SECTION-B

MATHEMATICS

1. In an entrance test, there are multiple choice questions. There are four possible answers to each question, of which one is correct. The probability that a student knows the answer to a question is 90%. If he gets the correct answer to a question, then the probability that he was guessing is

(a) $\frac{1}{40}$ (b) $\frac{1}{39}$ (c) $\frac{1}{37}$ (d) $\frac{2}{43}$

2. If $\pi/2 < x < \pi$, then $\int x\sqrt{\frac{1+\cos 2x}{2}}dx =$

(a) $\cos x + x\sin x + C$ (b) $-\cos x - x\sin x + C$
(c) $\sin x + x\cos x + C$ (d) $x\sin x - \cos x + C$

3. A rectangle with one side lying along the x-axis is to be inscribed in the closed region of the xy plane bounded by the lines $y = 0$, $y = 3x$ and $y = 30 - 2x$. The largest area of such a rectangle is

(a) 135/8 (b) 45 (c) 135/2 (d) 90

4. If $f : R \to R$ be a function defined by $f(x) = 4x^3 - 7$. Then

(a) f is one-one -into (b) f is many-one - into
(c) f is many-one onto (d) f is bijective

5. $\sim((\sim p) \wedge q)$ is equal to

(a) $p \vee (\sim q)$ (b) $p \vee q$
(c) $p \wedge (\sim q)$ (d) $\sim p \wedge \sim q$

6. With the usual notation $\int_1^2 \left([x^2] - [x]^2\right)dx$ is equal to

(a) $4 + \sqrt{2} - \sqrt{3}$ (b) $4 - \sqrt{2} + \sqrt{3}$
(c) $4 - \sqrt{2} - \sqrt{3}$ (d) none of these

7. The general solution of $x(1+y^2)^{1/2}dx + y(1+x^2)^{1/2}dy = 0$ is

(a) $\cos^{-1}x + \cos^{-1}y = C$
(b) $x^2 + y^2 = (1+x^2)^{1/2} + (1+y^2)^{1/2} + C$
(c) $(1+x^2)^{1/2} + (1+y^2)^{1/2} = C$
(d) $\tan^{-1}x - \tan^{-1}y = C$

8. If $A = \begin{bmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{bmatrix}$, then A^{2008} is equal to

(a) A (b) A^{-1} (c) I_3 (d) 0

9. Three vertices of a parallelogram ABCD are A (3, –1, 2), B (1, 2, –4) and C (–1, 1, 2). The coordinates of fourth vertex D are

(a) (1, 1, 1) (b) (1, –2, 8)
(c) (2, –2, 6) (d) (1, 0, 2)

10. The value of $\int \frac{\sin x + \cos x}{\sqrt{1 - \sin 2x}}dx$ is equal to

(a) $\sqrt{\sin 2x} + c$
(b) $\sqrt{\cos 2x} + c$
(c) $\pm(\sin x - \cos x) + c$
(d) $\pm\log(\sin x - \cos x) + c$

11. The equation of the plane containing the line $\frac{x+1}{-3} = \frac{y-3}{2} = \frac{z+2}{1}$ and the point (0, 7, – 7), is

(a) $x + y + z = 2$ (b) $x + y + z = 3$
(c) $x + y + z = 0$ (d) None of these

12. The co-ordinates of the foot of perpendicular from the point A (1, 1, 1) on the line joining the points B (1, 4, 6) and C (5, 4, 4) are

(a) (3, 4, 5) (b) (4, 5, 3)
(c) (3, –4, 5) (d) (–3, –4, 5)

13. $(p \wedge \sim q) \wedge (\sim p \wedge q)$ is

(a) A tautology
(b) A contradiction
(c) Both a tautology and a contradiction
(d) Neither a tautology nor a contradiction

14. Two finite sets have m and n elements. The total number of subsets of the first set is 56 more than the total number of subsets of the second set. Then :

(a) m = 3, n = 6 (b) m = 6, n = 3
(c) m = 5, n = 6 (d) None of these

15. Let f be the function defined by

$$f(x) = \begin{cases} \frac{x^2 - 1}{x^2 - 2|x-1| - 1}, & x \neq 1 \\ 1/2, & x = 1 \end{cases}$$

(a) The function is continuous for all values of x
(b) The function is continuous only for x > 1
(c) The function is continuous at x = 1
(d) The function is not continuous at x = 1

16. The distance of the point (1, – 2, 3) from the plane $x - y + z = 5$ measured parallel to the line $\frac{x}{2} = \frac{y}{3} = \frac{z-1}{-6}$ is

(a) 1 (b) 2
(c) 4 (d) None of these

17. $\int \frac{x+\sin x}{1+\cos x}dx$ is equal to :

(a) $x\tan\frac{x}{2}+C$ (b) $\cot\frac{x}{2}+C$

(c) $\log(1+\cos x)+C$ (d) $\log(x+\sin x)+C$

18. The maximum value of $z = 6x + 8y$ subject to constraints $2x + y \le 30$, $x + 2y \le 24$ and $x \ge 0$, $y \ge 0$ is

