NRT/KS/19/2166

Bachelor of Science (B.Sc.) Semester—VI Examination

DISCRETE MATHEMATICS AND ELEMENTARY NUMBER THEORY

Optional Paper—2

(Mathematics)

Time : Three Hours] [Maximum Marks : 60

N.B. :— (1) Solve all the **FIVE** questions.

(2) All questions carry equal marks.

(3) Question Nos. **1** to **4** have an alternative. Solve each question in full or its alternative in full.

UNIT—I

1. (A) Given that X = {1, 2, 3, 4, 5, 6, 7, 8, 9, 10} and a relation on X is such that :

$R = \{(x, y)/x + y = 10; x, y \in X\}$.

Prove that R is neither reflexive nor transitive but symmetric. 6

(B) Let $(L, \leq)$ be a Lattice. For any a, b, c, $\in$ L, prove that :

$$b \leq c \Rightarrow \begin{cases} a * b \leq a * c \\ a \oplus b \leq a \oplus c \end{cases}$$ 6

OR

(C) In a bounded complemented distributive lattice $(L, *, \oplus)$, prove that :

$(a * b)' = a' \oplus b'$. 6

(D) Define the following graphs :

(i) Directed graph

(ii) Undirected graph

(iii) Mixed graph

(iv) Simple graph

(v) Multiple graph

(vi) Weighted graph. 6

UNIT—II

2. (A) If g is the g.c.d. of b and c, then prove that there exist integers x_0 and y_0 such that $g = bx_0 + cy_0$. 6

(B) Give that (a, 4) = 2 and (b, 4) = 2, prove that (a + b, 4) = 4. 6

OR

(C) Find all the solutions of :

$353x \equiv 254 \pmod{400}$. 6

(D) Find all integers that give the remainders 1, 2, 3 when divided by 3, 4, 5 respectively. 6

UNIT—III

3. (A) Evaluate the Legendre's symbol $\left(\frac{-35}{97}\right)$. 6

(B) Verify that $x^2 \equiv 10 \pmod{89}$ is solvable and solve it. 6

OR

(C) If p and q are distinct odd primes, then prove that :

$$\left(\frac{p}{q}\right)\cdot\left(\frac{q}{p}\right) = (-1)^{\frac{p-1}{2}\cdot\frac{q-1}{2}}$$ 6

(D) Prove that if p is an odd prime, then $x^2 \equiv 2 \pmod{p}$ has solutions if and only if $p \equiv 1$ or $7 \pmod{8}$. 6

UNIT—IV

4. (A) Find all integer solutions of $147x + 258y = 369$. 6

(B) Find all solutions in integers of $2x + 3y + 4z = 5$. 6

OR

(C) If u and v are relatively prime positive integers whose product uv is a perfect square, then prove that u and v are both perfect squares. 6

(D) Prove that :

(i) Every term in a Farey sequence is in reduced form.

(ii) The fractions in the sequence are listed in order of their size. 6

Question—V

5. (A) Find domain and range of the relation $R = \{(1, 2), (2, 3), (1, 3)\}$. 1½

(B) Define an anti-symmetric relation. 1½

(C) Prove that $(a^2, b^2) = c^2$, if $(a, b) = c$. 1½

(D) List all integers x in the range $1 \le x \le 100$ that satisfy $x \equiv 7 \pmod{17}$. 1½

(E) Find quadratic residues of 11. 1½

(F) Verify whether the congruence $x^2 \equiv 2 \pmod{61}$ is solvable. 1½

(G) Define Primitive Pythagorean Triplet with one example. 1½

(H) Show that $12x + 3y = 1$ is not solvable. 1½