{3z-5x}=\dfrac{dz}{4x-3y}$ are circles. 6

(B) Verify that the equation $y(1+z^2)dx - x(1+z^2)dy + (x^2+y^2)dz = 0$ is integrable and find its primitive. 6

OR

(C) Show that a necessary and sufficient condition that there exists between two functions u(x, y) and v(x, y) a relation F(u, v) = 0, not involving x or y explicitly is that $\dfrac{\partial(u, v)}{\partial(x, y)} = 0$. 6

(D) Form the partial differential equation by eliminating arbitrary constants and functions from the following equations :

(i) $(x-a)^2 + (y-b)^2 + z^2 = 1$; a and b are arbitrary constants.

(ii) $z = f(x+ay) + \phi(x-ay)$; f and ϕ are arbitrary functions and a is a constant. 6

UNIT—II

2. (A) Find the general integral of the partial differential equation

$$(2xy-1)p + (z-2x^2)q = 2(x-yz)$$

and also the particular integral which passes through the line x = 1, y = 0. 6

(B) Show that the equations f(x, y, p, q) = 0, g(x, y, p, q) = 0 are compatible if :

$$\frac{\partial(f, g)}{\partial(x, p)} + \frac{\partial(f, g)}{\partial(y, q)} = 0.$$

Further, verify that the equations p = P(x, y), q = Q(x, y) are compatible if $\dfrac{\partial P}{\partial y} = \dfrac{\partial Q}{\partial x}$. 6

OR

(C) Find the general integral of the linear partial differential equation :

$$x(x^2+3y^2)p - y(3x^2+y^2)q = 2z(y^2-x^2).$$ 6

(D) Prove that an equation of the form

$$x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} + z\frac{\partial u}{\partial z} = f\left(\frac{\partial u}{\partial x}, \frac{\partial u}{\partial y}, \frac{\partial u}{\partial z}\right)$$

is solvable by Jacobi's method. Hence solve the equation :

$$\left(\frac{\partial u}{\partial x} - \frac{\partial u}{\partial y} - \frac{\partial u}{\partial z}\right)\left(x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} + z\frac{\partial u}{\partial z}\right) = 1.$$ 6

UNIT—III

3. (A) Solve the partial differential equation $(D'^2 + DD' + D')z = \sin(2x - 3y);\ D = \frac{\partial}{\partial x},\ D' = \frac{\partial}{\partial y}$. 6

(B) Solve $(D^2 - 4DD' + 4D'^2)z = 2\cos x - y^2$. 6

OR

(C) Solve the partial differential equation :

$$y^2 \frac{\partial^2 z}{\partial y^2} - y\frac{\partial z}{\partial y} = x \cdot y^2.$$ 6

(D) Solve $(D - 3D' - 2)^2 z = e^{2x} \tan(3x + y)$. 6

UNIT—IV

4. (A) Find the shortest curve joining the two points (x_1, y_1) and (x_2, y_2) in the plane. 6

(B) Find the extremal of the functional $I[y(x)] = \int_0^1 (xy' - y'^2)dx;\ y(0) = 1,\ y(1) = \frac{1}{4}$. 6

OR

(C) Find the general extremals of the functional $I[y(x), z(x)] = \int_{x_1}^{x_2} (y'^2 - 2y^2 + 2yz - z'^2)dx$. 6

(D) Derive Euler-Ostrogradsky equation for the functional $I = \iint_D \sqrt{1 + p^2 + q^2}\, dxdy;\ p = \frac{\partial z}{\partial x},\ q = \frac{\partial z}{\partial y}$. 6

Question—5

5. (A) Solve the equation :

$$3y^2z^2dx + 4z^2x^2dy + 5x^2y^2dz = 0$$ 1½

(B) Transform the equation $(y + z)dx + (z + x)dy + (x + y)dz = 0$ into a homogeneous equation in y and x using substitution $z = x + ky$, k is a constant. 1½

(C) Find complete integral of $\sqrt{p} + \sqrt{q} = 2x,\ p = \frac{\partial z}{\partial x},\ q = \frac{\partial z}{\partial y}$. 1½

(D) Find complete integral of the equation $pqz = p^2(xq + p^2) + q^2(yp + q^2)$. 1½

(E) Find particular integral of the partial differential equation $(D^2 - 4D'^2)z = e^{4x - 2y}$. 1½

(F) Solve $\frac{\partial^2 z}{\partial x^2} = \frac{\partial z}{\partial x}$. 1½

(G) Find the distance of order zero between the functions $y = x^3$ and $y = x^2$ on $[0, 2]$. 1½

(H) Show that extremal of the functional $I[y(x)] = \int_0^1 y'^2 dx;\ y(0) = 0,\ y(1) = 1$ is $y = x$. 1½