(a) 90 (b) 120 (c) 96 (d) 240

19. $\int_{\pi/3}^{\pi/2} x\sin(\pi[x]-x)\,dx$ is equal to :

(a) $\frac{1}{2}+\frac{\pi}{6}$ (b) $1-\frac{\sqrt{3}}{2}+\frac{\pi}{6}$

(c) $-\frac{1}{2}-\frac{\pi}{6}$ (d) $\frac{\sqrt{3}}{2}-1-\frac{\pi}{6}$

20. The general solution of the equation $\tan\theta+\tan 4\theta+\tan 7\theta=\tan\theta\tan 4\theta\tan 7\theta$

(a) $\theta=\frac{n\pi}{4}$ (b) $\theta=\frac{n\pi}{12}$

(c) $\theta=\frac{n\pi}{6}$ (d) None of these

21. For non zero, non collinear vectors $\vec{p}$ and $\vec{q}$, the value of $[\hat{i}\ \vec{p}\ \vec{q}]\hat{i}+[\hat{j}\ \vec{p}\ \vec{q}]\hat{j}+[\hat{k}\ \vec{p}\ \vec{q}]\hat{k}$ is

(a) $\vec{0}$ (b) $2(\vec{p}\times\vec{q})$

(c) $(\vec{q}\times\vec{p})$ (d) $(\vec{p}\times\vec{q})$

22. Let $f(\theta)=\sin\theta(\sin\theta+\sin 3\theta)$ then

(a) $f(\theta)\ge 0\ \forall\theta\in R$ (b) $f(\theta)\le 0\ \forall\theta\in R$

(c) $f(\theta)\ge 1\ \forall\theta\in R$ (d) $f(\theta)\le 1\ \forall\theta\in R$

23. The maximum value of $z = 5x + 2y$, subject to the constraints $x + y \le 7$, $x + 2y \le 10$, $x, y \ge 0$ is

(a) 10 (b) 26 (c) 35 (d) 70

24. The volume of the greatest cylinder which can be inscribed in a cone of height 30 cm and semi-vertical angle 30° is

(a) $4000\pi/3\ cm^3$ (b) $400\pi/3\ cm^3$

(c) $4000\pi/\sqrt{3}\ cm^3$ (d) None of these

25. The equation of the plane through the line $x+y+z+3=0=2x-y+3z+1$ and parallel to the line $\frac{x}{1}=\frac{y}{2}=\frac{z}{3}$, is

(a) $x-5y+3z=7$ (b) $x-5y+3z=-7$

(c) $x+5y+3z=7$ (d) $x+5y+3z=-7$

26. Let $\vec{a}$, $\vec{b}$ and $\vec{c}$ be non-coplanar unit vectors equally inclined to one another at an acute angle θ. Then $[\vec{a}\ \vec{b}\ \vec{c}]$ in terms of θ is equal to :

(a) $(1+\cos\theta)\sqrt{\cos 2\theta}$

(b) $(1+\cos\theta)\sqrt{1-2\cos 2\theta}$

(c) $(1-\cos\theta)\sqrt{1+2\cos\theta}$

(d) None of these

27. General solution of the equation $\sin 2x-\sin 4x+\sin 6x=0$ is

(a) $\frac{n\pi}{4}$ or $n\pi\pm\frac{\pi}{6}$ (b) $n\pi$ or $n\pi\pm\frac{\pi}{3}$

(c) $n\pi\pm\frac{\pi}{4}$ (d) $n\pi$ or $2n\pi\pm\frac{\pi}{4}$

28. A focus of an ellipse is at the origin. The directrix is the line $x = 4$ and the eccentricity is $\frac{1}{2}$. Then the length of the semi-major axis is

(a) $\frac{8}{3}$ (b) $\frac{2}{3}$ (c) $\frac{4}{3}$ (d) $\frac{5}{3}$

29. The locus of a point that is equidistant from the lines $x+y-2\sqrt{2}=0$ and $x+y-\sqrt{2}=0$ is

(a) $x+y-5\sqrt{2}=0$ (b) $x+y-3\sqrt{2}=0$

(c) $2x+2y-3\sqrt{2}=0$ (d) $2x+2y-5\sqrt{2}=0$

30. A ray of light coming from the point (1, 2) is reflected at a point A on the x-axis and then passes through the point (5, 3). The co-ordinates of the point A is

(a) $\left(\frac{13}{5}, 0\right)$ (b) $\left(\frac{5}{13}, 0\right)$

(c) $(-7, 0)$ (d) None of these

31. If $x \in \mathbf{R}-\{0\}$, then $\tan^{-1}\left(\frac{\sqrt{1+x^2}+\sqrt{1-x^2}}{\sqrt{1+x^2}-\sqrt{1-x^2}}\right)$

