NRT/KS/19/2135

Bachelor of Science (B.Sc.) Semester—V Examination
MATHEMATICS
Optional Paper—1
(Analysis)

Time : Three Hours] [Maximum Marks : 60

N.B. :— (1) Solve all the **FIVE** questions.
(2) All questions carry equal marks.
(3) Question Nos. **1** to **4** have an alternative. Solve each question in full or its alternative in full.

UNIT—I

1. (A) Show that any function f(x) defined on symmetrically placed interval can be written as the sum of an even function and an odd function. Hence, show how to write $f(x) = x + x^2 + x^3$ as per above statement. 6

(B) Find the Fourier series for the function of period 2π defined by

$$f(x) = \begin{cases} x + \dfrac{\pi}{2} & , \quad -\pi < x \le 0 \\ \dfrac{\pi}{2} - x & , \quad 0 < x < \pi \end{cases}$$

and deduce that at x = 0,

$$\frac{\pi^2}{8} = \frac{1}{1^2} + \frac{1}{3^2} + \frac{1}{5^2} + \ldots\ldots$$ 6

OR

(C) Find the Fourier series for the function f(x) defined by $f(x) = |x|, -2 \le x \le 2$. 6

(D) Obtain the half range Fourier cosine series for the function f(x) = sin x in $0 \le x \le \pi$. Hence show that $\sum_{n=1}^{\infty} \frac{1}{4n^2 - 1} = \frac{1}{2}$. 6

UNIT—II

2. (A) Define Lower R-S integral and Upper R-S integral for the function f with respect to α on [a, b]. Also prove the relation $\underline{\int_a^b} f\, d\alpha \le \overline{\int_a^b} f\, d\alpha$. 6

(B) If f is continuous on [a, b] then prove that $f \in R(\alpha)$ on [a, b]. 6

OR

(C) Let $f \in R$ on [a, b]. For $a \le x \le b$, put $F(x) = \int_a^x f(f)\, dt$. If f is continuous at a point x_0 of [a, b], then prove that F is differentiable at x_0 and $F'(x_0) = f(x_0)$. 6

(D) If $f \in R(\alpha)$ on [a, b] and if a < c < b, then prove that $f \in R(\alpha)$ on [a, c] and $f \in R(\alpha)$ on [c, b]. Hence, show that

$$\int_a^c f\, d\alpha + \int_c^b f\, d\alpha = \int_a^b f\, d\alpha.$$ 6

UNIT—III

3. (A) If n is real, then show that $r^n[\cos n\theta + i \sin n\theta]$ is analytic except possible when r = 0 and that its derivative is $n r^{n-1}[\cos(n-1)\theta + i \sin(n-1)\theta]$. 6

(B) Show that $u = \frac{1}{2} \log(x^2 + y^2)$ is harmonic and determine its harmonic conjugate. 6

OR

(C) If $u = (x-1)^3 - 3xy^2 + 3y^2$, determine v so that u + iv is an analytic function of x + iy. 6

(D) If u and v are harmonic in a region R, then prove that $\left(\frac{\partial u}{\partial y} - \frac{\partial v}{\partial x}\right) + i\left(\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y}\right)$ is analytic in R. 6

UNIT—IV

4. (A) Let a rectangular domain R be bounded by lines x = 0, y = 0, x = 2, y = 3 in z-plane. Determine the region R' of w-plane into which R is mapped under the transformation $w = \sqrt{2}\, e^{\frac{i\pi}{4}} . z + (1 - 3i)$. 6

(B) Show that every general bilinear transformation can be considered as a combination of the transformations of translation, rotation, stretching and inversion. 6

OR

(C) Show that $w = \frac{5-4z}{4z-2}$ transforms the circle |z| = 1 into a circle of radius unity in w-plane and find the centre of the circle. 6

(D) Find the bilinear transformation which maps the points z = –2, 0, 2 into the points w = 0, i, –i respectively. 6

QUESTION—V

5. (A) Find the Fourier series coefficient b_n for the function f(x) = x, $-\pi \le x \le \pi$. 1½

(B) Obtain Fourier coefficient a_1 for the function

$f(x) = \cos \pi x, -1 \le x \le 1$. 1½

(C) If $f_1(x) \le f_2(x)$ on [a, b], then prove that

$$\int_a^b f_1\, d\alpha \le \int_a^b f_2\, d\alpha.$$ 1½

(D) State the fundamental theorem of calculus. 1½

(E) Prove that $u = y^3 - 3x^2y$ is a harmonic function. 1½

(F) Prove that an analytic function with constant real part is constant. 1½

(G) Show that w = iz + i maps half plane y > 0 onto half plane u < 0. 1½

(H) Find the fixed points of the bilinear transformation

$$w = \frac{z+1}{z-1}.$$ 1½