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NJR/KS/18/3135

Bachelor of Science (B.Sc.) Semester-V (C.B.S.) Examination

ANALYSIS

Paper—1

(Mathematics)

Time : Three Hours] [Maximum Marks : 60

N.B. :— (1) Solve all five questions.

(2) All questions carry equal marks.

(3) Questions 1 to 4 have an alternative. Solve each question in full or its alternative in full.

UNIT–I

1. (A) Find the Fourier Series for the 2π – periodic function

$f(x) = \pi - x$, on the interval $-\pi < x < \pi$. 6

(B) Find Fourier Cosine series of the function

$f(x) = \cos x$ in the half range $0 < x < \pi$. 6

OR

(C) Find the Fourier series of 2L – periodic function f(x) in the interval (–L, L) using the substitution

$t = \frac{\pi x}{L}$, where $-\pi \le t \le \pi$ and $-L \le x \le L$. 6

(D) Find the Fourier series expansion for the function defined by :

$$f(x) = 1 + x, \quad -1 \le x < 0$$
$$= 1 - x, \quad 0 \le x \le 1.$$ 6

UNIT–II

2. (A) Prove that $U(P^*, f, \alpha) \le U(P, f, \alpha)$, where P^* is a refinement of the partition P of [a, b], α is monotonically increasing function on [a, b] and f is bounded function on [a, b]. 6

(B) If f, g $\in R(\alpha)$ on [a, b], then prove that :

(i) $fg \in R(\alpha)$ and

(ii) $\left|\int_a^b f\, d\alpha\right| \le \int_a^b |f|\, d\alpha$. 6

OR

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(C) If $f \in R$ on [a, b] and if there is a differentiable function F on [a, b] such that $F'(x) = f(x)$, $x \in [a, b]$, then prove that

$$\int_a^x f(x)\,dx = F(b) - F(a).$$ 6

(D) If f is a continuous function on [a, b], then prove that $f \in R(\alpha)$ on [a, b]. 6

UNIT–III

3. (A) If f(z) is an analytic function of z, then prove that

$$\left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}\right)|f(z)|^2 = 4\,|f'(z)|^2 .$$ 6

(B) If $u = x^3 - 3xy^2$, then show that there exists a function $v(x, y)$ such that $w = u + iv$ is analytic in a finite region. 6

OR

(C) If $u = \dfrac{\sin 2x}{\cosh 2y + \cos 2x}$, then find the corresponding analytic function $f(z) = u + iv$. 6

(D) Show that, if $w = f(z) = u(x, y) + iv(x, y)$ is an analytic function at any point $z = x + iy$ of its domain D, then in polar form the Cauchy-Reimann equations are

$$\frac{\partial u}{\partial r} = \frac{1}{r}\frac{\partial v}{\partial \theta} \text{ and } \frac{\partial v}{\partial r} = -\frac{1}{r}\frac{\partial u}{\partial \theta}$$

Hence, find p if the function $f(z) = r^2 \cos 2\theta + ir^2 \sin p\theta$ is analytic. 6

UNIT–IV

4. (A) If there are two distinct invariant points p and q, then show that the bilinear transformation may be put in the form :

$$\frac{w - p}{w - q} = k\left\{\frac{z - p}{z - q}\right\}, \text{ where } k = \frac{a - cp}{a - cq}.$$ 6

(B) Find the bilinear transformation which maps the points $z_1 = 2$, $z_2 = i$, $z_3 = -2$ into the points $w_1 = 1$, $w_2 = i$, $w_3 = -1$. 6

OR

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(C) Show that the transformation $w = \dfrac{2z + 3}{z - 4}$ transforms the circle $x^2 + y^2 - 4x = 0$ into the straight line $4u + 3 = 0$. 6

(D) Prove that the cross ratio remains invariant under a bilinear transformation. 6

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QUESTION–V

5. (A) Obtain the Fourier series for the function $f(x) = 1$ on $(-\pi, \pi)$. 1½

(B) Find the Fourier coefficient a_n for the function $f(x) = |x|, -2 \le x \le 2$. 1½

(C) Prove that every constant function is Riemann Stiettjes integrable. 1½

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(D) Let $\alpha(x) = x, \forall x \in [a, b]$ be a monotonic increasing function, then find $\sum_{i=1}^{n} \Delta\alpha_i$. 1½

(E) Examine analyticity of the function $e^x (\cos y - i \sin y)$. 1½

(F) Find whether the function $u = x^3 - 3xy^2 + 3x^2 - 3y^2 + 1$. 1½

(G) Find fixed points of the bilinear transformation $w = \dfrac{3iz + 1}{z + i}$. 1½

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(H) Define conformal transformation. 1½

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