(a) $\frac{1}{2}\cos^{-1}(x^2)$ (b) $\frac{\pi}{2}+\frac{1}{2}\cos^{-1}(x^2)$

(c) $\frac{\pi}{4}+\frac{1}{2}\cos^{-1}(x^2)$ (d) None of these

32. If $y = x^{x^2}$, then $\frac{dy}{dx}$ is equal to

(a) $(2\ln x)$ (b) $(2\ln x+1)$

(c) $(\ln\ln x+1)x^{x^2}$ (d) None of these

33. In a triangle ABC, $\angle C = 90°$, then $\frac{a^2 - b^2}{a^2 + b^2}$ is equal to :
(a) $\sin(A + B)$ (b) $\sin(A - B)$
(c) $\cos(A + B)$ (d) $\sin\left(\frac{A-B}{2}\right)$

34. The internal angles of a convex polygon are in A.P. The smallest angle is 120° and the common difference is 5°. The number to sides of the polygon is
(a) 8 (b) 9 (c) 10 (d) 16

35. In a binomial distribution n = 5, P(X= 1) = 0.4096 and P (X = 2) = 0.2048, then the mean of the distribution is equal to
(a) 1 (b) 1.5 (c) 2 (d) 2.5

36. The equation of tangent to the curve $y = \sin^{-1}\frac{2x}{1+x^2}$ at $x = \sqrt{3}$ is
(a) $y = -\frac{1}{2}(x - \sqrt{3})$
(b) $y - \frac{\pi}{3} = -\frac{1}{2}(x - \sqrt{3})$
(c) $y + \frac{\pi}{3} = -\frac{1}{2}(x - \sqrt{3})$
(d) None of these

37. Let A, B be two events such that the probability of A is $\frac{3}{10}$ and conditional probability of A given B is $\frac{1}{2}$. The probability that exactly one of the events A or B happen equals
(a) $\frac{1}{2}$ (b) $\frac{1}{6}$ (c) $\frac{3}{10}$ (d) $\frac{7}{10}$

38. If the line passing through $P(1, 2)$ making an angle with the x-axis in the positive direction meets the pair of lines $x^2 + 4xy + y^2$ at A and B, then $PA \cdot PB =$
(a) 13/3 (b) 13/6 (c) 11/6 (d) 11/3

39. If the curves $ay + x^2 = 7$ and $x^3 = y$ cut orthogonally at (1, 1), then the value of a is.
(a) 5 (b) 6 (c) 7 (d) 8

40. The value of $\cos\left(2\cos^{-1}x + \sin^{-1}x\right)$ at $x = \frac{1}{5}$ is
(a) $-\frac{2\sqrt{6}}{5}$ (b) $-2\sqrt{6}$ (c) $-\frac{\sqrt{6}}{5}$ (d) None

41. Which of the following is logically equivalent to $\sim(\sim p \Rightarrow q)$
(a) $p \wedge q$ (b) $p \wedge \sim q$
(c) $\sim p \wedge q$ (d) $\sim p \wedge \sim q$

42. The area of the region bounded by the curves $y = |x - 2|, x = 1, x = 3$ and the x-axis is
(a) 4 (b) 2 (c) 3 (d) 1

43. If the slope of the tangent at (x, y) to a curve passing through $\left(1, \frac{\pi}{4}\right)$ is given by $\frac{y}{x} - \cos^2\left(\frac{y}{x}\right)$, then the equation of the curve is :
(a) $y = \tan^{-1}\log(e/x)$
(b) $y = e^{1+\cot(y/x)}$
(c) $y = x\tan^{-1}\log(e/x)$
(d) $y = e^{1+\tan(y/x)}$

44. A fair coin is tossed 99 times. If X is the number of times head occurs, P(X= r) is maximum when r is
(a) 49 or 50 (b) 50 or 51
(c) 51 (d) None of these

45. The fourth term of an A.P. is three times of the first term and the seventh term exceeds the twice of the third term by one, then the common difference of the progression is
(a) 2 (b) 3 (c) $\frac{3}{2}$ (d) –1

46. The eccentricity of the hyperbola $x^2 - 3y^2 = 2x + 8$ is
(a) $\frac{2}{3}$ (b) $\frac{1}{3}$ (c) $\frac{2}{\sqrt{3}}$ (d) $\frac{3}{2}$

47. The differential equation representing the family of curves $y^2 = 2c\left(x + \sqrt{c}\right)$, where c is a positive parameter, is of
(a) order 3 (b) order 2
(c) degree 3 (d) degree 4

48. If $f(x) = \frac{1}{1-x}$, the number of points of discontinuity of $f\{f[f(x)]\}$ is :
(a) 2 (b) 1 (c) 0 (d) infinite

49. If $x = a(\cos t + t\sin t)$ and $y = a(\sin t - t\cos t)$, then $\frac{d^2y}{dx^2}$ is
(a) $\sec^3 t$ (b) $at\sec^3 t$
(c) $\frac{\sec^3 t}{at}$ (d) $\sec^2 t$

50. The number of solutions of equation $x_2 - x_3 = 1, -x_1 + 2x_3 = 2, x_1 - 2x_2 = 3$ is
(a) zero (b) one (c) two (d) infinite