NKT/KS/17/5138

Bachelor of Science (B.Sc.) Semester-IV (C.B.S.) Examination

MATHEMATICS

Paper—I

(M_7 Partial Differential Equation and Calculus of Variation)

Time : Three Hours] [Maximum Marks : 60

Note :— (1) Solve all the **FIVE** questions.

(2) All questions carry equal marks.

(3) Question Nos. **1** to **4** have an alternative. Solve each question in full or its alternative in full.

UNIT—I

1. (A) Find the integral curves of the equations :

$$\frac{dx}{cy - bz} = \frac{dy}{az - cx} = \frac{dz}{bx - ay}$$

and show that they are circles. 6

(B) Verify the Pfaffian differential equation :

$(y^2 + z^2)dx + xydy + xzdz = 0$

is integrable and solve it. 6

OR

(C) Verify the equation :

$z(z + y^2)dx + z(z + x^2)dy - xy(x + y)dz = 0$

is integrable and find its primitive. 6

(D) Prove a necessary and sufficient condition that there exists between two functions u(x, y) and v(x, y) a relation F(u, v) = 0 not involving x or y explicitly is that :

$$\frac{\partial(u, v)}{\partial(x, y)} = 0.$$ 6

UNIT—II

2. (A) Find the general integral of the linear partial differential equation :

$z(xp - yq) = y^2 - x^2$. 6

(B) Show that the equations :

$xp - yq = x$ and $x^2p + q = xz$

are compatible and hence find their solution. 6

OR

(C) Find the complete integral of the partial differential equation :

$$p^2q^2 + x^2y^2 = x^2q^2(x^2 + y^2)$$

by using Charpit's method. 6

(D) Show that a complete integral of the equation $f\left(\frac{\partial u}{\partial x}, \frac{\partial u}{\partial y}, \frac{\partial u}{\partial z}\right) = 0$ is $u = ax + by + \theta(a, b)z + c$ where a, b, c are arbitrary constants and $f(a, b, \theta) = 0$. Hence find complete integral of the equation :

$$\frac{\partial u}{\partial x} + \frac{\partial u}{\partial y} + \frac{\partial u}{\partial z} = \frac{\partial u}{\partial x} \cdot \frac{\partial u}{\partial y} \cdot \frac{\partial u}{\partial z}.$$ 6

UNIT—III

3. (A) Solve :

$$\frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2} = -4\pi(x^2 + y^2).$$ 6

(B) Solve : $(D^2 - 3DD' + 2D'^2)z = \sin(x - 2y)$

where $D = \frac{\partial}{\partial x}, D' = \frac{\partial}{\partial y}$. 6

OR

(C) Solve :

$$\frac{\partial^2 z}{\partial x^2} - \frac{\partial^2 z}{\partial x \partial y} - 2\frac{\partial^2 z}{\partial y^2} + 2\frac{\partial z}{\partial y} + 2\frac{\partial z}{\partial x} = \sin(2x + y).$$ 6

(D) Solve :

$$x^2 \frac{\partial^2 z}{\partial x^2} + 2xy \frac{\partial^2 z}{\partial x \partial y} + y^2 \frac{\partial^2 z}{\partial y^2} = x^m y^n$$

by using $x = e^u$ and $y = e^v$. 6

UNIT—IV

4. (A) Prove the necessary condition for an extremum of a functional $I[y(x)] = \int_{x_0}^{x_1} F(x, y, y')\,dx$ where function F and its partial derivatives with respect to x, y and y' upto second order are continuous in the domain D of a plane is $F_y - \frac{d}{dx}F_{y'} = 0$. 6

(B) Find the extremal of the functional :

$I[y(x)] = \int_0^{\pi/2} (y'^2 - y^2)\,dx$ that satisfies the boundary conditions y(0) = 0, y(π/2) = 1. 6

OR

(C) Find the extremal of the functional :

$$I[y(x)] = \int_{x_0}^{x_1} [(y'')^2 - 2(y')^2 + y^2 - 2y \sin x]\, dx .$$ 6

(D) Find Euler-Ostrogradsky equation for the functional :

$$I[u(x, y, z)] = \iiint_D \left[\left(\frac{\partial u}{\partial x}\right)^2 + \left(\frac{\partial u}{\partial y}\right)^2 + \left(\frac{\partial u}{\partial z}\right)^2 + 2uf\right] dxdydz .$$ 6

5. (A) Form a partial differential equation by eliminating arbitrary constants from the equation : z = (x + a) (y + b). 1½

(B) Eliminate the arbitrary function f from the equation $z = xy + f(x^2 + y^2)$ and obtain a partial differential equation. 1½

(C) Find the general integral of the linear partial differential equation p + q = z. 1½

(D) Show that the equations f(x, y, p, q) = 0 and g(x, y, p, q) = 0 are compatible if $J_{xp} + J_{yq} = 0$. 1½

(E) Solve the partial differential equation $\frac{\partial^2 z}{\partial x \partial y} = 0$ by integration. 1½

(F) Find P.I. of $(D^2 - 2DD' + D'^2)z = \tan(x + y)$. 1½

(G) Find the distance of order zero between the functions

$y = xe^{-x}$ and y = 0 on [0, 2]. 1½

(H) Let $I[y(x)] = \int_0^1 [y(x)]^2\,dx$ be a functional. If $y(x) = e^x$, then find I[y(x)]. 1½